SANGAKU PROOFS

A Japanese Mathematician at Work

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In memory of Yamada Hisao

Preface

This short book is an interpretive translation of all twenty-two problems in fascicle 14 and first four problems in fascicle 15 of *Saijō-ryū sanpō kantsū jutsu* (Inductive calculation techniques of the Saijō school) 最上流算法貫通術 (1793) by Aida Yasuaki 会田安明 (1747–1817).[1] Aida was a prolific mathematician, and this work, though not his most famous,[2] is worthy of careful study for several reasons.

First, each fascicle treats a difficult problem or group of related problems by explaining how to analyze their simplest cases and systematically work from their solutions to more complex variations.[3] This is, I believe, the reason for the word *kantsū* "induction" in the title.[4] I have therefore not altered Aida's organization of the material to accommodate modern standards of exposition.

Second, Problem 20 proves an elegant version of a celebrated theorem by the illustrious Lazare Carnot (1753–1823). I chose to translate all of fascicle 14 because it contains this particular problem, and continued with fascicle 15 to give a flavor of Aida's methodical approach in this work. Its fourth problem is an appropriate high note on which to end because it opens the door to

[1] Aida's personal name is sometimes given as Yasuakira or Anmei, the Sino-Japanese reading of the characters 安明. FASCICLE refers to a bundle of wide, back-folded sheets stitched along their free edges together with endpapers. Each fascicle is typically less than a centimeter thick, and many books consisted of several fascicles.

[2] One reason for this is that *Sanpō kantsū jutsu* survives only in manuscript, according to Kotera Hiroshi, whose bibliographic knowledge of *wasan* and personal collection of Edo period books on *wasan* is second to none.

[3] There are 63 numbered fascicles, but no. 30 has two parts, and there is a supplement to no. 36. Aida wrote a table of contents of 36 pages in 1805, which states that the main work consists of 65 fascicles, 2,168 back-folded sheets (hence 4,336 pages), and 1,472 problems. PDF scans of Aida's works can be accessed at http://repo.lib.yamagata-u.ac.jp/.

[4] The word *kantsū*, coined by Aida (Satō et al. 2009: 172–75), was perhaps contrived to make a *kanbun* pun (*kantsū jutsu* "inductive techniques" = *kan tsūjutsu* "penetrating common techniques [of others]," Kotera 2013: 24).

the study of how *wasanka* 和算家 (practitioners of Japanese mathematics, or *wasan*) interacted.

Carnot's version of the theorem states that the sum of the signed distances from the circumcenter of a triangle to the midpoints of its sides equals the sum of its circumradius and inradius, $R+r$. "Signed distances" are specified because one must distinguish three cases. In an acute triangle, $R+r=m_a+m_b+m_c$ (Figure 1a, left), in a right triangle, $m_a=0$, so $R+r=m_b+m_c$ (Figure 1a, right), and, in an obtuse triangle, $R+r=m_a-m_b+m_c$ (Figure 1b, left—note the minus sign). The Japanese counterpart of this theorem (Figure 1b, right) avoids the need to distinguish cases by referring to the perpendicular distances from the midpoints of the sides to the circumcircle.[5] Regardless of the kind of triangle circumscribed, $2R-r=s_a+s_b+s_c$.

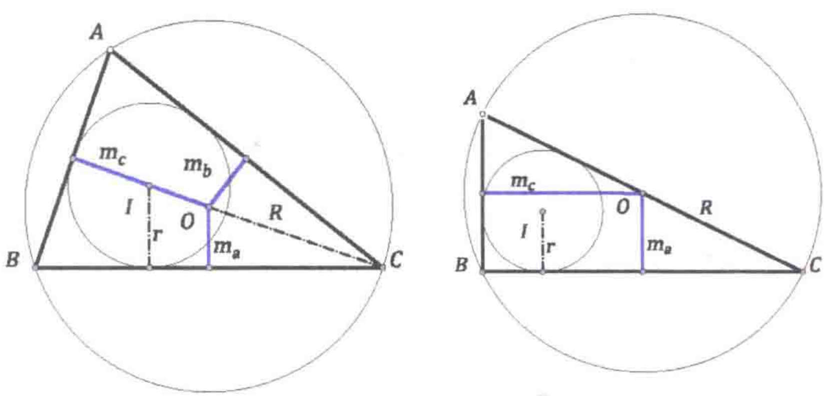

Figure 1a. Carnot's theorem, acute and right angle cases

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[5] A handy term for this distance is Latin SAGITTA "arrow" (plural *sagittae*). Y. Sawayama used it in 1906 in showing that the Japanese theorem generalizes to a result very close to the celebrated Japanese cyclic polygon triangulation theorem (Mikami 1910: 153; Honsberger 1985: 24–26).

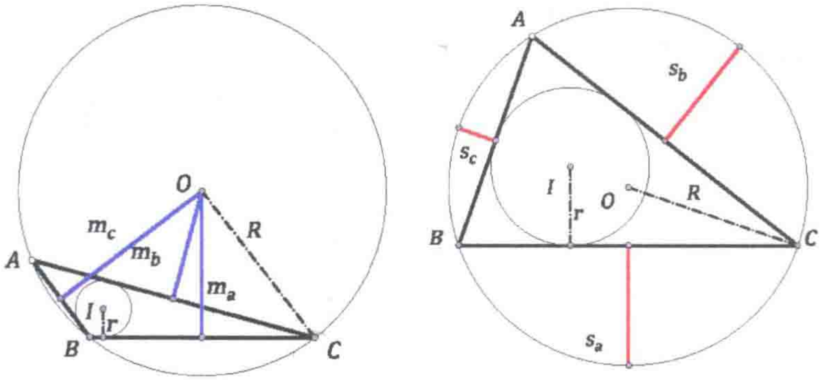

Figure 1b. Carnot's theorem, obtuse case, and Aida's variant

Third, Problem 21 suggests a solution using inversive geometry, though Aida does not pursue that line of attack. Quite a bit has been written about inversion, which could have been but was not used by *wasanka*; I offer my thoughts on this matter in Appendix A.

Fourth, Aida's career sheds light on many aspects of society, education, and communication in Japan during the late Edo period (1603–1867). A native of Yamagata in the Tōhoku (northeastern) region, he worked as a civil engineer for the Tokugawa government on water projects in the capital (Edo, present-day Tōkyō), but lost his position in 1786 when the shōgun Ieharu died and was succeeded by Ienari. This turn of events was in part precipitated by a public feud between Aida and the mathematician Fujita Sadasuke藤田貞資 (or定資 or定賢, 1734–1807). Fujita, who had refused to accept Aida as a student and was at that time the foremost exponent of the Seki school of *wasan*. Aida's criticism of his work and the establishment of his own Saijō school[6] forced other mathematicians to take sides, and left bad

[6] Fujita and Aida's own teacher, the remarkable Honda Toshiaki 本田利明 (1734–1821), were both academic descendants of famous Seki Takakazu 関孝和 (1642–1708) (Satō et al. 2009: 246). That did not stop Aida from choosing a name for his school that exploited the fact that Chinese characters typically take

feelings all around. This episode highlights the fact that mathematics was one of those fields of cultural development socially organized according to the quasi-familial *iemoto* 家元 or family master system.

Conceptually, the *iemoto* of some school of an art or craft was its founder; his school was (though not always literally) his family. The *iemoto* system subsumed a wide range of cultural endeavors including the tea ceremony, flower arrangement, formal etiquette, many individual theatrical and musical arts, and scholarly studies, in which determining excellence could be a highly subjective matter, as well as medicine, various martial arts, the games of *go* and *shōgi*, and mathematics, in which success could be defined somewhat more objectively. Consequently, while succession to the role of *iemoto* was ideally hereditary from father to son, it was not uncommon for an *iemoto* who lacked suitable heirs but wanted to continue the family name to adopt an outstanding extrafamilial disciple, particularly in the latter fields.[7] The ability to solve a difficult *wasan* problem independently was evidence of talent, and demanded recognition regardless of the youth, gender, or family occupation of the solver. While I do not know of a female mathematical prodigy in Japan comparable to, say, the famous Marie-Sophie Germain (1776–1831), there is ample evidence that local groups of devotees of *wasan* included women, bright children, and people not of the *samurai* class.[8] The word "local" here is important, for such groups met all over the country.[9] It is notable that, after losing his government job, Aida

multiple readings in Japanese. Mogami 最上 was the old name for the part of present-day Yamagata prefecture where Aida was born; *saijō* "highest" 最上 speaks for itself.

[7] This liberal attitude toward adoption carried over into the Meiji period (1868–1912) and beyond. It is easy to overemphasize the fact that Japan under the Tokugawa was a stratified neo-Confucianist class society, with *samurai* at the top ruling farmers, artisans, merchants, and various groups of outcastes. Adoptions were not uncommon when it suited the interests of the powerful.

[8] A popular book on *wasan* of 1775 entitled *Sanpō shōjo* (The girl calculator), discussed at length in Kotera 2009, has inspired at least one modern novel.

[9] In this sense, they were a type of the *shijuku* or private academy (Rubinger 1982) that played a critical role in sustaining higher education throughout the Edo period.

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travelled back to his native region, and trained many students who set up their own academies there. This part of Japan was, and sometimes still is, looked down upon by residents of the imperial and shōgunal capitals (Kyōto and Tōkyō), but has nonetheless been the birthplace of some of the most eminent leaders of 20th-century Japanese science, literature, and public affairs.[10]

Fifth, many devotees of recreational mathematics around the world know about the Edo period geometry problems, now widely called *sangaku* 算額, thanks to pathbreaking books coauthored by Fukagawa Hidetoshi and the late Daniel Pedoe, the British geometer John F. Rigby, and, most recently, Tony Rothman. The sheer graphic beauty of the figures representing these problems has been enough to earn them a place in the pages of *Scientific American*. Without detracting in any way from either these authors' contributions or the visual attractiveness of the figures, a deeper understanding of what the *wasanka* were actually doing requires correcting misconceptions that some readers may form from the visually attractive *sangaku*.

In the first place, not all *wasan* problems were geometric, and although the problems now known as *sangaku* are couched geometrically, their solutions involve, for the most part, elaborate exercises in algebra.[11] The question of CONSTRUCTING the figures is almost never discussed. Mathematicians like Aida must have had some theory of exact constructions, for otherwise they could not be sure that the more complicated figures they proposed actually existed. My own guess is that construction was a topic reserved for oral instruction by the *iemoto*.

Furthermore, all secondary sources comment on the obscurity of the writing style used in *sangaku* problems. Although portions of some *sangaku* texts are written in a form of Edo period vernacular Japanese, the material studied here and most other *wasan* texts are written in *kanbun*, that is, in literary Chinese as it had come to be used in Japan. Just as the academic Latin used by such mathematicians as Newton, Euler, and even Gauss until fair-

[10] To mention just a few, Nitobe Inazō, Tanakadate Aikitsu, Hara Takashi, Miyazawa Kenji, and Dazai Osamu all hailed from Tōhoku.

[11] For a helpful overview that situates *sangaku* within the larger Japanese mathematical tradition, see Ogawa 2001.

ly late in his career, was quite different from the classical language of Cicero and Vergil, so too the *kanbun* of Aida and others was a specialized variety of literary Chinese marked by numerous peculiarities, made all the more recondite by the obsessive desire of *wasanka* to write every sentence with as few characters as possible. And just as many people who can read and understand the translations of Newton or Euler in modern languages would be hard put to deal with the originals, so too most well-educated Japanese today find *wasan* texts cryptic. Yet, linguistically speaking, they are not excessively hard once one understands the conventions they employ. Indeed, the difficulties that chiefly stymied me in doing this translation were slips of the brush, which could only be detected and confidently corrected when I worked through the solution of each problem. As previously noted, though many Edo period *wasan* books were printed from woodblocks, the Yamagata University Library copy of *Sanpō kantsū jutsu* is handwritten—whether by Aida himself or by a student is unclear.

Manuscripts and woodblock-printed books[12] are important sources of information about *wasan*, not just the surviving *sangaku*, or plaques, after which geometry problems of the kind discussed are now commonly known. *Sangaku* "framed calculations" 算額 were, strictly speaking, a kind of *ema* "votive plaque" 絵馬, which one can still see hanging on the walls or under the eaves of (Shintō) shrines and (Buddhist) temples all over Japan.[13] The custom is to inscribe a wish on a plaque and hang it in a venerated place, later adding words of thanks on the reverse if the wish is granted. Nowadays, one sees hundreds of small wooden plaques hung up by students praying to pass college entrance examinations, job-seekers, the lovelorn, people hoping that a relative will recover from an illness, and so on. The sometimes quite large and beautifully colored *sangaku* of the Edo period often contained multiple problems and were challenges to other *wasan*

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[12] Kornicki 1998 is the definitive work in English on the history of the book in Japan.

[13] *Ema* have to date been found in all the modern prefectures covering the domains (*han*) controlled by the daimyō of the Edo period except Kumamoto, Miyazaki, Shimane, and Tottori.

aficionados, and many Japanese amateurs hunt for, photograph, and sometimes restore them. But our knowledge of *wasan* is by no means limited to what these *sangaku* show, and there may be special insights into the history of education, communication, and literacy in Edo period Japan waiting to be discovered through the study of *wasan* books because of the technical nature of their subject matter.

Finally, an appreciation of the mathematical sophistication of late Edo period Japanese is important for understanding how and why Japan made such a rapid and successful transition in almost every sphere of life after the restoration of contact with the West. Most well-read students of Japan today are aware that Japanese technological and intellectual progress did not come to a virtual standstill when the Tokugawa adopted a policy of national seclusion (*sakoku* 鎖国) in 1635, yet the old idea lingers on that Japan was caught off-guard by Commodore Perry in the 1850s, and dashed, in a sometimes ridiculous mad scramble, to learn the ways of the West. One of the lessons of *wasan* is that Japan was in many ways well-prepared for its abrupt encounter with the powers of the Industrial Revolution. Trigonometric functions and the full theory of calculus were new but by no means a complete surprise to Japanese schooled in *wasan*. An interesting historical question is whether, by comparison, the Chinese, from whom the seeds of *wasan* had come, were as ready as the Japanese for the cultural upheaval that sustained contact brought to East Asia.

I would only add that this small study is also a personal labor of love. It is often noted in the secondary literature that *wasanka* thought about problems for months, sometimes years—"day and night," as the saying goes—until they solved them. In 2009, finding current-events magazines and news programs on public radio too depressing to endure while walking or riding the bus to and from work, I developed the habit of keeping an unsolved *sangaku* problem in my head, usually one mentioned by Fukagawa and Pedoe. Each problem was the intellectual equivalent of a tangy lemon drop, a guilty pleasure that could be unobtrusively indulged in at boring meetings or while having to wait idly for one reason or another. I would jot down good ideas I came up with on slips of paper until, at length, I had a solution. A solution found in this way brought me real joy. I collected and wrote up solutions and posted them on my webpage. As a result, I became acquainted with several people on-line, all of whom have asked to remain anonymous, but for whose suggestions, criticisms, and friendship I am deeply grateful.

I cannot, however, fail to mention Shimano Tatsuo and Kotera Hiroshi, who graciously met personally with me to talk about *kanbun* and *sangaku* while I was in Japan in November 2013 as a Visiting Professor at the National Institute for Japanese and Linguistics (NINJAL). Professor Shimano's webpage http://www5.ocn.ne.jp/~jyorin/index.htm, which offers many valuable links to information on *wasan*, contains carefully annotated transcriptions and *kundoku* interpretations of the *kanbun* prefaces to dozens of important *wasan* books. Kotera's outstanding website http://www.wasan.jp/index.html includes images of a huge number of surviving or restored *sangaku* and a vast archive of *wasan* materials. I sincerely hope that this small contribution inspires English-reading students of mathematics, Japan, or both—especially young ones—to acquire the linguistic and mathematical skills needed to pursue the highly rewarding study of the achievements of the Edo period *wasanka* that these fine scholars have generously shared with the world through the web.

My thanks also to Fukagawa Hidetoshi, whom I had the pleasure of getting to know in person after this book was in press.

Introduction

To understand the geometric connection between the two problems Aida analyzes in fascicle 14 of *Sanpō kantsū jutsu*, it helps to know two basic constructions, though Aida does not discuss them. In what follows, I will use these conventions: uppercase letters represent points, lowercase letters line-segment lengths. A circle with center O is designated (O). If P lies on its circumference and $OP = r$, we can include this information by writing either $(O)P$ or $(O)r$.

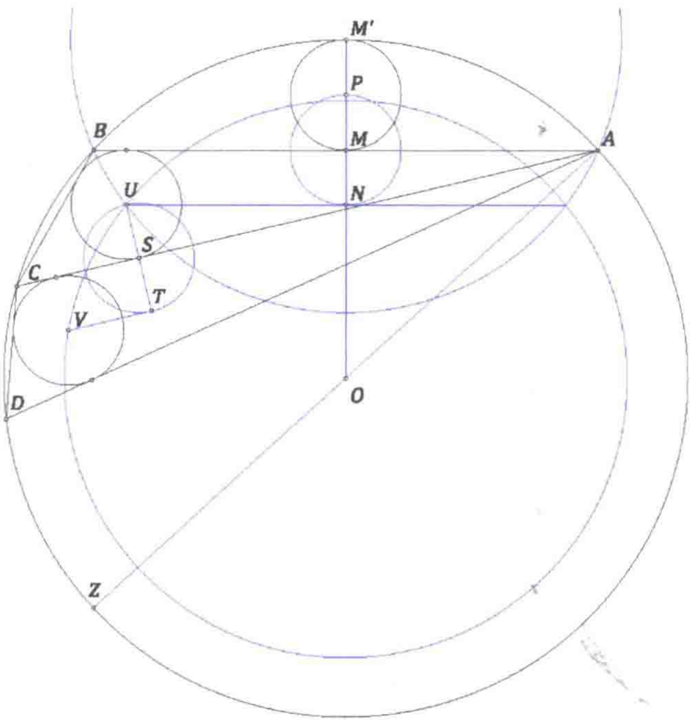

Figure 2. Basic constructions

Suppose that M is the midpoint of chord AB of circle $(O)A$ and that $(P)r$ touches AB at M and (O) at M', as shown in Figure 2.

1. To construct a triangle ABC circumscribed by (O) with incircle $(U)r$, find N, the second point where $(M)P$ meets OP, and draw a line l parallel to AB through N. Let U be the point where $(M')A$ meets l near B. Then $(U)r$ is tangent to AB, and the other tangents to (U) from A and B meet at C on (O).

2. To construct ACD with incircle $(V)r$ in the same circumcircle as ABC with incircle $(U)r$, find T, the second point where $(S)r$ meets US, and draw l parallel to AC. Then V is the point where $(O)U$ mets l near C.

The validity of the first construction follows from two lemmas:

1. The bisector of angle A in triangle ABC with circumcircle (O) passes through the midpoint of the arc BC that does not include A.

2. If M is the midpoint of an arc subtended by chord BC in circle (O), then the arc of $(M)B$ inside (O) is the locus of the incenters of all the triangles with side BC circumscribed by (O).

The second construction follows from the fact that the distance from the circumcenter to the incenter of any triangle is $\sqrt{R(R-2r)}$, where R is the circumradius and r is the inradius. Hence, for constant R and r, the incenters of the triangles all lie on a circle, and one can construct a fan of as many of them as one pleases. In Figure 2, we just show two.

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