INTERMEDIATE ALGEBRA

A JUST-IN-TIME APPROACH

Alice Kaseberg

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Preface

INTERMEDIATE ALGEBRA: A JUST-IN-TIME APPROACH is based on my long-time involvement in mathematics education, extensive teaching experience at the junior high, high school, and community college levels, and degrees in business administration, mathematics, and engineering science. Although somewhat less formal and less dependent on symbolic manipulation than many books, this text does not make for an easy course. Students are asked to integrate ideas from algebra and geometry, to make connections among tabular, graphical, and symbolic information, and to apply mathematics to real-world settings.

"STUDENTS ARE ASKED TO INTEGRATE IDEAS FROM ALGEBRA AND GEOMETRY, TO MAKE CONNECTIONS AMONG TABULAR, GRAPHICAL, AND SYMBOLIC INFORMATION, AND TO APPLY MATHEMATICS TO REAL-WORLD SITUATIONS."

The college developmental algebra population comprises at least four different types of students: those who have never had algebra, those who have been out of school for a while and need to relearn forgotten math skills, those who are fresh out of high school but need to review, and those who failed algebra in high school. Each of these groups may benefit from an approach to algebra characterized by the following concerns:

- What purpose does algebra serve?
- How much can students reasonably retain?
- Why do students forget?
- Why do students fail algebra in the first place?

"STUDENTS WITH SOME ALGEBRA BACKGROUND...WILL BE ESPECIALLY SURPRISED TO FIND AN APPROACH THAT IS NOT A REHASH OF THEIR HIGH SCHOOL ALGEBRA."

Intermediate Algebra: A Just-in-Time Approach is intended for college students in any of the four categories above who have passed a placement test or completed an Introductory Algebra course. The text is sufficiently different in its approach to provide new learning and thinking experiences for students with some algebra background. These students will be especially surprised to find an approach that is not a rehash of their high school algebra. The text is also sufficiently complete in its presentation and practice of basic skills to accommodate the student with only an Introductory Algebra background.

PEDAGOGY

Objectives

The learning outcomes for each section are listed at the beginning of the section. They serve as summaries for students and instructors. If students get distracted by the complexities of the applications, the objectives return focus to what is important.

Warm-ups

The Warm-up at the beginning of each section is designed to serve as a class opener, reviewing important concepts and linking prior and current topics. Warm-ups tend to be skill-oriented; the problems generally connect to the algebra needed to solve text examples. I am indebted to Steve Givant, of Mills College, who introduced me to the technique of using key parts of the lecture as class warm-ups. In the classroom setting, warm-ups permit examples to flow more evenly, and in the textbook setting, they make reading easier for the student.

"EXAMPLES ARE SELECTED TO EMPHASIZE NOT ONLY THE CONNECTION BETWEEN MATH AND THE REAL WORLD BUT ALSO THAT BETWEEN MATH AND THE STUDENT'S WORLD."

Applications

Nontrivial applications are drawn from life experiences, as well as from agriculture, art, business, economics, engineering, environmental science, health, household math, science, sports, statistics, transportation, travel, etc. Examples are selected to emphasize not only the connection between math and the real world but also that between math and the student's world.

To remind students to pick up a pencil and try to solve the worked examples, the solution to each in-text example is preceded by a pencil icon: 

Tables and Graphs

The text makes extensive use of data in tabular form. Tables encourage organization of information and promote pattern observation. They also prepare students for spreadsheet technology. Where appropriate, a graph related to a table underscores the connections among algebra, geometry, statistics, and the real world. Input-output tables and their corresponding equations and graphs are employed to emphasize the fact that algebra is the transition language between arithmetic and analysis.

Think about It

"Think about it" questions are included within the reading material to encourage students to relate examples to prior material, to extend examples, and to practice verbalization skills. Answers to the questions are provided in the adjacent margin of the Annotated Instructor's Edition.

Calculators

The graphing calculator is required. Graphs and tables are provided in many examples so that students who are just learning the technology are not handicapped by a lack of understanding of the calculator. Graphs in the text are not reproductions of those made by graphing calculators because the calculator resolution is not sufficient to provide clear details and smooth curves.

Suggestions for use of graphing calculators in general are included throughout the text. Graphing calculator keystrokes for the TI-82 and TI-83 are provided in a few Graphing Calculator Technique boxes, identified by the graphing calculator icon:  . For quick reference to techniques, look under "Calculator, graphing" in the Glossary/Index.

"INPUT-OUTPUT TABLES AND THEIR CORRESPONDING EQUATIONS AND GRAPHS ARE EMPLOYED TO EMPHASIZE THE FACT THAT ALGEBRA IS THE TRANSITION LANGUAGE BETWEEN ARITHMETIC AND ANALYSIS."

Exercise Sets

A spiral curriculum is built into the exercises. Exercises in many sections use the same settings. This repetition builds familiarity with material and makes connections among seemingly isolated topics.

Within many exercise sets, students are directed back to examples in the text and asked to try the same examples with different numbers or to extend the ideas presented. This encourages students to read the text and find patterns and underlying relationships. Students need your encouragement to believe that this type of activity is worthwhile.

Small-Group Work

Some sections contain exploration material presented in question or activity form. These are intended to be done in class in small groups. It is important to emphasize how students improve their own understanding by helping others.

Projects

Most exercise sets contain Projects, which are intended for group work or individual effort. These may be more complicated problems related to the topic at hand or an activity-based problem using manipulatives, the calculator, a computer, or the Texas Instruments calculator-based laboratory. Projects may include applications that require research outside class. Projects suited to in-class group work are highlighted with this icon: 

"TO KEEP STUDENTS ENGAGED AND BUILD THEIR CONFIDENCE, A MID-CHAPTER TEST IS INCLUDED IN EACH CHAPTER."

Mid-Chapter Test

To keep students engaged and build their confidence, a Mid-Chapter Test is included in each chapter. This test gives students the opportunity to check their progress.

Chapter Summary, Review Exercises, and Test

Every chapter ends with a Chapter Summary, Chapter Review Exercises, and a Chapter Test.

Final Exam Review

A Final Exam Review, containing exercises requiring both short and long answers, follows Chapter 7. The review is divided by chapter and may be used as a source of additional exercises or cumulative review material.

Glossary/Index

For student and instructor convenience, in the combined Glossary/Index at the very end of the text, essential vocabulary is defined and referenced by page.

ANCILLARIES TO ACCOMPANY THE TEXT

The extensive ancillary package for this text includes the following:

The ***Annotated Instructor's Edition*** (0-534-95667-X) contains a complete answer to each exercise right in the text, adjacent to the exercise. Annotations in the margin offer teaching hints and strategies.

The ***Student Solutions Manual*** (0-534-95668-8) by Cindy Rubash and John Thickett provides complete step-by-step solutions for all odd-numbered exercises.

Assessment Materials (0-534-95669-6) by Alice Mullaly includes test items, sample tests, and a project for each chapter.

EXP Test (for DOS: 0-534-34584-0; for Windows: 0-534-34585-9) and ***ESA Test*** (for Macintosh: 0-345-34586-7) are computerized versions of the assessment materials.

Algeblocks, Volume 1 (for Macintosh: 0-534-95144-9; for Windows: 0-534-95146-5) and Volume 2 (for Macintosh: 0-534-95443-X; for Windows: 0-534-95456-X), is manipulative software.

I have also made text-specific videos illustrating the instruction of concepts from the text.

KASEBERG DISCUSSION LIST

To join an e-mail discussion group with other instructors of Introductory and Intermediate Algebra, send an e-mail message to

majordomo@list.thomson.com

and in the body of your message type

subscribe bc_kaseber_discuss

You will be automatically signed up. To post messages to the list, send them to

bc_kaseber_discuss@list.brookscole.com

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Alice Kaseberg

To the Student

"JUST-IN-TIME" is an industrial engineering term that describes a modern inventory management scheme in a manufacturing plant. Materials used in the manufacturing process are scheduled for purchase and delivery at the precise moment at which they are needed. The old inventory method of accumulating huge stockpiles of all manufacturing materials was costly, and during the recessions of recent decades many manufacturers either changed to the just-in-time inventory method or went out of business.

Intermediate Algebra: A Just-in-Time Approach presents material for an Intermediate Algebra course. "Just in time" describes this algebra in three ways. First, it refers to the fact that, as in the Introductory Algebra text, review material is presented just as it is needed instead of being placed at the beginning of the course. Second, it reflects the book's novel approach to the study of algebra, in which you work on real-world problems, learning principles and procedures of algebra just in time as you need them. Third, it describes the book's new curriculum, which allows you to concentrate on what you need to know in this age of modern technology, just in time for the twenty-first century.

A tri-fold approach—numeric, graphic, and symbolic—is utilized in concept development and equation solving. Throughout the text, numerical investigations lead to discovery of patterns and graphing with the calculator. A graphing calculator is required. You will use the calculator to graph, to convert data into equations, and to solve equations. To find graphing calculator techniques quickly, look under "Calculator, graphing" in the Glossary/Index.

BEGINNING THIS COURSE

A number of different items for you to think about as you begin this course follow.

Learning Styles

Because you and your classmates have different cultural backgrounds, with a wide variety of past and present life experiences, no single example or presentation will appeal to all of you. Consider how you best learn directions to a destination—in words over the phone, from a map, or from having been there with someone else. As a student of algebra, you may prefer words (a verbal approach), drawings, pictures, and graphs (a visual approach), getting up and moving around (a kinesthetic approach), or a combination of these approaches. To be successful in mathematics, you need to know your learning style and focus on those ways of learning information that best fit your style. It is advantageous, in the long run, if you also begin to learn in the other styles. To help you achieve success with algebra, *Intermediate Algebra: A Just-in-Time Approach* presents concepts in as many ways as space permits.

Independent Thinking

Although you will find the examples helpful, this text is designed to encourage your independent thinking. Look for patterns and relationships; discover concepts for yourself; seek out applications that are meaningful to you. The more involvement you have with the material, the more useful it will be and the longer you will remember it.

Groups

You are encouraged to work with others throughout this course. One of the most important benefits of working with other people is that you clarify your own understanding when you explain an idea to someone else. This is especially true for the kinesthetic learner.

Alternative Approaches

Those of you who have had algebra before may find many familiar concepts in the text. Some concepts will appear just the way you learned them the first time; others will be presented quite differently. You are being asked to learn alternative approaches, not to discard your former skills.

Alternative approaches are important for several reasons. Your old way may not work in all situations. The new way may help introduce a later concept; it may be more efficient or give more useful results. Acknowledgment of alternative approaches validates your own discoveries. New and often better ways to do mathematics are being discovered all the time.

Are You in the Right Course?

Each mathematics course has one or more prerequisite courses. Having passed a placement test does not ensure that you remember everything you need to be successful. If you have studied the background material recently, then usually with time, effort, confidence, and patience you will be able to learn the new material. If you have had a semester or a quarter or a summer break since your last math course, it is necessary to review. If the review provided in the text is not sufficient for you to remember the skills you need, you should immediately seek advice from your instructor or outside help. Use your prior book as a reference. If you took the prerequisite course more than a year ago, you may want to retake it before going on. It is normal to forget material that you don't use.

Getting a Good Start

To succeed you need to attend class, read the book before class, and do the homework in a timely manner. Plan your study time. Some students set up their schedules to have the hour after math class free, to review notes and start the assignment.

Success also depends on being prepared with the proper equipment: a graphing calculator, a six-inch ruler also marked in centimeters, and graph paper.

Homework

Keep in mind that your first homework paper is a "grade application," just as a cover letter and resume are part of a job application. First impressions count. Neatness and completeness make a lasting impression on the instructor. So does having homework ready to turn in as you walk into class.

Maximize the benefit from doing the exercises. For example, suppose the answer to an exercise appears in the back of the book as "10." If "10" is all you write, it may be all you remember about the problem. Even showing the steps

$$9 + 3 - 2 = 10$$

will tell you very little when you look at the paper again a week later. To obtain the maximum benefit, first state the problem, then work the problem, and then write a sentence describing what you did in your own words, such as

> If $f(x) = x^2 + x - 2$, then $f(3) = 3^2 + 3 - 2 = 9 + 3 - 2 = 10$.
> The notation $f(3)$ means to let $x = 3$ in the expression given by $f(x)$.

Now, get on with the course. Come back to the following suggestions if you run into difficulty at a later time.

KEEPING UP THE *GOOD* WORK

Stuck on the Homework?

Sometimes we get too close to a problem and overlook the obvious. A fresh point of view may help. Work on assignments as early as possible, right after class or early in the evening or weekend. This gives you the option of going back later and spending more time on a difficult problem. Make notes to yourself on homework papers. Highlight problems that were difficult and that you want to review again later. Summarize the solution method in your own words so you won't forget how to do the problem later. Highlight formulas or key steps.

Obtain help from your teacher or resource center as you need it. Don't wait until just before a test.

Falling Behind?

If you find yourself falling behind, let your instructor know that you are trying to catch up. Set up a plan that allows two to three days for each missed assignment. Most important, do current assignments first, even if you have to skip a few problems because you missed material. Work immediately after the class session. By doing the current assignment first you will stay with the class. If you gradually complete missed work, within a reasonable amount of time you will be completely caught up. Do not skip class because you are behind or confused.

Forgetting Material?

Many students select one or two exercises from each section and write them on $3'' \times 5''$ cards, with complete solutions on the back. These "flash cards" may then be shuffled and practiced at any time for review. Cards provide an excellent way to study for tests and the final exam. Include vocabulary words in your card set.

STRATEGIES FOR TAKING TESTS

Prepare Yourself Academically

1. Attend class, and do the homework completely and regularly. If there are exercises in the homework that you do not know how to do, get help—from a classmate, the teacher, or another appropriate source.
2. Work under time pressure on a regular basis. Set yourself a limited amount of time to do portions of the homework. Use a time limit when doing review exercises or the practice tests at the middle and end of each chapter. Working in one- or two-hour blocks of time is usually more productive than spending all afternoon and evening on math one day a week.

Prepare Yourself Physically

3. Get a good night's sleep. Being rested helps you think clearly, even if you know less material.

Prepare Yourself Mentally

Psych yourself up. This is especially important if you have your test later in the day.

4. If you have a test at 8:00 A.M., use the last few minutes before bedtime to get everything ready for the next day. Make your lunch, set your books or pack on a chair by the door, set out your umbrella or appropriate weather gear, and make sure you have change for the bus or train or that your car's tires, battery, and gasoline level are okay.
5. Plan 10 or 15 minutes of quiet time before the test. Try to arrive early, if possible.
6. Imagine yourself taking the test.
   - **a.** Imagine writing your name on the test.
   - **b.** Imagine reading through the test completely to see where the instructor put various types of questions.
   - **c.** Imagine writing notes, formulas, or reminders to yourself on the test.
   - **d.** Imagine working your favorite type of problem first.

Take the Test Right

7. Arrive early. Be ready—pencil sharpened, homework papers ready to turn in.
8. Concentrate on doing steps **a**–**d** in item 6 above.
9. Work quickly and carefully through those problems you know how to solve. Don't spend excessive time on one problem until you have tried every problem.

INTERMEDIATE ALGEBRA

A JUST-IN-TIME APPROACH

Contents

CHAPTER ONE

Functions

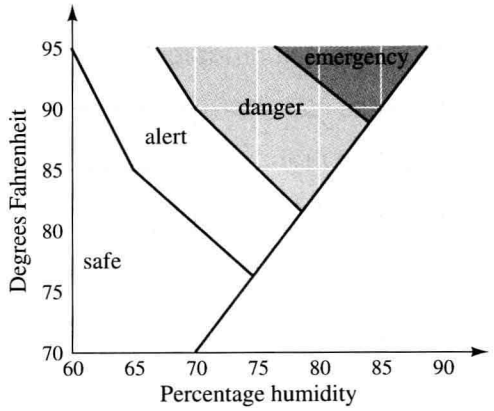

Figure 1 *Temperature-humidity chart*

HOW DO YOU FEEL when you exercise on a hot, muggy day? On a cool, dry day? The temperature-humidity chart in Figure 1 illustrates a relationship between weather conditions and exercise. The *safe* region on the chart indicates the temperature and humidity levels when it is safe to exercise. Other regions alert the reader to use caution, indicate danger to health, or warn that emergency medical conditions could result.

The relative safety of exercising is related to, or depends on, both temperature and humidity. The dependence of safety on temperature and humidity makes the chart an example of a special relationship called a

(continued)

function. This chapter covers the definition of functions, notation for writing functions, reading and making graphs of functions, and linear functions. The chapter closes with a section on arithmetic sequences (functions with positive integers as inputs). Because graphs play an important role throughout the course, we begin with a review of important vocabulary and coordinate graphing. We return to the temperature-humidity chart in Section 1.1.

1.0 INTEGER REVIEW AND COORDINATE GRAPHS

OBJECTIVES Identify sets of numbers, including integers and real numbers. ♦ Add, subtract, multiply, and divide integers and real numbers. ♦ Identify properties of real numbers. ♦ Identify x- and y-axes, quadrants, origin, coordinates, and ordered pairs. ♦ Build input-output tables from equations. ♦ Plot input-output pairs as coordinates on coordinate graphs. ♦ Match inequalities and their number line graphs. ♦ Identify the meaning of *and* and *or* in inequality statements.

TIPS FOR STUDENTS

Use the objectives to help you consider these questions:

What is it that I need to learn from this section?

Do I know it?

How does this section fit in with prior concepts?

WARM-UP

1. Locate the numbers $\{-3, 6, -2, -4, -6\}$ on a number line.
2. Which number is farther from zero, -4 or 6? Show why on your number line from Exercise 1.
3. If the distance from 0 to 1 is 1 unit, how far is it from -4 to 6? ♦

STUDENT NOTES

Each chapter begins with a zero section, and each section begins with a set of exercises called a Warm-up. The purpose of the zero section and the Warm-up is to spread review throughout the text and help you maintain skills. The Warm-up reviews skills or concepts needed within the section. No answers are provided to Warm-ups; compare your results with your classmates'. The symbol ♦ designates the end of a Warm-up or of a solution to an example.

This section reviews several ideas about real numbers: operations with positive and negative numbers, properties of real numbers, coordinate graph representation of ordered pairs, and inequalities.