Numerical Simulations of Heat Transfer and Fluid Flow on a Personal Computer

Transport Processes in Engineering, 3

Numerical Simulations of Heat Transfer and Fluid Flow on a Personal Computer

Incorporating Simulation Programs on Diskette

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ELSEVIER
Amsterdam – London – New York – Tokyo 1993

ELSEVIER SCIENCE PUBLISHERS B.V.,
Sara Burgerhartstraat 25
P.O. Box 211, 1000 AE Amsterdam, The Netherlands

ISBN 0-444-89811-5 (Hardcover)
ISBN 0-444-89812-3 (Paperback)

This book is printed on acid-free paper.

Printed in The Netherlands

Numerical Simulations of Heat Transfer and Fluid Flow on a Personal Computer

TRANSPORT PROCESSES IN ENGINEERING SERIES

Advisory Editor: A.S. Mujumdar, Department of Chemical Engineering, McGill University, Montreal, Quebec, Canada

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S. Kotake and K. Hijikata

Preface

Numerical simulations of transport processes in flows have been developed mainly in the aeronautical and atmospheric sciences. The majority of problems that have been investigated in these areas of research concern flows around a body, or *external flows*. Computational studies of flows in the interior of a body, which are important in mechanical engineering and related fields, have appeared relatively recently.

The basic equations of heat transfer and fluid flow have been well established. The chief interest is in how to solve them. The real issue here is associated with the boundary conditions. In the case of external flows, the boundary conditions are usually given in a comparatively simple manner in the infinite far-field and on the surface of a body. On the other hand, for internal flows, one needs to specify the boundary conditions on the inner surfaces of a closed space. Therefore, the proper prescription of boundary conditions and their appropriate modeling are the most crucial factors that influence computed results. It is extremely difficult to reproduce faithfully physical phenomena of interest by numerical computations without using suitable models of actual boundary conditions, no matter how accurately the basic equations are solved.

In the majority of textbooks and references on numerical analysis, including those which cover Computational Fluid Dynamics (CFD), discussion has been centered almost exclusively on the solution methods for equations. Such materials may serve as excellent sources of information when one is required to perform accurate and efficient computations. However, they are often of little use in an attempt to understand the relevant transport processes.

Hence, to conduct numerical simulations of fluid flow and heat transfer, one should have basic understanding of the physical phenomena of importance, the possible effects of boundary conditions on them, the influential parameters of the flow problem under investigation, and so on. Since the transport processes of interest are controlled by boundary conditions rather than by the basic equations, successful computations cannot be expected unless in-depth understanding of the pertinent physics is secured. This implies that one should conduct experiments in order to undertake computations.

One may wonder if it is possible to carry out such experiments by computations: namely, *numerical experiments*. It is most likely that, when a numerical analyst who is unfamiliar with physical phenomena happens to acquire reasonably precise numerical predictions of a heat transfer or related problem, he has succeeded in his numerical experiments.

Some of the latest personal computers are comparable in their capabilities to the mainframe machines that were considered to be standard ten years ago. These powerful computing facilities now make it possible to perform numerical experiments and, thereby, help us in appreciating significant transport processes. Such experiences are of immense usefulness in building up foundations for conducting more complex and accurate numerical calculations.

These ideas have motivated us to plan a book in the present form. To accomplish our final goal, the principal emphasis has been placed on simple and straightforward numerical techniques, rather than on overly sophisticated matters. Consequently, the apparatus and conditions of experiments, so to speak, may be readily modified so that improved and enlarged knowledge of the phenomena under investigation may be gained.

The present book should be of value to the following students and industrial practitioners:

- Those who are familiar with numerical simulations and who attempt to solve numerical fluid flow and heat transfer problems for the first time.
- Those who have the essential understanding of transport processes in heat transfer and fluid flow and who are considering computational studies on the subject without practical experience in numerical analysis.
- Those who are already conducting research on numerical heat transfer and who wish to consolidate a firmer basis on related topics.

The features of this book are that

1. It enables the reader to learn fundamental concepts and methodologies of numerical heat transfer and fluid flow by hands-on experience in solving basic problems on heat and mass transfer on a personal computer.
2. A systematic description is presented throughout the entire book. At the same time, any one of the chapters is nearly self-contained for partial referencing. Care is exercised to allow the reader to grasp the contents of each chapter at a glance.
3. The computer programs used for solution of example problems are written in the BASIC computer language. This is believed to be highly adaptable to the user environment provided by most personal computers. Responses due to the changes in parameters and boundary conditions can be examined with ease.
4. Owing to the above considerations, the phenomena of interest handled within the scope of the present book are rather limited. However, attention is paid such that more complicated and extended problems can be tackled.
5. Hence, we do not deal with combustion and radiation heat transfer in an explicit manner. However, the approaches taken in this book will suffice to account for these effects, as long as they may be regarded as classes of problems that involve heat generation.

Finally, our special thanks go to Professor Takao Nagasaki of the Tokyo Institute of Technology for his substantial help in writing the computer programs included in this book. The assistance of Mr. Hidenori Onda of the Maruzen Book Company in the planning and editorial processes of the book is also acknowledged.

Summer 1988

Susumu Kotake
Kunio Hijikata

Preface to the English Edition

Our original aim for writing the Japanese language version of this book was to provide the reader with an opportunity to conduct 'numerical experiments for computations', through which he/she can find a way to the phenomenological understanding of heat transfer and fluid flow processes. Through easy-to-follow examples on fundamental topics of heat transfer and fluid flow discussed in the text, the reader is taken to the point where he/she can actually experience the entire course of work for finding out what the most predominant and controlling factors are that govern the phenomena of interest. It is also possible to know what the most suitable and pertinent numerical methods are to solve the problem. Such experiences allow the reader to build a sufficient foundation for overviewing more practical engineering fields. They will also be useful for developing a manipulative competency with effective calculation schemes for solving the real-world problems.

The authors wish to express our great appreciation to Maruzen Publishing Co. for allowing us to translate and to publish our original Japanese-language book in English, although the English version has been slightly revised from the original text.

We also wish to acknowledge Dr. Toru Fusegi for his contribution for not only translating the text into English but improving the description of the book by providing us with numerous valuable suggestions. Thanks are also due Elsevier Science Publishers B.V. for making possible the publication of the English edition of the book.

June 1992

S. K.
K. H.

Translator's note

The basis of the present English translation is the original Japanese edition published in 1988 from Maruzen Book Company. In preparing the English text, rather extensive revisions were undertaken by keeping close contact with the authors so that their idea for improving presentation of the subjects of interest is reflected satisfactorily in the present book. Comments and suggestions directed to the Japanese language version have also been incorporated.

In a view toward better clarification of relevant transport processes, Chapters 5 and 11 have been thoroughly reorganized. For the latter, the task includes replacing three calculation codes for different turbulence models with a single program with which users can compute flow using a desired model. A good number of exercise problems have been appended. They will help enhancing readers' understanding of both simulated physical phenomena and numerical codes. When this book is used as a textbook or reference book for college engineering courses, these problems may serve as an appropriate source for the homework assignment to the course instructors.

The thirteen simulation programs contained in the attached floppy disk have been thoroughly checked and all of the results appearing in the text were generated newly for the present book. It is worth mentioning that very minor modifications of source codes are necessary in order to reproduce time history plots such as those shown in Figures 5-5 and 12-4. Both authors and I have spent considerable effort to exclude program bugs; however, in case the reader detect any inconsistency, we would be greatly appreciate it if you could inform us of the difficulty.

A further remark on the programs may deserve attention. For computer simulations of flow and heat transfer discussed in Chapters 5 through 12, two distinctively different numerical techniques are employed: the explicit scheme and the implicit scheme, for which details are available in Chapter 2. Using an explicit solver, field data at a new time level are substituted into dimensional arrays to update existing values at a prior time step. Hence, memory allocation can be kept minimum. This is an attractive feature especially if we must store all the data for the entire calculation field at a given time step. This is the case when we consider fluid motions accompanying recirculating flow, since local flow patterns influence the whole field. This appears to be the major motivation for employing an explicit method in Chapters 5, 7 - 9 and 12. Note that there is an upper limit for time increments to perform stable explicit calculations; refer to Chapter 2. Hence, if the steady state is of primary concern, we can reach the steady-state solution in fewer time steps using an implicit solver because we can assign a much larger time increment than is permitted to an explicit counterpart. Implicit computations are particularly preferable if the requirement for data storage is not excessively severe. This applies to the cases in which flow features along lines of constant cross-stream coordinates with respect to the main flow direction are unaffected by the downstream conditions. Consequently, computations can be processed by successively replacing field data at previous grid lines with those from new stations. This calls for only one-dimensional arrays in two-dimensional calculations. Implicit solvers are utilized in Chapters 6 and 11, which deal with boundary layer flow, and in Chapter 10 (secondary motion in a cross section of pipe flow).

I would like to express my sincere gratitude to Mr. Patrick E. Phelan of the Tokyo Institute of Technology for critically examining the entire manuscript and providing me with many valuable suggestions. I am also very grateful to Professor Shoichiro Nakamura of the Ohio State University for reading part of the manuscript. Comments from colleagues of both authors and mine are faithfully acknowledged. I am thankful to my family for their understanding and support throughout the course of this work.

Toru Fusegi

Using the Program Disks

The book contains two floppy disks, each of which stores a complete set of the simulation source codes to be discussed in the text. These programs are recorded in ASCII format and can be run either on IBM PC® or Macintosh® using Quick BASIC® (The programs run also with QBASIC that is supplied with the MS-DOS® 5.0 package). They can be edited with appropriate text-editing software. Refer to the relevant reference material of the software for detailed information. Macintosh users are advised not to attempt to change the window size while a program is running or to use multi-tasking.

Every pair of the programs with the same name have identical subroutines in which calculations are processed. Differences occur in program modules written for data in/output (I/O). One feature available on the Macintosh version only is the on-line Help menu. Default parameter values are preset in the programs for a sample run. The user can either adopt these data or modify any desired variables interactively.

The following program files are included in each disk:

BODYFLOW.BAS, CAVITYFL.BAS, CHANNEL.BAS, CONDUCT.BAS, DIFFCOM.BAS, DUCTFLOW.BAS, FEMFIN.BAS, MIXEDBDL.BAS, PREMIXCM.BAS, SECOFLOW.BAS, STEPFLOW.BAS, TURBFLOW.BAS, TURBPIPE.BAS, BODYFLOW.EXE, CAVITYFL.EXE, CONDUCT.EXE, MIXEDBDL.EXE, SECOFLOW.EXE, TURBPIPE.EXE

where *.BAS is an ASCII-saved source program and *.EXE refers to an executable module which has been compiled. If a memory error occurs, reduce the size of dimensional arrays and recompile. A graphic monitor (VGA) is required to display computed results.

Sample datafiles which can be loaded and used to produce various graphic outputs are also found on the disks:

BDL.DAT (for MIXEDBDL), SEC.DAT (for SECOFLOW), TBP.DAT (for TURBPIPE)

The user is urged to make a backup copy of the original disks. The authors, translator, or publisher will not assume responsibility for files erased from the disks or altered accidentally.

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CHAPTER 1

FUNDAMENTAL HEAT TRANSFER AND FLUID FLOW

1-1 Basic Equations

The basic equation system of heat transfer and fluid flow consists essentially of the continuity equation (the mass conservation equation) and the conservation equations of momentum and energy. In this book, we focus our attention to the solution of these equations on a personal computer in order to assist our understanding of the important basic transport processes of fluid flow and heat transfer. We will not consider overly complicated phenomena which are not relevant to our discussion. We restrict ourselves to physical phenomena which are present under the following conditions:

(1) Fluids are incompressible and Newtonian. We will not account for the variation of density unless it is responsible for the generation of buoyancy forces. Steady as well as unsteady features are dealt with.
(2) Physical properties of fluids are constant.
(3) Among various forms of the conservation of energy, only that of thermal energy is considered. Dissipation, which is an irreversible transformation from kinetic to thermal energy, will be neglected except for that occurring in turbulent flows.

Note that simple modifications to the computer programs appearing in succeeding chapters will allow us to accommodate effects due to (2) and (3). We recommend readers to attempt them as extended exercises. Notice that removal of the first assumption will lead us into situations which are beyond the scope of this book. Under such circumstances, we would have to handle additional physics such as the propagation of acoustic waves. They would demand a larger memory size owing to use of an increased number of grid points, a new computational algorithm to manage the effects of these processes, and so on.

Under these conditions, the following basic equations are obtained:

[Continuity equation]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1.1}$$

[Conservation equations of momentum]

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + \frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + g_x\rho$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)+\frac{\partial p}{\partial y}=\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+g_y\rho \tag{1.2}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)+\frac{\partial p}{\partial z}=\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+g_z\rho$$

[Conservation equation of energy]

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+Q \tag{1.3}$$

where u, v and w are the velocity components in the x, y and z directions, respectively, p is the pressure, ρ is the density, and T is the temperature. The components of gravitational acceleration in the x, y and z directions are denoted by g_x, g_y and g_z, respectively. The symbol μ is the fluid viscosity [kg/m·s], c_p is the specific heat at constant pressure [J/kg·K], and λ is the thermal conductivity [W/m·K]. The volumetric heat generation rate per unit volume is represented by Q. The momentum conservation equations are known as the Navier-Stokes equations. The reader is urged to refer to appropriate fluid mechanics textbooks for derivation of these equations.

In this book, we will principally analyze two-dimensional flows. There are the following two possible situations:

(1) The velocity component in the z-direction (w) is negligibly small compared to those in the other directions, i.e., u and v. Furthermore, u, v and T are not considered to be functions of z.
(2) Changes in u, v, w and T in the x-direction, say, are assumed to be known. In other words, the transport processes under investigation may be regarded as functions of y and z only.

When the first presumption is valid, we can further simplify the basic equations:

[Continuity equation]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1.4}$$

[Momentum conservation equation in the x-direction]

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\frac{\partial p}{\partial x}=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\rho g_x \tag{1.5a}$$

Unsteady term | Convection terms | Pressure term | Viscous terms | Buoyancy force term

[Momentum conservation equation in the y-direction]

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)+\frac{\partial p}{\partial y}=\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g_y \tag{1.5b}$$

Unsteady term | Convection terms | Pressure term | Viscous terms | Buoyancy force term

[Energy conservation equation]

$$\rho\, c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q \tag{1.6}$$

Unsteady term / Convection terms / Conduction terms / Heat Generation term

The physical significance of each term is also indicated in these expressions. When gravity has an appreciable effect on the pressure, it can be compensated for in the following manner. By noting that the pressure decreases vertically upward (in the negative x-direction, say) even in a stationary state, let us replace the pressure p with p_0:

$$p = p_0 + \rho^* g_x x \tag{1.7}$$

where ρ^* is a fixed reference density. The pressure and gravity terms in the basic equations are rewritten as

$$\cdots\cdots \frac{\partial p_0}{\partial x} = \cdots\cdots (\rho - \rho^*) g_x \tag{1.8}$$

If the variation of the density, ρ, depends only on the temperature, T, then,

$$\rho - \rho^* = -\rho\beta\,(T - T^*) \tag{1.9}$$

where β is the volumetric thermal expansion coefficient defined as $-(\partial \ln\rho/\partial T)$. For an ideal gas, we derive $\beta = 1/T$ from the equation of state $p = \rho RT$, where T is measured in an absolute temperature scale and R is the gas constant.

For generality, we usually nondimensionalize a given fluid flow and heat transfer problem with the aid of appropriate scales of length, time and other involved properties. Define the following nondimensional quantities (with a tilde, ~) by selecting U, L and ΔT as the reference scales for velocity, length, and temperature difference, respectively:

$$\tilde{t}=\frac{tU}{L},\ \tilde{u}=\frac{u}{U},\ \tilde{\mathrm{v}}=\frac{\mathrm{v}}{U},\ \tilde{x}=\frac{x}{L},\ \tilde{y}=\frac{y}{L},$$
$$\tilde{T}=\frac{T-T^*}{\Delta T},\ \tilde{p}=\frac{p_0}{\rho^* U^2},\ \tilde{Q}=\frac{Q}{\rho^* c_p{}^* \Delta T} \tag{1.10}$$

Then, we can rewrite the basic equations (1.4) - (1.6) as

[Continuity equation]

$$\frac{\partial \tilde{u}}{\partial \tilde{x}}+\frac{\partial \tilde{\mathrm{v}}}{\partial \tilde{y}}=0 \tag{1.11}$$