LATENT VARIABLES AND FACTOR ANALYSIS

VOLUME III

SAGE BENCHMARKS IN SOCIAL RESEARCH METHODS

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Alternative Approaches to Latent Variables

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Similar to EFA, PCA is a commonly used method for data reduction based on the re-dimensionalization of variables. Although EFA is a measurement model in which the observed data are modeled as the reflections of unobserved latent variables plus error (see Section 4), PCA is not strictly speaking concerned with measurement but is instead focused on data reduction: the transformation of a large number of observed variables into a smaller number of relevant latent variables. In PCA the observed variables are modeled as a straightforward mixture of the latent variables. The first three articles in this section all contrast PCA explicitly to EFA. Velicer and Jackson (1990) review the mathematical foundations of both techniques and argue that in practice the two techniques are broadly similar and largely interchangeable. But because PCA differs from EFA in its treatment of error, strong, clear components in PCA are usually the same as the strong, clear factors that would result from EFA. Where error predominates, the two techniques can produce widely divergent results. This is confirmed by Snook and Gorsuch (1989) in a Monte Carlo study comparing the performance of EFA and CFA in extracting signals in simulated data. With large amounts of data the two techniques converge. But even then they show that – when the EFA is in fact the true model – PCA results systematically overestimate factor loadings. Again, given the fact that PCA models ignore measurement error this is not surprising. Widaman (1993) is a review and reanalysis of Velicer and Jackson (1990) and Snook and Gorsuch (1989). Widaman concludes that PCA should not be used for measurement as such, which again is no surprise given that PCA is not primarily designed for measurement (even though it is often used for measurement, in many cases seeming inadvertently). Vyas and Kumaranayake (2006) apply PCA to household wealth data to produce a socioeconomic status (SES) variable. It could be argued that EFA or even CFA would have been more appropriate for this purpose, but the article provides a simple empirical run-through of how to use the technique.

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Component Analysis versus Common Factor Analysis: Some Issues in Selecting an Appropriate Procedure

Wayne F. Velicer and Douglas N. Jackson

Factor analysis and component analysis are two broad classes of procedures that share a common goal: to reduce a set of p observed variables to a set of m new variables ($m < p$). This reduction serves two different purposes. First, the pattern matrix **A** can be interpreted to describe the relationship between the original variables and the new variables. Second, scores for the m new variables can be derived to replace the original observed scores. These scores can be interpreted or employed as the basis of a subsequent analysis. A number of writers have drawn careful distinctions between these two broad classes of methods. After reviewing the distinctions, most writers recommend the use of factor analysis. In spite of these recommendations, principal component analysis, a type of component analysis, remains the most widely employed of the techniques (Glass & Taylor, 1966; Pruzek & Rabinowitz, 1981). The purpose of this article is to review some of the issues involved in the selection of one of these two classes of procedures.

The first section will describe the algebraic similarities and differences at the sample level. Four issues will then be discussed in the context of the analysis of data: (a) The degree of similarity between alternative solutions, (b) issues relating to the number of components retained, (c) problems with improper solutions in factor analysis, and (d) comparisons with respect to computational efficiency. Three broader theoretical issues will also be considered: (a) The factor indeterminacy problem, (b) the distinction between

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exploratory and confirmatory analysis, and (c) the contrast between latent and manifest variables. This article will not consider these issue(s) within the broader context of structural analysis, primarily because the theory is still developing and the limited number of empirical examples do not provide an adequate basis for discussing these issues. However, the choice between methods discussed in this article seems to involve the same issues (Bentler, 1980; Fornell & Bookstein, 1982) that would be involved in selecting between a factor analysis based approach such as LISREL (Jöreskog, 1970, 1978, 1981), EQS (Bentler, 1985) or COSAN (McDonald, 1978, 1980) and a component analysis based approach such as the PLS (Partial Least Squares) approach (Wold, 1966, 1982). It is clear that, at the present time, general procedures for structural analysis based on the factor analysis model are better developed and have been employed more extensively.

Algebraic Relations

An examination of the mathematical representations of the two approaches will serve to highlight the similarities and differences. This section will present only a brief description. Readers interested in a more detailed and rigorous derivation are referred to standard texts such as Gorsuch (1983), Lawley and Maxwell (1971), McDonald (1985), Meredith and Millsap (1985), Mulaik (1972) or the Schönemann and Steiger (1976) article on component analysis.

The basic problem for both factor analysis and component analysis involves the description of a set of p random variables $\eta' = (Y_1, Y_2, \ldots, Y_p)$ in terms of $m \leq p$ random variable $\zeta = (X_1, X_2, \ldots, X_m)$ and p residuals $\varepsilon' = (e_1, e_2, \ldots, e_p)$. Both factor analysis and component analysis can be expressed as a model of the form

$$\eta = \mathbf{A}\zeta + \varepsilon \tag{1}$$

where $\mathbf{A}$ is the $p \times m$ multiple regression pattern for optimally predicting the p variates in η from the m variates in ζ. In component analysis, ζ must be expressable as $\zeta = \mathbf{A}'\eta$, in which case the variance-covariance matrix of ε cannot be diagonal and of full rank. In factor analysis, the variance-covariance matrix of ε must be diagonal and of full rank.

Alternative methods result in different sample estimates of $\mathbf{A}$, the pattern matrix. For a sample of size N, the observed data can be represented as an $N \times p$ matrix $\mathbf{Y}$. When $\mathbf{Y}$ is expressed in deviation score form, a $p \times p$ sample covariance matrix $\mathbf{C}$ can be represented as

$$\mathbf{C}_{yy} = \mathbf{Y}'\mathbf{Y}/(N-1). \tag{2}$$

Component analysis will be defined as any eigen decomposition of a covariance matrix. The most widely employed version is principal component analysis where all observed variables are transformed to standard score form and the covariance matrix is now the $p \times p$ matrix of correlations, $\mathbf{R}$.

Principal component analysis can be expressed as

$$\mathbf{R} = \mathbf{L}_c \mathbf{D}_c^2 \mathbf{L}_c' \tag{3}$$

where $\mathbf{D}_c^2$ is an $m \times m$ diagonal matrix containing the largest m eigen values and $\mathbf{L}_c$ is the $p \times m$ matrix containing the corresponding eigen vectors. This approach is sometimes referred to as truncated principal component analysis, indicating that for some applications all p components may be retained. Alternatives to principal component analysis are described by Bartholomew (1984, 1985), Meredith and Millsap (1985) and Schönemenn and Steiger (1976).

A variation of principal component analysis that has received considerable attention is image component analysis (Guttman, 1953; Harris, 1962), which can be expressed as

$$\mathbf{S}^{-1}\mathbf{R}\mathbf{S}^{-1} = \mathbf{L}_1 \mathbf{D}_1^2 \mathbf{L}_1' \tag{4}$$

where $\mathbf{L}_1$ and $\mathbf{D}_1^2$ are the matrices of eigen vectors and values, respectively, and

$$\mathbf{S}^2 = \text{Diag}^{-1}\,(\mathbf{R}^{-1}). \tag{5}$$

The principal component pattern can be expressed as

$$\mathbf{A}_C = \mathbf{L}_c \mathbf{D}_c \tag{6}$$

and the image component pattern can be expressed as

$$\mathbf{A}_1 = \mathbf{S}\mathbf{L}_1 \mathbf{D}_1 \tag{7}$$

Factor analysis, as we have seen, can be viewed as a linear model relating manifest and (using the term loosely) latent variables with an important constraint on the variance-covariance structure of the latent variables. However, this version of the factor model fits a given η if and only if the variance-covariance matrix of the η variables may be written in the form

$$\Sigma_{\eta\eta} = \mathbf{A}\mathbf{A}' + \mathbf{U}^2. \tag{8}$$

Consequently, an alternative view of the factor analysis model is available. Specifically, the factor model fits η if and only if there exists a positive definite, diagonal matrix $\mathbf{U}^2$ which, when subtracted from $\Sigma_{\eta\eta}$ leaves a residual that is Gramian and of rank m.

Regardless of whether or not the factor model fits $\Sigma_{\eta\eta}$ in the population for a small number of factors, it is a virtual certainty that it will not fit a sample variance-covariance matrix $\mathbf{C}_{yy}$ perfectly. Hence, when working with a sample variance-covariance matrix, we fit the model

$$\mathbf{C}_{yy} = \mathbf{A}\mathbf{A}' + \mathbf{U}^2 + \mathbf{E} \tag{9}$$

where $\mathbf{E}$ is to be kept *as small as possible* by choosing $\mathbf{A}$ and $\mathbf{U}^2$ to minimize a loss-function. Different loss-functions yield different solutions, which often are subsumed under the generic name *common factor analysis.* Maximum Likelihood Factor Analysis (MLFA), which results when $\mathbf{A}$ and $\mathbf{U}^2$ are chosen to minimize the maximum likelihood loss function, will be considered as the exemplar of this approach. Alternative derivations from different rationale have produced the same results (Howe, 1955: Lawley, 1940, 1941; Rao, 1955) and comparative studies (Browne, 1968) support this preference. If the diagonality constraint ($\mathbf{A}'\mathbf{U}^{-2}\mathbf{A} = \text{Diagonal}$) is employed for MLFA, the result can be expressed as an eigen decomposition

$$\mathbf{U}^{-1}\mathbf{R}\mathbf{U}^{-1} = \mathbf{L}_\text{F}\mathbf{D}_\text{F}^2\mathbf{L}_\text{F}' \tag{10}$$

where $\mathbf{L}_\text{F}$ and $\mathbf{D}_\text{F}^2$ are the matrices containing, respectively, the eigen vectors and eigen values, and the corresponding factor pattern can be represented as

$$\mathbf{A}_\text{F} = \mathbf{U}\mathbf{L}_\text{F}(\mathbf{D}_\text{F}^2 - \mathbf{I})^{1/2}. \tag{11}$$

An examination of the two methods of analysis will demonstrate that they differ only to the extent that factor analysis involves a reduction of the variance or diagonal elements of the covariance matrix. Component analysis permits no operation on the diagonal elements that will not also affect the covariance or off-diagonal elements. It should be noted that the values of $\mathbf{U}^2$ are typically unknown and must be estimated from the data. Estimation in factor analysis always employs an iterative procedure where $\mathbf{U}^2$ is estimated by a numerical minimization algorithm, while $\mathbf{A}$ is estimated in closed form at each value of $\mathbf{U}^2$ found by the algorithm. The procedure continues until convergence occurs. The presence of the $\mathbf{U}^2$ matrix is also the source of the indeterminacy problem (Mulaik & McDonald, 1978; Steiger & Schönemann, 1978). The extent to which the algebraic differences in the two approaches result in practical differences in the solutions will be discussed in the next section.

Similarity of Solutions

An examination of the algebraic representations of the two methods of analysis has served to highlight the differences between them. However, when the same number of components or factors are extracted, the results from different types of component or factor analysis procedures typically yield highly similar results. Discrepancies are rarely, if ever, of any practical importance in subsequent interpretations.

The comparisons have either focused on the similarity between the derived scores produced by the two approaches or the similarity between the patterns produced by the two methods. Studies have involved both real data examples and artificial data sets.

Velicer (1976b) calculated the correlation between image component scores, principal component scores, and three alternative estimates of factor scores. Data from nine well known studies were employed. The correlations between the different types of scores were typically close to unity. More recently, Fava and Velicer (in press) confirmed these results in an extensive simulation study. The use of simulated data permitted a systematic variation of the degree of saturation, the sample size, and the population structure underlying the sample correlation matrix. When the components/factors are well defined, that is, an adequate number of variables possessing at least a moderate loading (.60), the correlation between the alternative types of scores typically exceeded .99. The only exceptions occurred in the poorly defined cases, a combination of low loadings and few variables per component, and even in these cases the correlations typically exceeded .90.

Velicer (1977) employed the same nine data sets in a comparison of image component, principal component, and factor patterns. The patterns were compared after a varimax rotation, after a promax rotation, and after a procrustes rotation to maximum similarity. Two types of comparisons were performed: a direct loading-by-loading comparison of the patterns and a summary statistic defined on the matrix of differences between the patterns. Velicer concluded that the "... patterns produced by each of the three methods are remarkably similar. Rotation position has little effect on the degree of similarity" (p. 20).

Velicer, Peacock, and Jackson (1982) performed an extensive comparison of component, image, and factor patterns using simulated data sets. Under all conditions, the patterns produced by all three methods were highly similar, typically with differences only in the second decimal of the loadings. The degree of similarity increased as sample size and saturation increased. In other words, improvements in the quality of the data increased the degree of similarity.

This similarity is easily illustrated. Following the procedure described by Velicer et al. (1982), a target pattern was used to construct a population correlation matrix. A sample correlation matrix ($N = 108$), generated by the Montanelli (1975) program, was analyzed by principal component analysis, image component analysis, and maximum likelihood factor analysis. Table 1 presents the target pattern and the pattern produced by each of the three methods.

An examination of Table 1 will illustrate the high degree of similarity between the three patterns. Although empirical studies generally show that image and principal component analysis are slightly more similar to each other than to factor analysis, and Velicer et al. (1982) found that factor analysis fits better to the factor target and component analysis fits better to component target, these differences are so small as to be of no practical importance.

A somewhat different conclusion was reached by Snook and Gorsuch (1989). On the basis of a simulation study, they concluded that component and factor analysis give discrepant solutions, particularly when the number of variables (p) is small. The component analysis loadings are described as systematically *inflated.* We view their conclusions as incorrect for two reasons. First, the differences in the numeric size of the loadings was what would be expected on the basis of the algebraic differences described in the previous section. In effect, the equivalence of the unique variance of one

Table 1: Target pattern and sample patterns from three methods

	Target pattern						*Varimax rotated MLFA pattern*					
	1	*2*	*3*	*4*	*5*	*6*	*1*	*2*	*3*	*4*	*5*	*6*
1	.80	.00	.00	.00	.00	.00	.83	−.05	.02	−.13	−.07	−.01
2	.80	.00	.00	.00	.00	.00	.78	−.17	.02	.01	−.01	.04
3	.80	.00	.00	.00	.00	.00	.79	−.09	−.00	−.06	−.09	.07
4	.80	.00	.00	.00	.00	.00	.77	.08	.17	−.12	−.15	.04
5	.80	.00	.00	.00	.00	.00	.82	−.01	.10	−.11	−.06	−.04
6	.80	.00	.00	.00	.00	.00	.82	−.02	.07	−.03	−.04	.09
7	.00	.80	.00	.00	.00	.00	−.04	.81	.16	.02	−.05	.07
8	.00	.80	.00	.00	.00	.00	−.06	.79	.03	−.06	−.03	.11
9	.00	.80	.00	.00	.00	.00	−.04	.86	.01	.01	.05	−.00
10	.00	.80	.00	.00	.00	.00	−.08	.79	.04	.05	.10	.15
11	.00	.80	.00	.00	.00	.00	.01	.82	−.03	.03	−.01	.06
12	.00	.80	.00	.00	.00	.00	−.05	.79	−.02	.02	.04	−.05
13	.00	.00	.80	.00	.00	.00	.09	−.05	.76	.09	.05	.05
14	.00	.00	.80	.00	.00	.00	.08	.04	.81	.06	.00	.03
15	.00	.00	.80	.00	.00	.00	.04	.00	.84	.08	.05	−.04
16	.00	.00	.80	.00	.00	.00	.04	.13	.79	.09	.06	−.05
17	.00	.00	.80	.00	.00	.00	.07	.06	.78	.12	.03	.00
18	.00	.00	.80	.00	.00	.00	.04	−.03	.82	.02	.03	.07
19	.00	.00	.00	.80	.00	.00	−.16	.03	−.07	.79	.14	−.12
20	.00	.00	.00	.80	.00	.00	−.13	.06	−.13	.80	.01	.02
21	.00	.00	.00	.80	.00	.00	−.04	.02	−.02	.80	.03	−.09
22	.00	.00	.00	.80	.00	.00	.05	−.01	.00	.79	.02	−.09
23	.00	.00	.00	.80	.00	.00	−.13	−.05	.07	.81	.05	−.10
24	.00	.00	.00	.80	.00	.00	−.05	.03	−.03	.79	.04	−.03
25	.00	.00	.00	.00	.80	.00	−.01	.11	.05	.10	.93	−.01
26	.00	.00	.00	.00	.80	.00	−.09	−.02	.03	.08	.80	.02
27	.00	.00	.00	.00	.80	.00	−.12	−.02	.02	−.05	.82	.06
28	.00	.00	.00	.00	.80	.00	−.07	.09	−.01	−.15	.75	.07
29	.00	.00	.00	.00	.80	.00	−.14	−.04	−.01	.03	.80	−.00
30	.00	.00	.00	.00	.80	.00	.03	−.01	.08	−.04	.81	.07
31	.00	.00	.00	.00	.00	.80	.01	−.02	.05	.00	.12	.76
32	.00	.00	.00	.00	.00	.80	.07	−.02	−.09	.03	.03	.79
33	.00	.00	.00	.00	.00	.80	.06	.12	.08	−.06	.03	.79
34	.00	.00	.00	.00	.00	.80	.03	.12	−.07	−.19	.03	.76
35	.00	.00	.00	.00	.00	.80	.07	.10	.10	−.12	.03	.81
36	.00	.00	.00	.00	.00	.80	.07	.10	.09	−.12	.04	.81

	Varimax rotated PCA pattern						*Varimax rotated ICA pattern*					
	1	*2*	*3*	*4*	*5*	*6*	*1*	*2*	*3*	*4*	*5*	*6*
1	.85	−.04	.02	−.12	−.06	−.01	.86	−.04	.02	−.12	−.06	−.01
2	.82	−.17	.01	.02	−.01	.04	.80	−.16	.01	.01	−.01	.04
3	.82	−.09	−.00	−.05	−.08	.08	.82	−.09	−.00	−.05	−.08	.08
4	.80	.08	.17	−.12	−.14	.04	.79	.07	.17	−.11	−.14	.04
5	.84	−.00	.09	−.10	−.06	−.04	.86	−.00	.10	−.10	−.06	−.04
6	.85	−.01	.06	−.03	−.03	.08	.85	−.01	.06	−.02	−.03	.09
7	−.03	.83	.16	.02	−.05	.07	−.03	.84	.16	.02	−.06	.07
8	−.05	.82	.02	−.06	−.02	.11	−.05	.82	.02	−.06	−.03	.11
9	−.03	.87	.00	.00	.05	−.00	−.04	.88	.00	.00	.05	−.00
10	−.08	.82	.03	.05	.09	.15	−.08	.82	.03	.05	.10	.15
11	.01	.84	−.03	.02	−.01	.06	.01	.84	−.03	.02	−.00	.05
12	−.05	.82	−.02	.01	.04	−.06	−.04	.81	−.02	.01	.04	−.05
13	.09	−.04	.80	.09	.05	.05	.09	−.04	.80	.09	.05	.05
14	.07	.03	.84	.06	.00	.03	.07	.03	.83	.06	.00	.03
15	.03	.00	.86	−.08	.05	−.04	.03	.00	.86	−.08	.05	−.04
16	.03	.13	.82	−.09	.06	−.06	.03	.13	.82	−.09	.06	−.06
17	.06	.05	.81	−.12	.02	.01	.06	.06	.81	−.12	.02	.00
18	.03	−.02	.84	−.02	−.03	.08	.03	−.03	.86	−.02	−.03	.08
19	−.15	.02	−.06	.81	−.15	−.12	−.15	.02	−.06	.81	−.15	−.12
20	−.12	.05	−.13	.83	−.01	.03	−.12	.05	−.13	.82	−.01	.03
21	−.03	.01	−.02	.83	.03	−.09	−.03	.01	−.02	.94	.03	.05
22	.05	−.01	−.00	.83	.02	−.09	.05	−.01	−.00	.82	.02	−.05

	Varimax rotated PCA pattern						Varimax rotated ICA pattern					
	1	2	3	4	5	6	1	2	3	4	5	6
23	−.13	−.05	.07	.84	−.05	−.10	−.13	−.04	−.07	.84	−.05	−.10
24	−.04	.03	−.02	.83	.04	−.93	−.04	.02	−.02	.82	.04	−.03
25	−.00	.11	.05	.10	.86	−.01	.00	.11	.05	.11	.87	.01
26	−.08	−.02	.02	−.07	.83	.02	−.08	−.02	.02	−.07	.83	.02
27	−.11	−.01	.02	−.05	.85	.06	−.11	−.02	.02	−.05	.85	.06
28	−.07	.09	−.01	−.05	.80	.07	−.07	.08	−.00	−.05	.79	.07
29	−.14	−.04	−.00	.02	.83	−.00	−.14	−.04	−.01	.02	.82	−.00
30	.03	−.01	.07	−.03	.85	.07	.03	−.01	.07	−.03	.84	.07
31	.00	−.02	.04	.00	.12	.81	.00	−.02	.04	.00	.12	.79
32	.07	−.01	−.09	.04	−.04	.83	.07	−.01	−.09	.03	−.04	.83
33	.06	.12	.08	−.04	.03	.83	.06	.12	.07	−.05	.03	.83
34	.02	.11	−.07	−.19	.03	.80	.02	.11	−.07	−.19	.03	.79
35	.06	.10	.09	−.11	.03	.84	.07	.10	.09	−.11	.03	.85
36	−.03	.04	.02	−.11	.03	.84	−.03	.04	.02	−.11	.03	.85

variable must be included in forming the linear composite for a component analysis. A numeric example from Velicer et al. (1982) will serve to illustrate this. "Consider the case of six variables which all have loadings of .80 on a factor. The total common variance for the set is 3.84 and the unique variance is 2.16. The unique variance of one variable (.36) is added to the common variance for component analysis, for a total of 4.20. Divided among the six variables, this results in a loading for each of the six variables in component analysis of .837." (p. 387) This is the source of the differences reported by Snook and Gorsuch and the differences are, as expected, greater for small variable problems. However, there is no reason to believe that differences of the magnitude reported have any effect in practice on conclusions regarding which factor loadings are salient.

Second, the use of the word *inflated* is perjorative and misleading. It involves the implicit assumption that the factor loading is the *correct value.* It would be equally appropriate but similarly misleading to describe the factor loadings as *deflated.* A more appropriate procedure for assessing the stability of results under small variable conditions would be to compare each sample pattern to its corresponding population pattern, a procedure followed by Velicer and Fava (1987, 1990) which resulted in a very different set of conclusions.

In any case, it does not seem appropriate to select a procedure on the basis of the very small differences in the numeric values of the loadings. There is no reason to believe that the differences reported will result in differences in interpretation for the patterns. Arguments that the slightly larger loading are *better* or *worse* are not likely to convince anyone. None of the studies provide support for the persistent belief that the methods produce highly discrepant solutions. One explanation for the possible source of discrepant solutions may lie in the area of overestimation of the number of components, a topic that is discussed in the next section.

Number of Components (Factors) Retained

An area that has received too little attention is the determination of the number of components or factors to retain. Rigorous and accurate procedures for determining the number of factors are now available, but inaccurate procedures are still widely employed. If factor analysis is the procedure of choice, at least one of the following three situations is assumed to exist: (a) The number of factors is known a priori, (b) the asymptotic chi-square statistic will accurately determine how many factors to retain, or (c) the problem is trivial and of no interest. For component analysis a diversity of alternative procedures have been proposed, including Bartlett's (1950, 1951) test of the equality of the last $p-m$ eigen values, Cattell's (1966) scree procedure, Horn's (1965) parallel analysis procedure, Jackson and Morf's (1973) approach to the reliability of components, Kaiser's (1960) eigen value greater than unity rule, and Velicer's (1976a) minimum average partial (MAP) correlation procedure. The most widely employed of these is the Kaiser criterion. Indeed, some critics (Comrey, 1978) of component analysis assume that principal component analysis implicitly involves the use of the Kaiser rule.

The Kaiser rule has been criticized for retaining too many components (Browne, 1968; Cattell & Jaspers, 1967; Linn, 1968) and by one investigator, for retaining too few (Humphreys, 1964). Cliff (1988) has questioned the justification for this rule. Recent investigations of the accuracy of alternative methods for determining the number of components (Hakstian, Rogers, & Cattell, 1982; Zwick & Velicer, 1982, 1986) found that the Kaiser rule was the *least* accurate of the procedures studied. Typically, the number of components retained equalled one third of the number of original variables as determined by the Kaiser rule, irrespective of the actual number of components underlying the sample (Revelle & Rocklin, 1979; Zwick & Velicer, 1982; 1986). Because the Kaiser rule is the default value in most computer programs, overextraction has been a typical problem in many published studies. However, the problem of overextraction has received little attention.

Possibly the problem of overextraction has received little attention because of the advice given by textbooks in the area, ranging from the early work of Thurstone (1947) four decades ago to more recent references such as that of Cattell (1978), that overextraction is not a problem, only underextraction. Comrey (1978) recently took issue with this viewpoint, pointing out that overextraction followed by a varimax rotation will result in the last retained factors being inappropriately inflated at the expense of the earlier major factors, thus distorting the interpretation. In essence, unreplicable factors are created by degrading well-defined factors.

The number of components question may be related to the conflict over the similarity-dissimilarity of factor analysis and component analysis solutions. Dziuban and Harris (1973) found large differences between the patterns produced by alternative methods. The Kaiser rule was employed and