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QUANTUM GRAVITY 2
A Second Oxford Symposium

Edited by C. J. Isham,
R. Penrose, and
D. W. Sciama

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A SECOND OXFORD SYMPOSIUM

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C. J. ISHAM
R. PENROSE
AND
D. W. SCIAMA

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PREFACE

The union of quantum theory with Einstein's general theory of relativity remains one of the major unattained goals of present-day theoretical physics. In February 1974, our first Oxford Symposium on Quantum Gravity was held, and we took stock of the ideas that were current at that time, publishing an account of them in our first Quantum Gravity volume.* In April 1980 a second conference on quantum gravity was held in Oxford, this time under the generous sponsorship of the Nuffield Foundation. Six years had passed since our first meeting and we sought to review and analyse the developments that had occurred in the intervening years. The second set of lectures is published in the present volume. Some of these provide review articles covering general or specific areas of interest whilst others are of a more technical nature, reflecting the increasing complexity and sophistication of current mathematical techniques.

Significant advances have been made in several branches of theoretical physics during the past decade. In particular, and of considerable relevance to quantum gravity, has been a renaissance in quantum field theory following the discovery that Yang–Mills theory is renormalizable if the masses of the particles are introduced via a spontaneous symmetry-breaking mechanism. The Salam–Ward, Weinberg, and Glashow theories of weak and electromagnetic interactions fall into this category and provide an important unification of two of the fundamental forces of nature. This has raised again the hope that all the basic interactions may be incorporated in one grand unified theory. The concept of unification has now permeated many approaches to quantum gravity, in contrast to much of the older work where attempts to quantize the gravitational field were usually made in isolation.

Perhaps the most striking example of this attitude is the theory of supergravity – a subject which has entirely arisen since the first Oxford meeting. This theory necessarily involves

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Quantum gravity, an Oxford symposium (ed. C.J. Isham, R. Penrose, and D.W. Sciama). Oxford University Press, Oxford (1975).

fields other than the gravitational. Much effort is now being expended in an attempt to show that its most sophisticated version (n = 8 supergravity) can be employed as a genuine grand unified theory. A hopeful feature of supergravity is the disappearance of some of the infinities which have plagued conventional quantum field theory, by mutual cancellations between the various fields. The existence of infinitely many uncontrollable infinities had been a seemingly fatal flaw in conventional approaches to quantizing the gravitational field, so the hope that supergravity may resolve this problem is one of its main attractions.

Another significant modern development which has arisen out of the quantization of Yang–Mills theory is the study of non-perturbative phenomena. Many workers in quantum gravity have long held that the perturbative approach commonly employed – the expansion of the metric tensor around Minkowski space – is quite inappropriate in a theory that is as intrinsically non-linear as general relativity. Consequently the non-perturbative methods of Yang–Mills theory have been adapted to the gravitational case, providing one of the motivations for the study of gravitational instantons.

This has given a new impetus to the investigation of the role played by topology. The topological structure of the solutions of non-linear equations has received much attention of late. In the case of general relativity, this pertains to the topology of space-time itself. In the early formative years of quantum gravity John Wheeler consistently emphasized the significance of space-time topology. But it is only relatively recently that physicists have become aware of the mathematical tools that are necessary to implement these ideas; and many groups now actively study this subject from a wide variety of viewpoints.

At our 1974 meeting, we were greatly privileged to witness the announcement of Stephen Hawking's important discovery that, via quantum mechanical processes, black holes radiate particles with a thermal distribution. This has led, in the intervening years, to an intense study of quantum field theory in fixed background space-times, raising a number of fundamental conceptual and technical problems. Much physical insight has been gained, particularly in the close relation between the Hawking effect and the thermal state observed by a detector moving with a constant acceleration through a quantum vacuum. Interesting developments in the mathematical theory have also occurred, leading to the study of thermal Green's functions in space-times with a complex periodicity, particularly in relation to gravitational instantons, where the above topological ideas play a key role. The Hawking effect inevitably relates also to quantum gravity studies of the very early universe, and it has a number of astrophysical implications and interrelations with particle physics. These developments are in many ways exciting; yet mysteries remain, and some of the deeper issues are still unresolved, such as those which relate the Hawking effect to time-asymmetry questions in physics.

While it is clear that many of the techniques of 1980 are radically different from those of 1974, the prime objective remains the same – to attempt to obtain a unification of general relativity and quantum theory, two of the great intellectual achievements of this century. There is still the mystery of the Planck length, $\left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}} \sim 10^{-33}$ cm, at which the structure of space-time and quantum effects would become inextricably intertwined. The feeling that, at this distance, some profound change in our conceptual understanding of the physical world would be necessary, motivates and inspires much of the research into quantum gravity. But it may be that some completely new approach to the subject is required, our attempts to date representing primitive gropings in some not quite correct directions. Perhaps some modification of the ideas of quantum mechanics will of necessity accompany any eventual successful union of quantum theory with general relativity. Maybe such a union could lead to a resolution of some of the inherent interpretative difficulties of present-day quantum mechanics. Perhaps, instead (or as well), the ideas of space-time structure need overhauling, one possible approach to such questions being provided by the twistor programme. We do not know where the future will lead us. Yet we feel that much of the work to date has been exciting, and that some of it has yielded significant insights that are here to stay.

We hope very much that by publishing this volume we shall assist young researchers who wish to enter the field and take up the challenge of tackling one of the greatest problems of modern theoretical physics.

C.J. Isham
R. Penrose
D.W. Sciama

CONTRIBUTORS AND ADDRESSES

C.J. Isham		*The Blackett Laboratory,* *Imperial College,* *London SW7 2BZ*
T.W.B. Kibble		*The Blackett Laboratory,* *Imperial College,* *London SW7 2BZ*
M.J.Duff		*Physics Department,* *Imperial College,* *London SW7 2BZ*
G.T. Horowitz		*Institute of Advanced Study,* *Princeton.*
D. Deutsch		*Department of Astrophysics,* *South Parks Road,* *Oxford*
S.J. Avis		*Department of Applied Mathematics and Theoretical Physics,* *Silver Street,* *Cambridge*
N.D. Birrell		*Logica Limited,* *64 Newman Street,* *London W1A 4SE*
	formerly	*Department of Mathematics,* *King's College,* *Strand, London WC2R 2LS*
P.C.W. Davies		*Department of Theoretical Physics,* *University of Newcastle-upon-Tyne*
D.W. Sciama		*Department of Astrophysics,* *South Parks Road,* *Oxford*
	and	*Center for Relativity,* *University of Texas,* *Austin, Texas 78712*
R.M. Wald		*Enrico Fermi Institute,* *University of Chicago,* *Chicago, Illinois 60637*
P. Penrose		*Mathematical Institute,* *University of Oxford,* *24-29 St. Giles,* *Oxford.*

M.J. Rees — *Institute of Astronomy, University of Cambridge, The Observatories, Madingley Road, Cambridge*

G.G. Ross — *Theoretical Physics Department, University of Oxford, 1 Keble Road, Oxford*

J.B. Hartle — *Department of Physics, University of California, Santa Barbara, CA 93106*

K. Kuchař — *Department of Physics, University of Utah, Salt Lake City, Utah 8412*

C.N. Pope — *Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge*

A. Ashtekar — *Physics Department, Syracuse University, Syracuse, N.Y. 13210*

M.R. Brown — *Department of Astrophysics, University of Oxford, South Parks Road, Oxford*

B.S. DeWitt — *Department of Physics, University of Texas, Austin, Texas 78712*

P. van Nieuwenhuizen — *Institute for Theoretical Physics, State of University of New York at Stony Brook, Long Island, N.Y. 11794*

S. Ferrara — *CERN, Geneva 23, 1211 Switzerland*
and *Laboratori Nazionali di Frascati, INFN, Frascati, Italy*

K.S. Stelle — *CERN, Geneva 23, 1211 Switzerland*

P.C. West — *Department of Mathematics, King's College, Strand, London WC2R 2LS*

R.S. Ward — *Department of Mathematics,*
Trinity College,
Dublin

J.S. Bell — *CERN,*
Geneva 23,
1211 Switzerland

B. Mielnik — *Institute of Theoretical Physics,*
Warsaw University,
Hoza 69, Warszawa, Poland

R.C. Jennison — *Electronics Laboratories,*
The University,
Canterbury, Kent CT2 7NT

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QUANTUM GRAVITY—AN OVERVIEW

C.J. Isham

Blackett Laboratory, Imperial College, London SW7

1. INTRODUCTION

Six years have passed since the last Oxford Symposium on Quantum Gravity. During this time there have been radical changes in attitudes and methodology. Many of the theoretical physicists who have traditionally regarded quantum gravity as esoteric and rather pointless will now grudgingly admit that not only is it technically fascinating, but it may even impinge directly on their own, superficially remote, specialities. This significant change arises mainly from the way in which large portions of particle-physics based quantum field theory have been enthusiastically adapted to the gravitational situation. Somewhat unexpectedly, this has not been at the expense of the geometric or topological aspects of the classical theory of general relativity, but instead reflects the absorption of modern mathematical ideas into conventional quantum field theory.

The advances in theoretical physics that have been so instrumental in influencing research in quantum gravity are as follows:

1. There has been a general rapid evolution in quantum field theory following t'Hooft's discovery in the early 1970s that the Yang–Mills Higgs–Kibble theory could be successfully renormalized. This led to a dramatic shift in attitude of the particle physics community and the sterile S-matrix dominated 1960s gave way to the swinging second quantized 1970s to such an extent that the great majority of theoretical particle physicists now work in quantum field theory.

Of particular interest has been the emphasis on non-perturbative methods, as reflected in the studies of solitons, monopoles, and instantons. A striking feature is the way in which simple topological ideas have played an important role and indeed these days a basic knowledge of algebraic topology is almost indispensible. These developments have been avidly followed by workers in quantum gravity and the relevant techniques rapidly absorbed.

2. Related to this renaissance has been the evolution of unified field theories. There is the successful Salam/Weinberg unification of weak and electromagnetic interactions and much effort is being expended in the development of a grand unified theory (GUT) incorporating strong interactions. Indeed quantum chromodynamics (the Yang–Mills based theory of strong interactions) has been intensively investigated by itself and is responsible in part for the desire to develop non-perturbative methods. It is natural to speculate that gravity can be added to this list of forces to give a true unification of subatomic physics. It is notable that the current GUT mass scale is set at 10^{14}–10^{15} GeV; which is not that far removed from the 10^{18} GeV energy that characterises real quantum gravity effects (see below).

3. A totally new subject that has evolved since 1974 is supergravity. This arises naturally as the local gauge version of bose–fermion supersymmetry and possesses considerable attractions. For example many of the divergences that plague conventional quantum field theory mutually cancel, bose v. fermion, in a supersymmetric model. Also supergravity, when extended by a type of internal symmetry group, may lead in a natural way to a rigidly prescribed unified theory of gravitation and other forces.

4. Finally, but of the utmost importance, Hawking's announcement in 1974, of the quantum mechanically induced thermal radiation of black holes, led to an explosion of interest in quantum theory in various types of gravitational background and heralded the dawn of a new and exciting era in quantum gravity research.

In spite of this extensive and sometimes frantic activity, I can confidently state that there is as yet no viable quantum theory of gravity. Attitudes towards this regrettable situation vary widely. One school maintains that it is just a matter of time and that the problem will be cracked within the framework of 'conventional' quantum field theory, albeit perhaps with the aid of technical tools that are as yet undiscovered.

However a long held contrary belief is that general relativity and quantum theory are intrinsically incompatible and that, rather than merely developing technique, what is required is some fundamental breakthrough in our understanding of the relationships between space-time structure and quantum processes. Adherents of the former opinion tend to regard the second attitude as being rather defeatist and unproductive. However it should be admitted that, although the potentialities are still considerable, all conventional, or semiconventional methods *have* failed to work and it does no harm occasionally to reconsider precisely what is meant by quantizing gravity (Section 2).

Like many other branches of theoretical physics, quantum gravity is subject to fads and fancies and it would be a mistake to assume that all the earlier work is irrelevant or that the trends of the moment will continue into the future. A new student of the subject would be well advised to study some of the older ideas, as discussed for example in the various review articles contained in the proceedings of the first Oxford quantum gravity conference (Isham, Penrose, and Sciama 1975). Other useful references are Ashtekar and Geroch (1974); B.S. DeWitt (1970); Bergmann and Komar (1962); J.A. Wheeler (1963, 1968).

2. WHAT IS A 'QUANTUM THEORY OF GRAVITY'?

The question 'what exactly do you expect a quantum theory of gravity to look like?' if posed to a selection of physicists is likely to result in quite a spread of replies. Subdividing the question into

(a) What do you expect from such a theory? What will it do for us?

(b) How will it affect the rest of physics?

(c) What sort of technical apparatus should be used?

will not noticeably narrow the range of opinions. Indeed the past and present, sometimes heated, disagreements on methodology can often be traced to pronounced differences in the expectations of the protagonists, concerning precisely what in principle can be extracted from the theory they are trying so hard to construct.

There are however certain *a priori* questions which everyone will agree are worth asking. For example, in a quantum theory of gravity:

1. How much of the technical and conceptual structure of classical general relativity do we expect to retain? In particular, one thinks of the underlying smooth C^∞ manifold, the metric tensor, the local field equations $G_{\mu\nu}-\Lambda g_{\mu\nu} = T_{\mu\nu}$, the global topological properties of the manifold and global metric features such as lightcone structure and the existence of event horizons. It is very difficult to judge how many of these classical concepts should be present, in some form or another, in a quantized theory.

2. How much of the technical and conceptual structure of conventional quantum field theory do we expect to retain? For example, does the usual idea of a local quantum field $\hat{\phi}(x)$ make any sense at all or should we decide from the outset that, at a Planck scale, space-time structure is not that of a smooth manifold and therefore the local properties of fields become very unconventional? Similarly, what remains of the local commutativity of quantum fields in a theory where the lightcone is determined by the metric tensor, which is itself a dynamical variable? Perhaps the most crucial question of all concerns the role of perturbation theory. The notion of quanta (gravitons in our case) arises basically as a property of the Fock space quantization of a free field propagating in Minkowski space. It reappears in a conventional interacting theory through the Feynman—Dyson expansion of the Green's functions and S-matrix elements and the associated idea of the completeness of the asymptotic scattering states. This notion is not obviously appropriate in quantum gravity and indeed conventional perturbative methods of this type lead to a non-renormalizable theory. Ideally one wants to get away from perturbing in the Newtonian coupling constant since, even classically, this is known to be a dubious procedure. The situation is somewhat reminiscent of that in QCD, where the main struggle is to show why quarks and gluons do *not* appear as quanta, i.e. as asymptotic particles. It is perhaps attitudes to perturbation theory and to the highly nonlinear nature of Einstein's equation that most sharply distinguish the different schools of thought in quantum gravity.

Aside from these technical considerations, it has frequently been maintained that the conceptual structure of the Copenhagen interpretation of quantum mechanics is quite inappropriate when gravity is involved. Basically this is because the supposed 'classical background' has itself become part of the dynamical system. The most famous attempt to overcome this is the Everett—Wheeler 'many universe' theory. It has even been suggested that the Einstein equations are so highly non-linear, that the linear superposition principle of conventional quantum mechanics must be removed before a satisfactory quantum gravity theory can be constructed.

3. Do we expect to be able to quantize general relativity plus any collection of matter fields or must some special selection be made? This question is more important than it looks at first sight and again its answer sharply distinguishes different approaches. In the early work on quantum gravity most attempts were concerned with the gravitational field alone. On the other hand modern ideas on supergravity and unification suggest that only a special mixture of gravity and matter will work.

4. What sort of physical predictions can we expect to make from a quantum gravity theory? Indeed are there likely to be *any* experimental tests at all? If the answer to the second question is no, then we are engaged in a rather peculiar intellectual exercise which has no contact with the external world and whose ultimate aim is only internal consistency. This is more like pure mathematics than theoretical physics.

5. What is the significance of the Planck length $(G\hbar/C^3)^{\frac{1}{2}} \approx 10^{-33}$ cm ($= 10^{28}$ eV, 10^{-44} s, 10^{32} K) which must presumably arise in any theory which unifies gravity (G) with quantum mechanics ($\hbar$). The general opinion is that 'something odd' happens to space-time structure at this tiny distance or time, but opinions differ as to how this 'something' manifests itself. Indeed, should it come out of the theory (e.g. by cutting off divergent Feynman graph integrals), or by using an appropriate technical structure (e.g. a space-time lattice), should it in some way be fed in from the outset?

We shall return to these questions when discussing the various approaches that are currently being pursued with a view