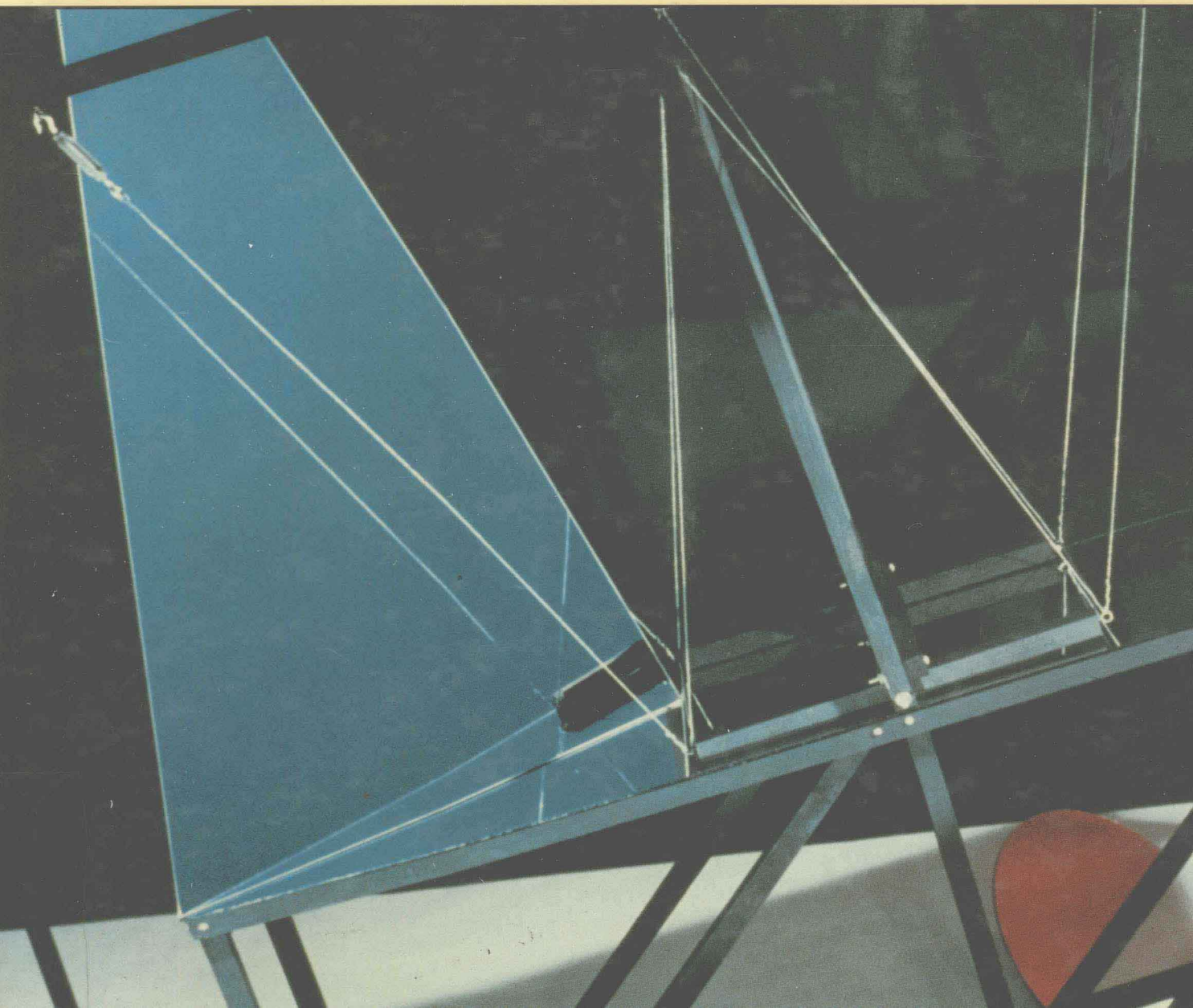
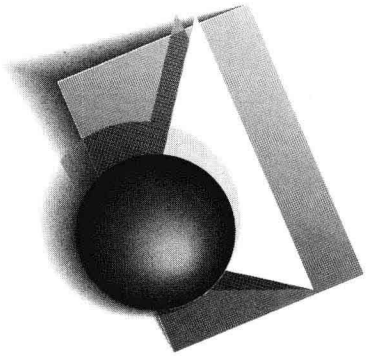
STANLEY I. GROSSMAN

COLLEGE TRIGONOMETRY

SECOND EDITION





College Trigonometry

Second Edition

Stanley I. Grossman

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To Aaron, Kerstin, and Erik

One learns by doing the thing; for though you think you know it, you have no certainty until you try.
SOPHOCLES

Preface

During the past few years the study of trigonometry has become increasingly important for students in almost every academic discipline for at least two reasons. First, periodic phenomena in biology, physics, and economics are described in terms of the trigonometric functions. Second, trigonometry is an indispensable prerequisite for calculus.

Many different kinds of students study trigonometry in college, but they fall mainly into two groups: recent high school graduates and returning students who may have been out of school for many years. Many students in both groups look upon trigonometry with less than unbridled enthusiasm. The comments ''I never liked math'' and ''I haven't seen any math for years'' are heard often by every trigonometry instructor.

College Trigonometry, Second Edition, contains all the trigonometric material that is needed to meet the requirements for other college disciplines and to satisfy prerequisites for further mathematical study. I have taught this topic many times at the University of Montana — a university with large numbers of both recent high school students and nontraditional students (the average age of our students is 27). Consequently, I wrote this book with the needs and concerns of these students in mind.

First, I have made the material more accessible by providing large numbers of examples with every step included. There should be no mystery in algebraic computations and this book has none. Second, I have provided many realistic applications so that students will understand that they are studying trigonometry because it is useful to them, not because it is a requirement for a degree. Finally, I have attempted to make the material interesting. In a number of cases I have included historical notes that should add spice to the subject.

Above all, it is my goal that students will both learn to appreciate the value of trigonometry and, most importantly, gain the confidence to succeed in their mathematical studies. I really believe that every university student can learn mathematics if he or she works hard doing problems and is given sufficient help and encouragement.

Changes in the Second Edition

I have made a number of changes which, I believe, will make this book more effective as a teaching tool. Here are the major ones:

1. Captions: Captions have been added to every example and figure in the text. This will make it easier for students to determine exactly what is being done in each example and will be very helpful when tackling the problem sets. The captions on figures clearly describe exactly what each figure illustrates.

2. Readiness Checks: Each problem set begins with up to five "Readiness Check" problems. These *true-false* and *multiple-choice* problems are intended to test students' understanding of the material they have just read and do not involve difficult computations. The student who can solve these is ready to tackle the exercise set that follows. Answers to the Readiness Check problems are given at the bottom of the last page on which they occur in each section — not at the back of the book.

3. New Exercises: The second edition has about 3,200 exercises — about 1,000 more than in the first edition. In addition, about 400 drill problems from the first edition have been replaced here.

4. New Graphs: This edition has almost 800 figures, about double the number of the previous one. These are used in new ways as well. See the discussion under "Figures" on page x.

5. New Introduction to the Trigonometric Functions: I changed the way I introduce the trigonometric functions. Now all six functions are defined with reference to a right triangle in Section 2.2. This is the way they were probably introduced in high school and should be more familiar to the student. In Section 2.3 I then define the sine and cosine functions using a circle of radius 1 and then extend this definition to circles of radius r (on page 80). Finally, I give the circular definitions of the other four trigonometric functions in Section 2.5. I believe that this change will make the study of trigonometry easier and more natural to students.

6. Analytic Geometry: A new chapter (Chapter 5) entitled Conic Sections and Parametric Equations has been added. Topics include the parabola, the ellipse, the hyperbola, translation and rotation of axes, and parametric equations. There are many applications of conics in the real world.

7. Two New Sections: In addition to the new chapter, there are two new sections:

a. The Dot Product of Two Vectors, Section 4.6.

b. Equations Involving Exponential and Logarithmic Functions, Section 7.5.

8. Use of the Graphing Calculator: Many students now have access to calculators that can, if used properly, provide accurate graphs of a great number of functions. In Appendix A I have shown students how to use their calculators effectively to draw graphs of functions, sketch conic sections and polar graphs, find zeros of polynomials, and solve other types of equations. Example 14 on page A.23 shows the limitations of such calculators by discussing a

polynomial whose graph *cannot* accurately be sketched on a calculator. The appendix is written to be used with any calculator now available.

In nine sections within Chapters 3, 5 and 7, I have added problems that are intended for solution on a graphing calculator. I made the deliberate choice to limit the use of the graphing calculator to those sections where it is appropriate, rather than to integrate it throughout the text. This gives the instructor an option. I stress that ownership of a graphing calculator is *not required* for use with this book.

Organization

Chapter 1 is introductory and is intended as a review of the most important topics in college algebra. To understand trigonometry, it is necessary to know something about functions and their graphs. Therefore, a review of the material in Chapter 1 will help the student who has not been exposed to it recently.

The trigonometric functions are introduced in Chapter 2. As described above, the six trigonometric functions are defined using a right triangle in Section 2.2. In Section 2.3 $\sin\theta$ and $\cos\theta$ are first defined in terms of the unit circle and then redefined using a circle of radius r. This section also shows that the triangular and circular definitions give the same values if θ is an acute angle. Section 2.8 contains a number of important applications of right triangle trigonometry. These applications follow a discussion of inverse trigonometric functions (Section 2.7) because many applied problems cannot be solved if one does not know how to compute inverse trigonometric values.

Chapter 3 discusses trigonometric identities, graphs, and equations. There are now *two* sections on graphs involving sine and cosine functions (Sections 3.6 and 3.7). In the graphing sections as well as in Section 3.5, students are asked to solve problems on a graphing calculator.

Chapter 4 is devoted to applications of trigonometry. The central chapters of the book (Chapters 2, 3, and 4) contain five sections that are filled with unique and interesting applications (Sections 2.8, 3.3, 4.3, 4.5 and 4.6).

Chapter 5 is a new chapter on analytic geometry. Topic coverage was described under item #6 on the previous page.

Chapter 6 contains an introduction to complex numbers. Sections 6.2 and 6.3, on polar form and complex roots, present an important application of the two basic trigonometric functions.

Exponential and logarithmic functions are described in Chapter 7. Section 7.6 contains a large and diverse number of realistic applications of exponential growth and decay.

Features

Examples As a student, I learned algebra from seeing examples and doing exercises. There are 360 examples in this book. The examples include all the necessary steps so that students can see clearly how to get from "A" to "B." In many instances explanations are highlighted by colored notes to make steps easier to follow.

Exercises The text includes about 3,200 exercises — both drill and applied. More difficult problems are marked with an asterisk (*) and a few especially difficult ones are marked with two (**). In my opinion, exercises provide the most important tool in any undergraduate mathematics textbook. *If you don't work problems, you won't learn the mathematics*. Or, to quote a button popular at mathematics meetings,

MATH IS NOT A SPECTATOR SPORT.

Readiness Check Problems Each problem set contains multiple-choice and true-false questions that require relatively little computation. Answers to these problems appear on the bottom of the page on which the last such problem in the set appears. They are there to test whether the student understands the basic ideas in the section, and they should be done before tackling the more standard problems that follow.

Chapter Review Exercises At the end of each chapter I have provided a collection of review exercises. Any student able to do these exercises can feel confident that he or she understands the material in the chapter.

Chapter Summary Outlines A summary of the most important facts discussed in each chapter appears at the end of that chapter. Students should find these summaries useful, especially when studying for a test.

Applications We study trigonometry because of its great utility. This book has a large number and variety of applications. A list of these applications appears on pages xix–xx.

Figures There are approximately 800 figures in this book. Many of these appear in a standard way — as graphs of functions. Others are used in the problem sets to help the student understand how graphs change as functions change. There are also a number of new figures attached to applied problems to help the student visualize what he or she is expected to solve.

Use of the Graphing Calculator Students with access to a calculator that can draw graphs can learn how to use it more effectively by consulting Appendix A at the back of the book. For more details on this feature, see item #8 on page viii.

Warnings An important part of the teaching process is helping students to avoid making mistakes, especially those that are commonly made. In eighteen places in the book I provide warnings that illustrate common errors. Each warning illustrates the mistake and shows how it can be avoided.

Use of the Calculator Virtually all college and university students own or have access to a hand calculator. Problems that were computational monstrosities 20 years ago have become fairly easy with the aid of a calculator. I have used the calculator in many examples and have suggested its use in a number of exercises. Examples and problems that require the use of a calculator are marked with a  .

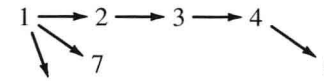
Precalculus Many students taking trigonometry will go on to study calculus. In several places in the book I have provided examples that do arise in calculus and have labelled them as such.

Focuses Studying trigonometry can be fun. In this book students can read about real people from the history of mathematics in the sketches that are headed "Focus on . . ." In these thirteen focuses students can learn, for example, about the first known instance of irrational behavior (p. 2), the history of π (p. 6), the origin of the trigonometric functions in Greece and India (p. 86), and the astonishing use of trigonometry by the Babylonians (p. 74). One focus beginning on page 411 contains no history but, rather, asks students to think about the assumptions inherent in a mathematical model.

Accuracy The success of a mathematics textbook largely depends on its accuracy. Galleys and page proofs were carefully checked for accuracy by me and four other mathematicians: Lynne Kotrous and Ray Plankinton at Central Community College, Platte Campus, in Nebraska, Paul Allen at the University of Alabama and Bruce Sisko at Belleville Area College. George Bradley at Duquesne University wrote many of the replacement drill problems and provided solutions to them. Lynne Kotrous, Ray Plankinton, and I solved all the odd-numbered problems in the book. Finally, all three of us proofread the typeset answers to make sure they were accurate.

The result is a book and answer section that is as clean as is within human ability to compile. However, if you do find an error in an answer or in the text, please send it to the publisher or to me; it will be corrected in the next printing.

Chapter Interdependence The following chart indicates chapter interdependence — that is, which later chapters depend on the student's having mastered earlier material.



Supplements

The answers to most odd-numbered problems appear at the back of the book. In addition, the following instructional aids are available from the publisher:

A **Student Solutions Manual** prepared by George Bradley and Daniel Barbush at Duquesne University contains chapter summaries and detailed solutions for all odd-numbered problems.

An **Instructor's Manual With Transparency Masters** also prepared by George Bradley and Daniel Barbush provides solutions for all the even-numbered problems and transparency masters of key figures from the text.

A **Computerized Test Bank,** of over 1200 multiple choice and open ended questions, prepared by Jan Wynn at Brigham Young University is available for IBM, Macintosh and Apple II computers. A **Printed Test Bank** of these questions is also available.

A & T Software is a computer software package referenced to *College Trigonometry*. Both interactive and tutorial, this software is available for use with IBM and compatible computers.

Videotaped Lectures prepared by Pat Stanley and Becki Bergs at Ball State University covers all the chapters of *College Trigonometry*. These lectures are referenced directly to the sections of this text.

A **Graphing Calculator Supplement** is available for purchase. Written by James Angelos of Central Michigan University, it explains how to use the Casio and TI graphing calculators and uses examples from this text.

An additional **Graphing Calculator Supplement,** written by Iris Fetta of Clemson University, also available for purchase, explores the use and value of the TI-81 graphing calculator.

Acknowledgements

Most of us really don't know what a book is like until it's been used in class and we get comments on how it works. I am grateful to the following individuals for their helpful comments on this second edition:

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A few problems in this book first appeared in *Mathematics for the Biological Sciences* (New York: Macmillan, 1974) written by James E. Turner and me. I am grateful to Professor Turner for permission to use this material.

I would also like to thank Eric Hays, of Hellgate High School in Missoula, Montana, for providing some of the navigation and force problems that appear in Chapters 2 and 4.

I wrote a great deal of this book while I was a research associate at University College London. I am grateful to the Mathematics Department at UCL for providing office facilities, mathematical suggestions, and, especially, friendship, during my annual visits there.

The book was produced by York Production Services in York, Pennsylvania. I wish to acknowledge the spectacular job done by York, in general, and by my Production Editor Kirsten Kauffman, in particular.

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Stanley I. Grossman

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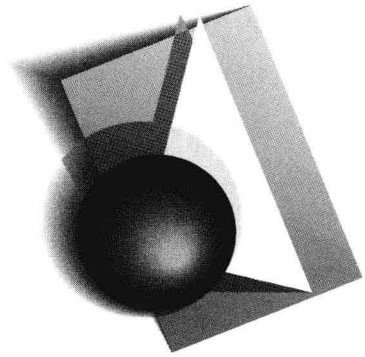

Chapter 1

A Review of Algebra

The study of trigonometry involves a number of important ideas in algebra, including the related concepts of functions and graphs. The purpose of this introductory chapter is to review these ideas.

1.1 The Real Number System

The collection of real numbers, denoted by $\mathbb{R}$, consists of the sets of natural numbers, integers, rational numbers, and irrational numbers. Real numbers can be represented on a **number line** in such a way that each point corresponds to exactly one real number and each real number corresponds to one point on the line. The number 0 (zero) is placed. Then the positive real numbers are placed at regular intervals to the right of 0 and the negative real numbers are placed at regular intervals to the left of 0.

The **natural numbers** (also called **positive integers** or **counting numbers**) are the numbers of counting: 1, 2, 3, 4, . . . (the three dots indicate that the string of numbers goes on infinitely). The number 2 is placed one unit to the right of 1 on the number line, the number 3 one unit to the right of 2, and so on. The natural numbers are denoted by N.

The **integers** consist of the natural numbers, their negatives (called the **negative integers**), and the number 0. The collection of integers is denoted by Z.

In Figure 1, we represent the integers 0, ± 1, ± 2, ± 3, and ± 4 on a **number line.**

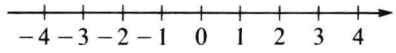

Figure 1 The part of the number line that includes the integers from -4 to 4.

A **rational number** is a real number that can be written as the quotient of two integers, where the integer in the denominator is not zero.