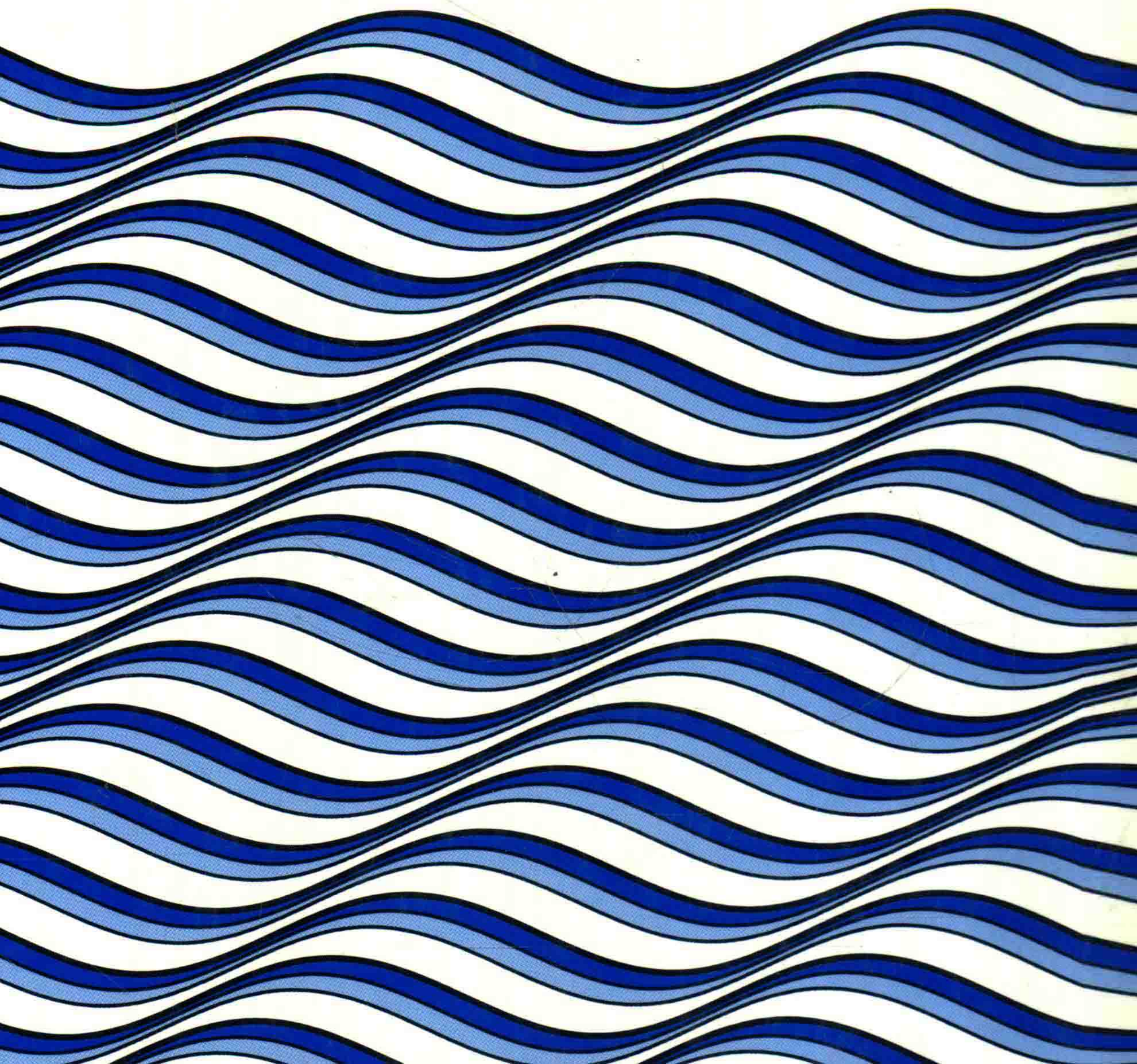
FOURIER ANALYSIS and IMAGING

Ronald N. Bracewell



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Fourier Analysis and Imaging

Preface

Digital signal processing has a short history and a future worth thinking about. The subject grew out of numerical analysis as once practiced with pencil and paper and later with electric calculators. On the digital side, familiar operations of finite differencing, numerical integration, correlation, convolution, statistical analysis, and spectrum analysis were all well developed in the days of hand calculation, as witnessed by publications of the Census Bureau, the Actuarial Society, and the Nautical Almanac Offices of seafaring nations. On the signaling side, use of electrical signals for information transfer goes back to telegraphy, where digital methods were present from the beginning; development was mainly in the telecommunications industry.

Binary digital signals have been in use since the predecessors of Morse code. They gained ground with the advent of the double-triode flip-flop, an electronic circuit that was stable in either of two states and could therefore be used as a counter. All the numerical operations could then be implemented and digital signals could be processed with the full versatility and speed of the modern computer. Naturally, the teaching of digital signal processing has developed within electrical engineering departments. Equally naturally, the focus has been on the waveforms arising in telecommunications (including television), radar, navigation, and computers.

Meanwhile the disciplines of radio astronomy, remote sensing, and space telemetry have developed electronic digital imaging practice that surpasses optical photography in both resolution and dynamic range. Applications in medical and other biological imaging, resource management, geographic systems, business, typography, art, geophysics, meteorology, and oceanography are expanding rapidly. Since the possessors of the necessary techniques are trained in electrical engineering, digital image processing is a growing sector of the old digital signal processing pie. This growth is bound to continue and will diversify in directions discussed in the Introduction (Chapter 1). Future imaging applications will call for new two-dimensional mathematical background going beyond basic electrical signal processing, which will nevertheless continue to be a prerequisite. Image engineering will become a substantial subfield of electrical engineering and computer science, with a distinctive flavor given by the indispensable infusion of finite (discrete) geometry, psychophysics of vision, and imaging practice from the many fields of application. Accommodating students from earth sciences and life sciences will be the hallmark of a basic course in image engineering as distinct from subsequent specialized courses in medical imaging, seismic imaging, radar imaging, etc., which need to be up-to-date in their specialty but do not need to be fundamental.

So wide are the applications of Fourier analysis to different forms of imaging that a fundamental treatment must begin by building a two-dimensional superstructure on whatever background of spectral analysis one already possesses. For a generation, electrical engineers have found that learning about Fourier's idea was one of their most durable investments, having application well beyond the confines of signal analysis. So also will it be found that two-dimensional Fourier analysis gives entrée to fields unconnected with imaging.

In preparing this book I typeset the text in TEX and used a Hewlett-Packard Integral PC with LaserJet IIIP printer for most of the drawings. The problems were tested on graduate students over the course of several years in a course that developed from origins in radio astronomical imaging. I had the privilege of associating with Joseph Pawsey's group at the Radiophysics Laboratory, Sydney, and with Martin Ryle's group at Cambridge, and I came to know the radio astronomers in France, Germany, Italy, the Soviet Union, and the United States. Pawsey and I coauthored the first book on radio astronomy. Many advances in image construction came from the explosive development of this subject. At Stanford, colleagues who shared my interest in image construction and influenced my understanding include Govind Swarup, A. Richard Thompson, Zvonko Fazarinc, T. Krishnan, L.R. d'Addario, Werner Graf, E.K. Conklin, Jacques Verly, John Villasenor, Domingo Mihovilović, and many students. What I know about painting was learnt from Reginald Earl Campbell. My colleagues Sylvia Plevritis and Ramin Samadani kindly criticized several chapters. I thank my wife Helen for her support and extraordinary patience.

It is a privilege to have worked in a department with a distinguished lineage of textbook writers, especially Frederick E. Terman and Hugh H. Skilling, whose work I have admired. Skilling used to express thanks for the liberal policy of Stanford University in encouraging publication, and I am happy to thank Joseph W. Goodman for maintaining that atmosphere.

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Contents

1 Introduction

Telecornmunication by radio shrank the world to a global village, and the satellite and computer have made imagery the language of that village. The creation of images was once mainly in the hands of artists and scribes. Their art works—the famous stone-age cave paintings, the engravings on stone, bone and tooth, and the paintings on bark and skin that have been lost—go back to the distant past. Their inscriptions are also images, though of a different kind. They are also very old, as witnessed by hieroglyphic writing on the walls of tombs and on papyrus in Egypt. In modern times artists and scribes were joined by photographers and printers as creators of images. A variety of artisans, playing a secondary role as artists, created a third kind of image: ornament manifested on pottery, in woven fabrics and carpets, in tile patterns, and in painted friezes. Builders and cartographers created a fourth kind of image, once relatively rare but nevertheless going back to antiquity: construction plans and maps.

Two-dimensional images also occur naturally: a shadow, the dappled light pattern under a tree, the optical image on a retina. Nature provided the motif for much abstract ornamentation, and our written letters and ideograms trace back to representations of nature.

The graven image is a three-dimensional generalization. Examples in art range from primitive ornamental abstract carving to sophisticated modern sculpture. Writing provides three-dimensional examples ranging from cuneiform impressions in clay tablets in Iraq to formal hieroglyphs incised in granite by the Egyptians, jade calligraphy in China, and inscriptions in marble in the elegant monumental alphabet devised by the Romans. Three-dimensional images today have become an important and growing part of the digital world. Examples include volumetric data sets produced for medical diagnosis and other purposes by x-ray tomography, magnetic resonance, positron-electron scanners, ultrasound, confocal microscopes, laser scanners, and supercomputer simulation.

Three-dimensional holographic images, which are of increasing importance, are presented by suitable illumination of holograms. Holograms themselves are usually two-dimensional analogue images (an exception is thin thermoplastic film embossed with corrugations). The construction of analogue holograms from image data sets is a reality, and no doubt composite digital holograms are not far behind. Video processing and multimedia products for mass consumption are also capable of three-dimensional image presentation and will encourage the development of stereoscopic techniques.

The main topic of this book is the two-dimensional image. The book explores the idea that a two-dimensional image is much the same as a mathematical function of two variables, and it explores a variety of phenomena that are susceptible to mathematical reasoning. Some of these quantitative phenomena, such as grey levels, dynamic range, resolution, spatial spectrum, perspective projection, and convolution, have wide significance and are therefore dealt with first. The canvas may be continuous or discrete. Discussion then branches into various specific kinds of image associated with techniques such as diffraction, tomography, interferometry, and range-Doppler mapping.

The creation of images has been moving progressively into the hands of those equipped with new tools: the cathode-ray tube and the laser printer under computer control. Many of us have learned to wield these tools without the accumulated wisdom of artists, typesetters, photographers, advertisers, and movie-makers, all of whom prepare visual material for the attention of others. Much of this wisdom is being reacquired, often painfully, and clearly a new synthesis is in the making. Michelangelo, if he were alive today in his workshop in Florence, might be cutting a titanium monument of extraordinary beauty with a laser-controlled interactive program that he was developing, causing severe competition for the younger generation. We get an inkling of things to come from the fractal images of Mandelbrot and the animated movies of Stephen Spielberg.

As Descartes taught, we should be skeptical. When we question things, our minds are stimulated to creativity. The questioning reader may already have asked, "Is an image really reducible to a function of two variables?" This question leads us to consider the color image. Of course, we know from color printing with ink that a set of four functions of two spatial variables gives the desired description; the four functions are, respectively, the densities of the yellow, cyan, magenta, and black imprints. This is a reassuring minor generalization, bolstering confidence that the mathematical approach is competent. But when you ask, "Are the inks deposited on the paper yellow first and black last and if so why?" you are raising sticky questions whose answers may make the mathematical part of you nervous. Modern practice was not arrived at by logical deduction but by empirical trial, sometimes so lengthy that the early steps have been forgotten. If and when the procedures of this sort of incrementally developed technology come to be understood, not only physics but psychophysics will prove to have been a significant factor. Only occasional allusions to psychophysical and psychological aspects of images can be made in this book, but the topic is introduced here to warn about the incompleteness of the mathematically based material for one wishing to become competent in image handling.

As a reminder of the limitations, take the case of a camera, which has difficulty making a realistic image of a gold ring with a pearl. The camera may capture a function of two variables—photographic density in x and y—very faithfully. It fails to capture surface gloss, metallic sheen, and translucency. The eye is very well aware of these additional variables. How the eye can see something that the camera cannot record will be discussed. Interestingly, some oil painters have been able to convey these nonphotographic surface qualities.

These are very interesting matters that image engineers are going to learn more about. Just as a hint of the phenomena involved, here are some suggestions for action. Get a flower, spray some water droplets on the petals, and put it in the sun; add a fluorescent garment and diamond ring if available. Take a photo and place a print alongside the original arrangement. Look at the solid object and the flat image from different directions; repeat while holding a card with a single pinhole close to your eye. Hold a card with a 5-mm square hole at arm's length, focus on the plane of the card, and use this tool to compare corresponding areas of the arrangement and its photograph. Catch a *camera obscura* image of an outdoor scene inside a cardboard carton and view it through an opening in the bottom. Turn your back on the scene and view it through your legs! Have the shadows changed color? Does the sky seem to have a different color when your head is upside down? Training in physics alone does not prepare you for this, or for what happens when you mix black and yellow paints, or for understanding why Newton reported seven colors in the solar spectrum. Get a prism and a slit and see these color bands for yourself. From experiments one can learn a lot about perception and prepare for the richness of experience that work with images will bring.

SUMMARY OF THE CHAPTERS

Chapter 2, which introduces the picture plane, mentions some of these interesting matters but is mainly concerned with the presentation of graphic images, with elementary operations on images, and with establishing terminology. Most of the operations are applicable to discrete images (those defined on a lattice of points rather than on the plane of continuous x and y) and to quantized images (which have digital function values); consequently, discrete examples are introduced freely, but the mathematics is kept general. The chapter makes relatively light reading. However, this book is written mainly for people equipped to handle computer images using more advanced mathematical methods, especially the Fourier transform in both its continuous and discrete forms. Some previous experience with Fourier analysis of signals will thus be helpful. Two-dimensional Fourier analysis, though, is somewhat different from analysis of signals into temporal vibrations, so the material has been written to be accessible to mathematically inclined students whether they have had previous experience with transform methods or not. The topics of the other chapters will now be summarized.

Several kinds of image involve Fourier transformation either directly, as with optical