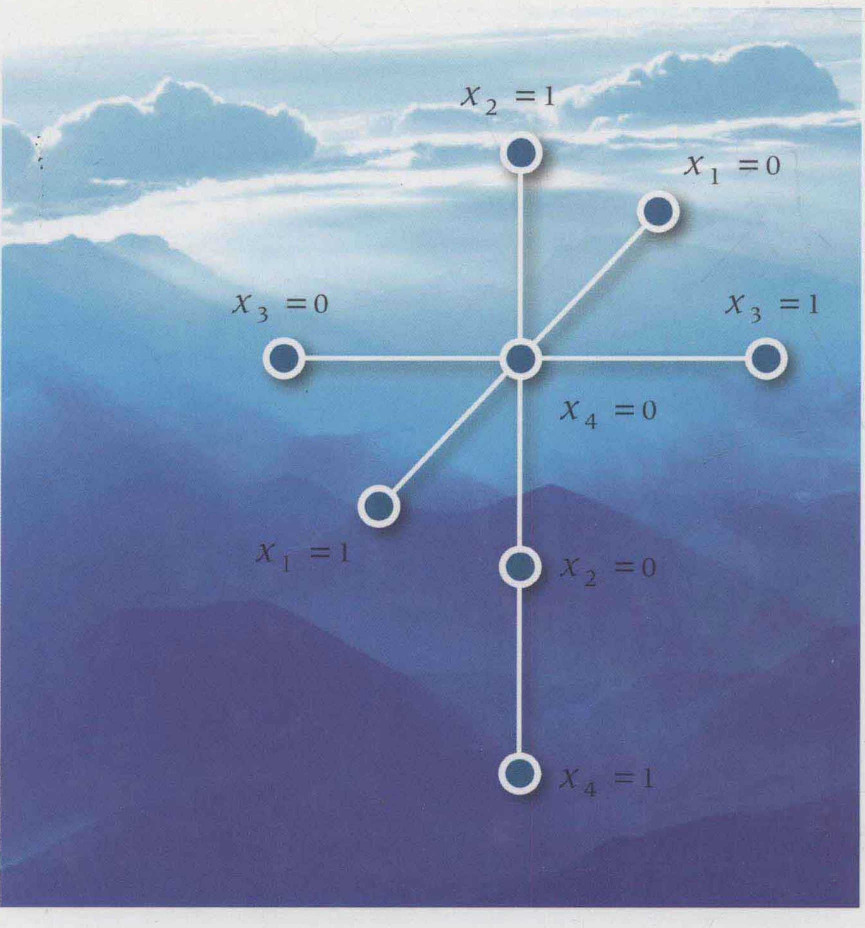
PHYSICS TEXTBOOK

Larry Silverberg

WILEY-VCH

Unified Field Theory for the Engineer and the Applied Scientist



Larry Silverberg

Unified Field Theory

For the Engineer and the Applied Scientist

WILEY-VCH

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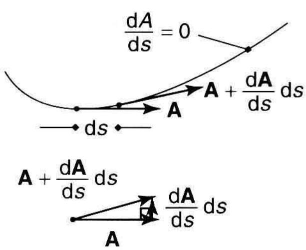
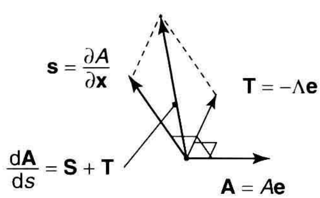
Unified Field Theory (circa 1950)

All the measurements are in a 4D coordinate system.
$A(x_1, x_2, x_3, x_4)$ = energy density field
$\mathbf{e}$ = unit vector in the observed direction ($dA/ds = 0$)

$\mathbf{A} = A\mathbf{e}$ = energy density field vector, ($d\mathbf{A}/ds$) = force density
$\mathbf{S}$ = mechanical force density, $\mathbf{T}$ = electrical force density
$\Lambda = 4 \times 4$ matrix of curls of $\mathbf{A}$





Governing equations for the energy density field

Scalar continuity: $$0 = \frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} + \frac{\partial A_3}{\partial x_3} + \frac{\partial A_4}{\partial x_4} \qquad (0 = \Box \cdot \mathbf{A})$$

Vector continuity: $$0 = \frac{\partial^2 \mathbf{A}}{\partial x_1^2} + \frac{\partial^2 \mathbf{A}}{\partial x_2^2} + \frac{\partial^2 \mathbf{A}}{\partial x_3^2} + \frac{\partial^2 \mathbf{A}}{\partial x_4^2} \qquad (\mathbf{0} = \Box^2 \mathbf{A}).$$

Larry Silverberg

Unified Field Theory

Preface

The unified field theory is a unified development of macrophysics. The development is based on four-dimensional geometry. Historically, the study of the mathematical structure of four-dimensional geometry led to the study of mathematical structures for microphysical phenomena, to attempts at grand unification (unification of the micro, macro, and celestial scales) and to the development of higher-dimensional mathematical structures not necessarily applied to physical phenomena. Research that was once a major interest of the theoretical physicist passed to the differential geometer, who has since built a strong mathematical foundation in hyper-dimensional field theories.

Today, a growing number of scientific investigations deal with intricate interdisciplinary problems at the macroscale. Consequently, the unified field theory has become relevant to the engineer and the applied scientist, not just to the differential geometer and other specialists. The purpose of this monograph was to translate the unified field theory into a language suitable for the broader engineering and scientific community.

Translating the unified field theory into the language of the scientist involved limiting the level of mathematics to calculus and vector algebra, and removing reliance on mathematical theorems and concepts that the scientist would not be aware of or that would require too much time to learn. It is also helpful to appreciate that the development is not a uniquely ordered set of theorems. There are a variety of ways to arrive at the results. Hence, translating the theory into the scientist's language involved redoing mathematical developments. Finally, it should be recognized that the scientist's demands are somewhat different from the differential geometer's demands. The scientist does not tend to be satisfied with a rigorous axiomatic development, instead, looks for the development to be intuitive as well as rigorous. For example, the differential geometer is likely satisfied with a development that begins with a 4D geometry in which 4D vectors satisfy a standard L_2 norm. This may not mean anything to the scientist. In the scientist's language, the quantities in the 4D space satisfy the Pythagorean theorem. But the scientist also asks, when introducing the fourth coordinate (time), why the Pythagorean Theorem would still apply. The scientist seeks motivating explanations that are grounded in intuitive ideas

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related to the conventions that have been established. It is not enough that a mathematical structure works.

Chapter 1 describes the 4D space. The focus is on the conventions that are accepted in geometry and the rationale used to extend 3D geometry to 4D geometry. Chapter 2 develops the theory of relativity from 4D geometry. Special attention is given to the relationship between geometric time, conventional time, and time measurement. Chapter 3 introduces the 4D field concept. The principle field considered is the energy density field. The energy density field is the field that unifies relativity, electrodynamics, and mechanics. Chapter 4 broadens our look at fields. It focuses on studying the basic ways that the energy density field can change. Chapter 5 develops the equations that govern the energy density field vector. It is explained why prediction can be regarded as an extrapolation process. Chapter 6 shows that the energy field density vector is essentially a 4D wave. The rectangular wave and the spherical wave are considered in some detail. Chapter 7 develops the field particle. The field particle, as described in this chapter, doubles as a wave. It is used as a building block for constructing the energy density field vector. Chapter 8 applies the unified field theory to electrodynamics. This chapter derives Maxwell's equations for free space, the concept of a medium, and the Lorentz force. Chapter 9 applies the unified field theory to mechanics. The principles of particle interaction are developed. Newton's second and third laws are derived, gravitation is developed, and then the concept of relativistic mass is discussed. Finally, the monograph ends with an essay about the evolution of science, the purpose being to place the unified field theory in the context of the development of science.

Raleigh, October 2008 *Larry Silverberg*

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1
4D Space

"A powerful theory is simple."

A powerful theory is simple. The simpler, the more powerful it becomes. The unified field theory starts with conventions that are accepted about the simplest and most basic aspects of observation. The space within which observations are made is just about as basic as you get. This chapter scrutinizes the convention of 4D space and develops some mathematical tools.

1.1
Convention

A point in ordinary 3D space is located by three independent coordinates. The distance between any two points is determined by the Pythagorean theorem. In an ordinary 4D space the added dimension is geometric time and the Pythagorean theorem is made to work in four dimensions.

Actually, there is the possibility of adopting any number of different geometries. For example, imagine a planar geometry that lies on the surface of a sphere. In this geometry, straight lines are arcs produced from intersections between the surface of the sphere and flat planes that cut through the center of the sphere. This particular geometry is called a *2D spherical geometry*. And there are others, too, although there is no need here to delve into them. Suffice it to point out that the geometry that is adopted depends on the problem being studied. It is really your decision as to the geometry in which to imagine reality. In other words, no single geometry is correct. On the other hand, say that you want to establish a "universal" geometry, in other words, a geometry that everyone prefers. Then, you would have to go out and persuade everyone to accept it. It is not the goal in this monograph to claim that the ordinary 4D geometry being used is universal. It is simply the one that was selected. It was selected by and large because it is easy and familiar.

Let us now begin the development. Imagine that you are at the beach watching a sun set. You would probably be willing to agree that at any given

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instant, the sun is located somewhere. This, of course, cannot be proven. The sun could be a hallucination or some other kind of figment of the imagination. Unless you accept certain rules about the things you observe, and unless the rest of us agree to accept these rules, it would be hopeless to proceed further. It follows that a community of like-minded people are required to agree that the things seen are located in the three dimensions of space and in the dimension of time. Indeed, the practice of interpreting everything as being located in the four dimensions of space and time is arguably the most basic convention established for the backdrop of reality. For now, suffice it to accept that space has three dimensions and time is another dimension. In this chapter the three dimensions of space and the dimension of time will be brought together to produce the 4D space.

The establishment of conventions is an evolutionary process that dates back to before recorded history. Today, only the results are being practiced. But at some point, conventions were established for a unit measurement of length and for a unit measurement of time. To establish these conventions, devices were manufactured to count multiples and fractions of units. The ruler and the clock were built to measure length and time. The spatial measurement is taken by placing the ruler up against a body and comparing a pair of coincident events on the body and the ruler. Similarly, the temporal measurement is taken by comparing a pair of coincident events. The ruler and the clock provide physical standards for measurement.[1)]

The idea of dimension is more recent, dating back some 500 years when an organized method was developed to coordinate or analyze measurements. The coordination method starts with the construction of a coordinate system. The coordinate system is a reference relative to which measurements are taken. The reference defines the starting point of a set of measurements and the directions along which they are taken. The most common type of coordinate system is called the *Cartesian coordinate system*, named after Renee Descartes [2]. Using a Cartesian coordinate system, measurements are taken in perpendicular directions.

1.2
Cartesian Coordinates

The location of the sun, or of any other event, is determined by four numbers. Three of them are referred to as the spatial coordinates x_1, x_2, and x_3 and one is the temporal coordinate x_4. The four numbers can be viewed as measurements along the axes of a four-dimensional coordinate system. Each

1) It is fascinating, although beyond the scope of this work, that these conventions produce, quite arbitrarily, spatial and temporal constants. The ruler takes the unit measurement of distance to be the same everywhere in space and the clock counts periodic events that are taken to have the same period throughout time. We are so accustomed to these constants that they appear to be natural even though they are really artificially constructed. In mathematics, this is called *congruence* [1].

axis is a dimension of 4D space. The four numbers collected together form the vector $\mathbf{x} = (x_1\ x_2\ x_3\ x_4)$. Whenever a measurement is taken along a direction that is not along one of the coordinate axes, a rule for the length of a measurement is needed. Drawing from the Pythagorean theorem, the *length* of the vector **x** is (see Figure 1.1)

$$\mathbf{x} = \sqrt{x_1^2 + x_2^2 + x_3^2 + x_4^2}. \tag{1.1}$$

Equation (1.1) is developed in detail in Figure 1.1. Figure 1.1a shows a proof of the Pythagorean theorem. Figure 1.1b extends the definition of length to 3D space. Figure 1.1c extends the definition of length to 4D space. Notice that length is extended from 2D space to 3D space the same way it is extended from 3D space to 4D space. A 4D geometry that defines length according to Equation (1.1) is referred to as an *ordinary geometry*.[2)]

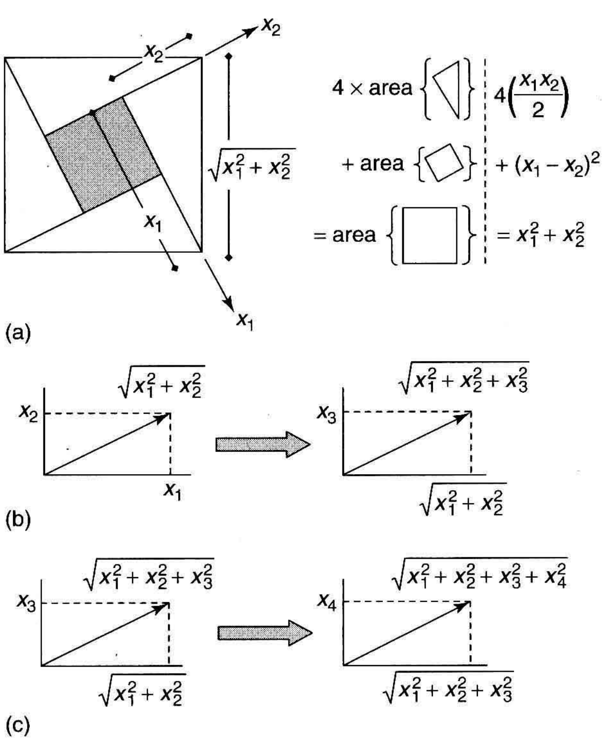

Figure 1.1 Length. (a) Proof of the Pythagorean theorem, (b) extension from 2D space to 3D space, and (c) extension from 3D space to 4D space.

2) A 4D geometry in which length is defined according to Equation (1.1) is also referred to as a 4D *Euclidean* geometry. Euclidean geometry is based on the works of Euclid [3]. Basic theorems in 4D geometry date back to the Greeks [4].

The proof of the Pythagorean theorem for 4D geometry treated the spatial coordinates and the temporal coordinate in the same manner, as though there is no real difference between them. But does that make sense? What assumptions did we make? To answer these questions, look at this proof more closely. Notice, to prove the Pythagorean theorem, that the area of a rectangle was first defined as the product of its sides. A notion of area was considered necessary to define length.

Area is basically defined as the number of unit squares in a rectangle. The Pythagorean theorem holds in a geometry in which area is defined this way. The reader should appreciate that there are other senses of length that are meaningful, too, although these senses of length would either not define area this way, or not define it at all. For example, imagine constructing a space from a grid of lines (see Figure 1.2). The distances between the lines can be infinitesimal or finite. It is fun to think of the lines as roads and the space as a city block. When traveling from **A** to **B** the length of travel is $a + b$, not $\sqrt{a^2 + b^2}$. This is another acceptable way to measure length even though it is not being adopted here.

Returning to the problem at hand, the question remains why must area be defined as the product of base and height when one of the coordinates is spatial and the other is temporal? In fact, is it even necessary to define area when one of the coordinates is temporal? And if so, how would we know whether this fourth coordinate is conventional time? The answer is that area would not necessarily be defined this way and that this definition of length does not necessarily make sense when one of the coordinates is temporal. Indeed, one should not view the temporal coordinate x_4 and conventional time t as identical. In the development below, the temporal coordinate x_4 is called *geometric time* and t is called *conventional time*. The temporal coordinate will be taken to be a fourth coordinate in an ordinary geometry, that is, a geometry that satisfies the Pythagorean theorem. However, the justification is predicated on the existence of a relationship between geometric time x_4

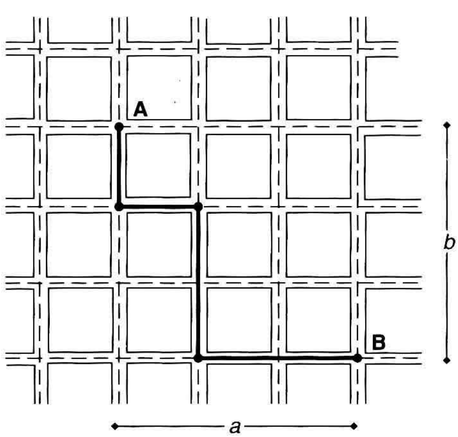

Figure 1.2 Road length.

and conventional time t, which will be developed shortly. From the outset, it should be understood that the immediate goal is to develop, as a convention, an ordinary 4D geometry with x_4 as the temporal coordinate. The basic question is whether there exists a relationship between x_4 and t and what it is.[3)]

1.3 Time as a Fourth Dimension

To develop the relationship between geometric time x_4 and conventional time t, we need to think more about how time is measured. Do we really know when an event actually occurs? Do we measure it directly and, if not, what do we actually measure? The development below is divided into three steps. In the first step, the relationship between conventional time and the images that we see is discussed. This step clarifies the difference between the time of a measurement and the time of an event. In the second step, the concept of the complex number is reviewed. The complex number is reviewed because of its central role and because there is a lot of confusion surrounding it. The aim of the review is also, in part, to trace the complex number back to its origin, so that this part of the development can be as intuitive as the rest. Finally, in the third step, the relationship between x_4 and t is exposed.

1.3.1 Images

Imagine that it is a clear night and you gaze up and see thousands of stars. The stars appear to be on the surface of a sphere, equidistant from you. In fact, this is precisely the sensation replicated in a planetarium, where stars are projected onto a spherical surface. But do the stars in the sky really lie on a spherical surface? The spatial image that you see is really of events that occurred at very different times, in some cases thousands of years apart, at very different distances.

Now, look down at your hand. The spatial image that you construct of it is formed from events that occurred almost simultaneously. It follows that depth perception and the images of the bodies that are formed in our minds are largely a result of relative spatial information. Spatial images of bodies are perceived by following changes in patterns. Without these patterns, our minds place the spatial images at equal distances to us to form images on a sphere.

When an event occurs, it takes time for the signal of the event to be communicated to your eye or to a clock. Time is counted in terms of the signal

3) The relationship between x_4 and t was the principle question answered in the theory of relativity. It was first studied soon after light was recognized to be a wave, upon which it was realized that the time during which an event is measured corresponds to the instant the signal reaches the clock and not when the event actually occurs [5]. The measured time was called *retarded* time. The theory of relativity provides a means for accounting for retarded time.

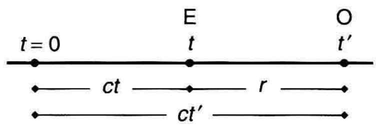

Figure 1.3 The communication line.

that reaches you. In fact, you do not know precisely where the signal comes from or the precise instant it occurs because events at different distances and at different instances can reach you at the same time. This is illustrated by the *communication line* shown in Figure 1.3.

The origin of the communication line is at time $t = 0$. An event E occurs at time t and observer O records the measurement at time t'. The spatial distance between event E at time t and the measurement by O at time t' is r. Time t' and time t are related by

$$ct' = r + ct, \tag{1.2}$$

where c is the speed of the signal. The observer only records the event at t'; the distance r, the time t of the event, and the speed c of the signal are not measured directly.

1.3.2 Complex Numbers

The communication line shown in Figure 1.3 is a primitive system showing time t and distance r on the same axis. This representation will be modified with the help of complex numbers. Using complex numbers, a geometric relationship between the time t of the event E and the distance r between the event and the measurement will be developed. The geometric relationship will produce a fourth dimension. Before proceeding with that, though, it will be instructive to review complex numbers.

A tremendous amount of confusion surrounds the complex number. After all, what is $\mathrm{i} = \sqrt{-1}$? The confusion that surrounds the complex number is traced to the way it became popular; it is a matter of history. However, there is nothing abstract about it as the following explains. Let us first recall that, before the coordinate system had become popular, geometry's principle role was to deal with shapes constructed from intersecting line segments, surfaces, and volumes. The line segments, surfaces, and volumes were represented by positive numbers and zero. Negative numbers are not necessary for the construction of shapes. In fact, not surprisingly, the earliest treatments with negative numbers and with complex numbers were met with skepticism.

As geometry evolved, directed line segments, called *rays*, were introduced to help with the analysis performed in geometry. The ray was introduced during the same period as the coordinate system because both kept track of direction.