

# Engineering Science

Sixth Edition

**W. Bolton**

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Published by Routledge  
2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN

and by Routledge  
711 Third Avenue, New York, NY 10017

*Routledge is an imprint of the Taylor & Francis Group, an informa business*

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First published by Newnes 1990

*British Library Cataloguing-in-Publication Data*

A catalogue record for this book is available from the British Library

*Library of Congress Cataloging in Publication Data*

Bolton, W. (William), 1933-

Engineering science / W. Bolton. – Sixth edition.

pages cm

Includes index.

1. Engineering. 2. Mechanical engineering. 3. Electrical engineering. I. Title.

TA145.B64 2015

620–dc23

2014034737

ISBN: 978-1-138-89843-1 (hbk)

ISBN: 978-1-138-82893-3 (pbk)

ISBN: 978-1-315-73797-3 (ebk)

Typeset in Univers by  
Servis Filmsetting Ltd, Stockport, Cheshire

# Engineering Science

Comprehensive engineering science coverage that is fully in line with the latest vocational course requirements:

- ▶ New chapters on heat transfer and fluid mechanics
- ▶ Topic-based approach ensures that this text is suitable for all vocational engineering courses
- ▶ Coverage of all the mechanical, electrical and electronic principles within one volume provides a comprehensive exploration of scientific principles within engineering.

*Engineering Science* is a comprehensive textbook suitable for all vocational and pre-degree courses. Taking a subject-led approach, the essential scientific principles engineering students need for their studies are topic-by-topic based in presentation. Unlike most of the textbooks available for this subject, W. Bolton goes beyond the core science to include the mechanical, electrical and electronic principles needed in the majority of courses.

A concise and accessible text is supported by numerous worked examples and problems, with a complete answer section at the back of the book. Now in its sixth edition, the text has been fully updated in line with the current BTEC National syllabus and will also prove an essential reference for students embarking on Higher National engineering qualifications and Foundation Degrees.

**W. Bolton** was formerly Head of Research and Development and Monitoring at BTEC. He has also been a UNESCO consultant and is author of many successful engineering textbooks.

# Preface

## Aims

This book aims to provide a comprehensive grounding in science relevant to engineers by:

- ▶ providing a foundation in scientific principles which will enable the solution of simple engineering problems;
- ▶ providing a platform for further study in engineering;
- ▶ providing the basic principles underlying the operation of electrical and electronic devices;
- ▶ providing the basic principles underlying the behaviour and performance of static and dynamical mechanical systems.

The breadth of its coverage makes it an ideal course book for a wide range of vocational courses and foundation or bridging programmes for Higher Education. The content more than covers the latest UK syllabuses, in particular the new specifications for Engineering in the BTEC National from Edexcel.

## Changes from the fifth edition

For the sixth edition, the book:

- ▶ has been completely reorganised and reset to give a 'less cluttered' appearance;
- ▶ includes extra material required to give the comprehensive coverage of *all* the science and principles at this level, omitting some material for which there is little demand at this level. Thus heat transfer and fluid mechanics have been included. The main omissions are engineering systems, parallel a.c. circuits and three-phase a.c.

## Structure of the book

The book has been designed to give a clear exposition and guide readers through the scientific principles, reviewing background principles where necessary. Each chapter includes numerous worked examples and problems. Answers are supplied to all the problems.

W. Boiton



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# Basics

## 1.1 Introduction

In this chapter there is a review of some basic engineering science terms and, as within this book and your studies you will come across many equations, the manipulation of equations and the units in which quantities are specified. Engineers make measurements to enable theories to be tested, relationships to be determined, values to be determined in order to predict how components might behave when in use and answers obtained to questions of the form 'What happens if?' Thus, there might be measurements of the current through a resistor and the voltage across it in order to determine the resistance. Thus, in this introductory chapter, there is a discussion of the measurement and collection of data and the errors that can occur.

## 1.2 Basic terms

### 1 *Mass*

The mass of a body is the quantity of matter in the body. The greater the mass of a body the more difficult it is to accelerate it. Mass thus represents the inertia or 'reluctance to accelerate'. It has the SI unit of kg.

### 2 *Density*

If a body has a mass  $m$  and volume  $V$ , its density  $\rho$  is:

$$\rho = \frac{m}{V}$$

Density has the SI unit of kg/m<sup>3</sup>.

### 3 *Relative density*

Relative density is by what factor the density of a substance is greater than that of water and is thus defined as:

$$\text{relative density} = \frac{\text{density of a material}}{\text{density of water}}$$

Since relative density is a ratio of two quantities in the same units, it is purely a number and has no units.

### Example

If the density of water at 20°C is 1000 kg/m<sup>3</sup> and the density of copper is 8900 kg/m<sup>3</sup>, what is the relative density of copper?

$$\text{Relative density of copper} = 8900/1000 = 8.9$$

**4 Force**

We might describe forces as pushes and pulls. If you pull a spring between your hands we can say that your hands are applying forces to the ends of the spring. If there is an unbalanced force acting on an object it accelerates. Force has the SI unit of the newton (N).

**5 Weight**

The weight of a body is the gravitational force acting on it and which has to be opposed if the body is not to fall. The weight of a body of mass  $m$  where the acceleration due to gravity is  $g$  is  $mg$ . Weight, as a force, has the SI unit of N.

**Example**

What is the weight of a block with a mass of 2 kg if the acceleration due to gravity is  $9.8 \text{ m/s}^2$ ?

$$\text{Weight} = mg = 2 \times 9.8 = 19.6 \text{ N}$$

**6 Pressure**

If a force  $F$  acts over an area  $A$ , the pressure  $p$  is:

$$p = \frac{F}{A}$$

It has the SI unit of  $\text{N/m}^2$ , this being given the special name of pascal (Pa).

**1.3 Manipulating equations**

The following are basic rules for manipulating equations:

- 1 Adding the same quantity to, or subtracting the same quantity from, both sides of an equation does not change the equality.
- 2 Multiplying, or dividing, both sides of an equation by the same non-zero quantity does not change the equality.

In general, whatever mathematical operation we do to one side of an equation, provided we do the same to the other side of the equation then the balance is not affected.

The term *transposition* is used when a quantity is moved from one side of an equation to the other side. The following are basic rules for use with transposition:

- 1 A quantity which is added on the left-hand side of an equation becomes subtracted on the right-hand side.
- 2 A quantity which is subtracted on the left-hand side of an equation becomes added on the right-hand side.
- 3 A quantity which is multiplying on the right-hand side of an equation becomes a dividing quantity on the left-hand side.
- 4 A quantity which is dividing on the left-hand side of an equation becomes a multiplying quantity on the right-hand side.

Suppose we have the equation  $F = kx$  and we want to solve the equation for  $x$  in terms of the other quantities. Writing the equation as  $kx = F$  and then transposing the  $k$  from the left-hand side to the right-hand side (or dividing both sides by  $k$ ) gives

$$x = \frac{F}{k}$$

**Example**

Determine  $L$  in the equation

$$2 = \sqrt{\frac{L}{10}}$$

Squaring both sides of the equation gives us the same as multiplying both sides of the equation by the same quantity since 2 is the same as  $\sqrt{(L/10)}$ :

$$2 \times 2 = \sqrt{\frac{L}{10}} \times \sqrt{\frac{L}{10}}$$

Hence, we have  $L = 40$ . We can check this result by putting the value in the original equation to give  $2 = \sqrt{(40/10)}$ .

Brackets are used to show terms are grouped together, e.g.  $2(x + 3)$  indicates that we must regard the  $x + 3$  as a single term which is multiplied by 2. Thus *when removing brackets, each term within the bracket is multiplied by the quantity outside the bracket*. When a bracket has a + sign in front of the bracket then effectively we are multiplying all the terms in the bracket by +1. When a bracket has a - sign in front of the bracket then we are multiplying all the terms in the bracket by -1. When we have  $(a + b)(c + d)$  then, following the above rule for the removal of brackets, each term within the first bracket must be multiplied by the quantity inside the second bracket to give:

$$a(c + d) + b(c + d) = ac + ad + bc + bd$$

As an example, consider the following equation for the variation of resistance  $R$  of a conductor with temperature  $t$ :

$$R_t = R_0(1 + \alpha t)$$

where  $R_t$  is the resistance at temperature  $t$ ,  $R_0$  the resistance at  $0^\circ\text{C}$  and  $\alpha$  a constant called the temperature coefficient of resistance. We might need to rearrange the equation so that we express  $\alpha$  in terms of the other variables. As a first step we can multiply out the brackets to give:

$$R_t = R_0 + R_0\alpha t$$

Transposing the  $R_0$  from the left-hand to the right-hand side of the equation gives:

$$R_t - R_0 = R_0\alpha t$$

Reversing the sides of the equation so that we have the term involving  $\alpha$  on the left-hand side, we then have

$$R_0\alpha t = R_t - R_0$$

Transposing the  $R_0 t$  from the left-hand to the right-hand side gives

$$\alpha = \frac{R_t - R_0}{R_0 t}$$

Consider the addition of two fractional terms  $a/b$  and  $c/d$ .

$$\frac{a}{b} + \frac{c}{d}$$

We proceed by multiplying the numerator and denominator of each fraction by the same quantity; this does not change the value of a fraction. The quantity is chosen so that both fractions end up with the same denominator. Thus:

$$\frac{a}{b} \times \frac{d}{d} + \frac{c}{d} \times \frac{b}{b} = \frac{ad + cb}{bd}$$

### Example

Rearrange the following equation for resistances in parallel to obtain  $R_1$  in terms of the other variables:

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}$$

Transposing  $1/R_2$  from the left-hand side to the right-hand side of the equation gives:

$$\frac{1}{R_1} = \frac{1}{R} - \frac{1}{R_2}$$

Arranging the fractions on the right-hand side of the equation with a common denominator, then:

$$\frac{1}{R_1} = \frac{R_2 - R}{RR_2}$$

We can invert the fractions provided we do the same to both sides of the equation. Effectively what we are doing here is transposing the numerators and denominators on both sides of the equation. Thus:

$$R_1 = \frac{RR_2}{R_2 - R}$$

### Example

The power  $P$  dissipated by passing a current  $I$  through a resistance  $R$  is given by the equation  $P = I^2R$ . Rearrange the equation to give  $I$  in terms of the other variables.

Writing the equation as  $I^2R = P$  and then transposing the  $R$  gives:

$$I^2 = \frac{P}{R}$$

Taking the square roots of both sides of the equation then gives:

$$I = \sqrt{\frac{P}{R}}$$

### 1.3.1 Units in equations

It is not only numerical values that must balance when there is an equation. The units of the quantities must also balance. For example, the area  $A$  of a rectangle is the product of the lengths  $b$  and  $w$  of the sides, i.e.  $A = bw$ . The units of the area must therefore be the product of the units of  $b$  and  $w$  if the units on both sides of the equation are to balance. Thus if  $b$  and  $w$  are both in metres then the unit of area is metre  $\times$  metre, or square metres ( $\text{m}^2$ ). The equation for the stress  $\sigma$  acting on a body when it is subject to a force  $F$  acting on a length of the material with a cross-sectional area  $A$  is:

$$\sigma = \frac{F}{A}$$

Thus if the units of  $F$  are newtons (N) and the area square metres ( $\text{m}^2$ ) then, for the units to be the same on both sides of the equation:

$$\text{unit of stress} = \frac{\text{newtons}}{\text{square metre}}$$

or, using the unit symbols:

$$\text{unit of stress} = \frac{\text{N}}{\text{m}^2} = \text{N/m}^2 = \text{Nm}^{-2}$$

This unit of  $\text{N/m}^2$  is given a special name of the pascal (Pa).

Velocity is distance/time and so the unit of velocity can be written as the unit of distance divided by the unit of time. For the distance in metres and the time in seconds, then the unit of velocity is metres per second, i.e.  $\text{m/s}$  or  $\text{m s}^{-1}$ . For the equation describing straight-line motion of  $v = at$ , if the unit of  $v$  is metres/second then the unit of the  $at$  must be metres per second. Thus, if the unit of  $t$  is seconds, we must have for the unit of acceleration:

unit of  $v$  = unit of  $at$

$$\frac{\text{metre}}{\text{second}} = \frac{\text{metre}}{\text{second}^2} \times \text{second}$$

Hence the unit of the acceleration  $a$  must be  $\text{m/s}^2$  for the units on both sides of the equation to balance.

### Example

When a force  $F$  acts on a body of mass  $m$  it accelerates with an acceleration  $a$  given by the equation  $F = ma$ . If  $m$  has the unit of  $\text{kg}$  and  $a$  the unit  $\text{m/s}^2$ , what is the unit of  $F$ ?

For equality of units to occur on both sides of the equation:

Unit of  $F$  = unit of mass  $\times$  unit of acceleration

$$\text{Unit of } F = \text{kg} \times \frac{\text{m}}{\text{s}^2} = \text{kg m/s}^2$$

This unit is given a special name of the newton (N).

### Example

The pressure  $p$  due to a column of liquid of height  $h$  and density  $\rho$  is given by  $p = h\rho g$ , where  $g$  is the acceleration due to gravity. If  $h$  has the unit of  $\text{m}$ ,  $\rho$  the unit  $\text{kg/m}^3$  and  $g$  the unit  $\text{m/s}^2$ , what is the unit of the pressure?

For equality of units to occur on both sides of the equation,

unit of pressure = unit of  $h \times$  unit of  $\rho \times$  unit of  $g$

$$\text{unit of pressure} = \text{m} \times \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}}{\text{s}^2} = \frac{\text{kg}}{\text{m} \times \text{s}^2}$$

Note that in the previous example we obtained the unit of newton (N) as being the special name for  $\text{kg m/s}^2$ . Thus we can write the unit of pressure as  $\text{N m}^{-2}$  or  $\text{N/m}^2$ . This unit of  $\text{N/m}^2$  is given a special name, the pascal (Pa).

### Example

For uniformly accelerated motion in a straight line, the velocity  $v$  after a time  $t$  is given by  $v = u + at$ , where  $u$  is the initial velocity and  $a$  the acceleration. If  $v$  has the unit  $\text{m/s}$ ,  $u$  the unit  $\text{m/s}$  and  $t$  the unit  $\text{s}$ , what must be the unit of  $a$ ?

For the units to balance we must have the unit of each term on the right-hand side of the equation the same as the unit on the left-hand side. Thus the unit of  $at$  must be  $\text{m/s}$ . Hence, since  $t$  has the unit of  $\text{s}$ :

$$\text{unit of } a = \frac{\text{unit of } at}{\text{unit of } t} = \frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}^2}$$

## 1.4 SI units

The *International System* (SI) of units has the seven basic units:

Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Also there are two supplementary units, the radian and the steradian.

Since in any equation we must have the units on one side of the equals sign balancing those on the other, we can form the SI units for other physical quantities from the base units via the equation defining the quantity concerned. Thus, for example, since volume is defined by the equation  $\text{volume} = \text{length cubed}$  then the unit of volume is that of unit of length unit cubed and so the metre cubed, i.e.  $\text{m}^3$ . Since density is defined by the equation  $\text{density} = \text{mass}/\text{volume}$ , the unit of density is the unit of mass divided by the unit of volume and thus  $\text{kg}/\text{m}^3$ . Since velocity is defined by the equation  $\text{velocity} = \text{change in displacement in a straight line}/\text{time taken}$ , the unit of velocity is unit of distance/unit of time and so is metres/second, i.e.  $\text{m}/\text{s}$ . Since acceleration is defined by the equation  $\text{acceleration} = \text{change in velocity}/\text{time taken}$ , the unit of acceleration is unit of velocity/unit of time and thus metres per second/second, i.e.  $\text{m}/\text{s}^2$ .

Some of the derived units are given special names. Thus, for example, force is defined by the equation  $\text{force} = \text{mass} \times \text{acceleration}$  and thus the unit of force = unit of mass  $\times$  unit of acceleration and is  $\text{kg m}/\text{s}^2$  or  $\text{kg m s}^{-2}$ . This unit is given the name newton (N). Thus 1 N is 1  $\text{kg m}/\text{s}^2$ . The unit of pressure is given by the defining equation  $\text{pressure} = \text{force}/\text{area}$  and is thus  $\text{N}/\text{m}^2$ . This unit is given the name pascal (Pa). Thus 1 Pa = 1  $\text{N}/\text{m}^2$ .

Certain quantities are defined as the ratio of two comparable quantities. Thus, for example, strain is defined as change in length/length. It is expressed as a pure number with no units because the derived unit would be  $\text{m}/\text{m}$ . Note that  $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$ , etc. are trigonometric ratios, i.e. a ratio of two sides of a triangle, and so have no units.

### Example

Determine the unit of the tensile modulus  $E$  when it is defined by the equation  $E = \text{stress}/\text{strain}$  if stress has the unit of Pa and strain is a ratio with no units.

Unit of  $E = (\text{unit of stress})/(\text{unit of strain}) = (\text{Pa})/(\text{no unit}) = \text{Pa}$

## 1.4.1 Powers of ten notation

The term *scientific notation* or *standard notation* is often used to express large and small numbers as the product of two factors, one of them being a multiple of ten. For example, a voltage of 1500 V can be expressed as  $1500 = 1.5 \times 1000 = 1.5 \times 10^3$  V and a current of 0.0020 A =  $2/1000 = 2.0 \times 10^{-3}$  A. The number 3 or  $-3$  is termed the *exponent* or *power*.

## 1.4.2 Unit prefixes

Standard prefixes are used for multiples and submultiples of units, the SI preferred ones being multiples of 1000, i.e.  $10^3$ , or division by multiples of 1000. Table 1.1 shows commonly used standard prefixes.

**Table 1.1** Standard unit prefixes

Multiplication factor		Prefix	
1 000 000 000	= $10^9$	giga	G
1 000 000	= $10^6$	mega	M
1000	= $10^3$	kilo	k
100	= $10^2$	hecto	h
10	= $10$	deca	da
0.1	= $10^{-1}$	deci	d
0.01	= $10^{-2}$	centi	c
0.001	= $10^{-3}$	milli	m
0.000 001	= $10^{-6}$	micro	$\mu$
0.000 000 001	= $10^{-9}$	nano	n
0.000 000 000 001	= $10^{-12}$	pico	p



**Example**

Express the capacitance of  $8.0 \times 10^{-11}$  F in pF.

$1 \text{ pF} = 10^{-12} \text{ F}$ . Hence, since  $8.0 \times 10^{-11} = 80 \times 10^{-12}$ , the capacitance can be written as 80 pF.

**Example**

Express the tensile modulus of 210 GPa in Pa without the unit prefix.

Since  $1 \text{ GPa} = 10^9 \text{ Pa}$ , then  $210 \text{ GPa} = 210 \times 10^9 \text{ Pa}$ .

**1.5 Measurements**

In making measurements, it is necessary to select the appropriate instrument for the task, taking into account the limitations of instruments and the accuracy with which it gives readings. Thus if you need to measure the mass of an object to a fraction of a gram then it is pointless using a spring balance since such an instrument cannot give readings to this degree of precision. The spring balance might have a scale which you think you can interpolate between scale markings to give a reading of a fraction of a gram, but it is unlikely that the calibration of the instrument is accurate enough for such interpolations to have any great significance.

With an instrument there is a specification of the accuracy with which it gives readings. The term *accuracy* is used for the extent to which a result might depart from the true value, i.e. the errors it might have. *Error* is defined as:

$$\text{error} = \text{measured value} - \text{true value}$$

Accuracy is usually quoted as being plus or minus some quantity, e.g.  $\pm 1 \text{ g}$ . This indicates that the error associated with a reading of that instrument is such that true value might be expected to be within plus or minus 1 g of the indicated value. The more accurate the measurement the smaller will be the error range associated with a measurement. In some situations the error is specified in the form of:

$$\text{percentage error} = \frac{\text{error in quantity}}{\text{size of quantity}} \times 100$$

Thus, for example, a mass quoted as  $2.0 \pm 0.2 \text{ g}$  might have its error quoted as  $\pm 10\%$ . In the case of some instruments, the error is often quoted as a percentage of the full-scale reading that is possible with an instrument.

**Example**

An ammeter is quoted by the manufacturer as having an accuracy of  $\pm 4\%$  f.s.d. on the 0 to 2 A scale. What will the error be in a reading of 1.2 A on that scale?

The accuracy is  $\pm 4\%$  of the full scale reading of 2 A and is thus  $\pm 0.08 \text{ A}$ . Hence the reading is  $1.2 \pm 0.08 \text{ A}$ .

**1.5.1 Sources of error**

Common sources of error with measurements are:

**1 Instrument construction errors**

These result from such causes as tolerances on the dimensions of components and the values of electrical components used in instruments, which are inherent in the manufacture of an instrument and the accuracy to which the manufacturer has calibrated it. The specification supplied by the manufacturer for an instrument will give the accuracy that might be expected under specified operating conditions.

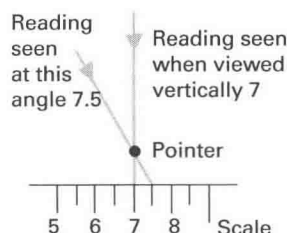


Figure 1.1 Parallax error

## 2 Non-linearity errors

In the design of many instruments a linear relationship between two quantities is often assumed, e.g. a spring balance assumes a linear relationship between force and extension. This may be an approximation or may be restricted to a narrow range of values. Thus an instrument may have errors due to a component not having a perfectly linear relationship. Thus in the specification supplied by a manufacturer for, say, a temperature sensor you might find a statement of a non-linearity error.

## 3 Operating errors

These can occur for a variety of reasons and include errors due to:

- (i) Errors in reading the position of a pointer on a scale. If the scale and the pointer are not in the same plane then the reading obtained depends on the angle at which the pointer is viewed against the scale (Figure 1.1). These are called *parallax errors*. To reduce the chance of such errors occurring, some instruments incorporate a mirror alongside the scale. Positioning the eye so that the pointer and its image are in line guarantees that the pointer is being viewed at the right angle. Digital instruments, where the reading is displayed as a series of numbers, avoid this problem of parallax.
- (ii) Errors may also occur due to the limited resolution of an instrument and the ability to read a scale. Such errors are termed *reading errors*. When the pointer of an instrument falls between two scale markings there is some degree of uncertainty as to what the reading should be quoted as. The worse the reading error could be is that the value indicated by a pointer is anywhere between two successive markings on the scale. In such circumstances the reading error can be stated as a value  $\pm$  half the scale interval. For example, a rule might have scale markings every 1 mm. Thus when measuring a length using the rule, the result might be quoted as  $23.4 \pm 0.5$  mm. However, it is often the case that we can be more certain about the reading and indicate a smaller error. With digital displays there is no uncertainty regarding the value displayed but there is still an error associated with the reading. This is because the reading of the instrument goes up in jumps, a whole digit at a time. We cannot tell where between two successive digits the actual value really is. Thus the degree of uncertainty is  $\pm$  the smallest digit.
- (iii) In some measurements the insertion of the instrument into the position to measure a quantity can affect its value. These are called *insertion errors* or *loading errors*. For example, inserting an ammeter into a circuit to measure the current can affect the value of the current due to the ammeter's own resistance. Similarly, putting a cold thermometer into a hot liquid can cool the liquid and so change the temperature being measured.

## 4 Environmental errors

Errors can arise as a result of environmental effects. For example, when making measurements with a steel rule, the temperature when the measurement is made might not be the same as that for which the rule was calibrated. Another example might be the presence of draughts affecting the readings given by a balance.

### Example

An ammeter has a scale with markings at intervals of 50 mA. What will be the reading error that can be quoted with a reading of 400 mA?

The reading error is generally quoted as  $\pm$  half the scale interval. Thus the reading error is  $\pm 25$  mA and the reading can be quoted as  $400 \pm 25$  mA.

## 1.6 Random errors

The term *random errors* is used for errors which can vary in a random manner between successive readings of the same quantity. This may be due to personal fluctuations by the person making the measurements, e.g. varying reaction times in timing events, applying varying pressures when using a micrometer screw gauge, parallax errors, etc., or perhaps due to random electronic fluctuations (termed noise) in the instruments or circuits used, or perhaps varying frictional effects.

Random errors mean that sometimes the error will give a reading that is too high, sometimes a reading that is too low. The error can be reduced by repeated readings being taken and calculating the mean (or average) value. The *mean* or *average* of a set of  $n$  readings is given by:

$$\text{mean } \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

where  $x_1$  is the first reading,  $x_2$  the second reading,  $\dots$   $x_n$  the  $n$ th reading.

The more readings we take the more likely it will be that we can cancel out the random variations that occur between readings. The *true value* might thus be regarded as the value given by the mean of a very large number of readings.

### Example

Five measurements have been taken for the resistance of a resistor: 20.1, 20.0, 20.2, 20.1, 20.1  $\Omega$ . Determine the mean value.

The mean value is obtained using the equation given above as:

$$\text{mean} = \frac{20.1 + 20.0 + 20.2 + 20.1 + 20.1}{5} = \frac{100.5}{5} = 20.1\Omega$$

## 1.7 Significant figures

When we write down the result of a measurement we should only write it to the number of figures the accuracy will allow, these being termed the *significant figures*. If we write 12.0 g for the mass of some object then there are three *significant figures*. However, if we quoted the number as 12, there are only two significant figures and the mass is less accurately known.

If we have a number such as 0.001 04 then the number of significant figures is 3 since we only include the number of figures between the first non-zero figure and the last figure. This becomes more obvious if we write the number in scientific notation as  $1.04 \times 10^{-3}$ . If we have a number written as 104 000 then we have to assume that it is written to six significant figures, the last 0 being significant. If we only wanted three significant figures then we should write the number as  $1.04 \times 10^5$ .

*When multiplying or dividing two numbers, the result should only be given to the same number of significant figures as the number with the least number of significant figures. When adding or subtracting numbers, the result should only be given to the same number of decimal places as the number in the calculation with the least number of decimal places.*

When the result of a calculation produces a number which has more figures than are significant, we need to reduce it to the required number of significant figures. This process is termed *rounding*. For example, if we have 2.05 divided by 1.30, then using a calculator we obtain 1.5769231. We need to reduce this to three significant figures. This is done by considering the fourth figure, i.e. the 6. If that figure is 5 or greater, the third figure is rounded up. If that figure is less than 5, it is rounded down. In this