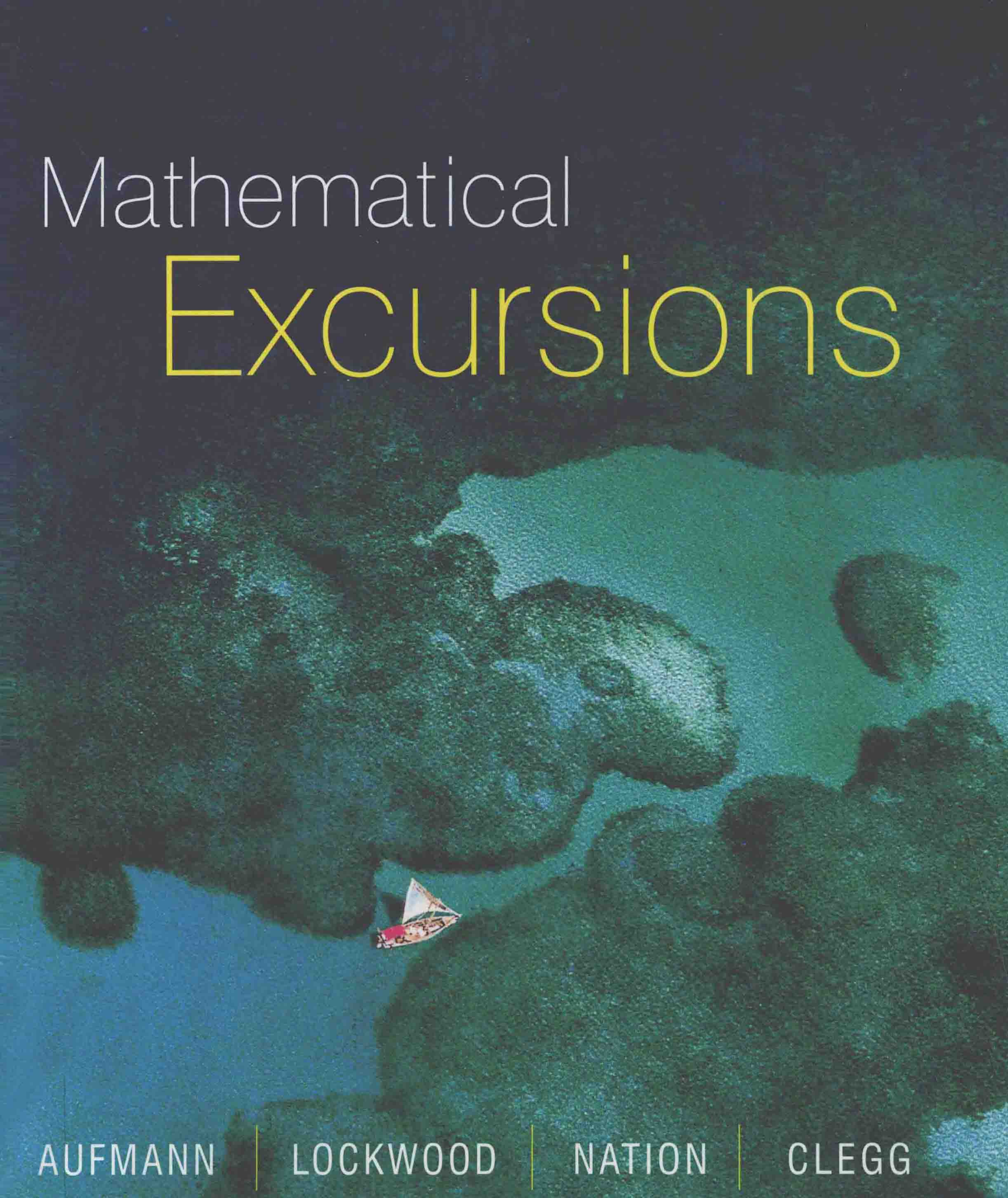
Mathematical Excursions



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Mathematical Excursions

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HOUGHTON MIFFLIN COMPANY
Boston New York

Publisher: Jack Shira
Development Manager: Maureen Ross
Sponsoring Editor: Lauren Schultz
Assistant Editor: Lisa Pettinato
Senior Project Editor: Tamela Ambush
Senior Production/Design Coordinator: Carol Merrigan
Editorial Assistant: Lisa Sullivan
Manufacturing Manager: Florence Cadran
Senior Marketing Manager: Ben Rivera
Marketing Associate: Alexandra Shaw

Cover Photographer: © 2002 Yann Arthus-Bertrand/Altitude

Photo credits are found immediately after the
Answer section in the back of the book.

Printed in the U.S.A.

Library of Congress Catalog Card Number: 2002109357

ISBNs:
Student Text: 0-395-72779-0
Instructor's Annotated Edition: 0-618-30254-9

23456789-VHP-07 06 05 04 03

PREFACE

Mathematical Excursions is about mathematics as a system of knowing or understanding our surroundings. It is similar to an English literature textbook, an Introduction to Philosophy textbook, or perhaps an Introductory Psychology textbook. Each of those books provide glimpses into the thoughts and perceptions of some of the world's greatest writers, philosophers, and psychologists. Reading and studying their thoughts enables us to better understand the world we inhabit.

In a similar way, *Mathematical Excursions* provides glimpses into the nature of mathematics and how it is used to understand our world. This understanding, in conjunction with other disciplines, contributes to a more complete portrait of our world. Our contention is that ancient Greek architecture is quite dramatic but even more so when the "Golden Ratio" is considered. That I. M. Pei's work becomes even more interesting with a knowledge of elliptical shapes. That the challenges of sending information across the Internet is better understood by examining prime numbers. That the perils of radioactive waste take on new meaning with a knowledge of exponential functions. That generally, a knowledge of mathematics strengthens the way we know, perceive, and understand our surroundings.

One theme around which this book is written is, "What if you wanted to know how to . . . , what would you need to know?" Using this strategy, a contemporary problem is introduced and then the relevant mathematics needed to solve that problem is developed. With the mathematics in place, the solution to the problem is presented and additional applications of the mathematics are illustrated. A second theme is to have you explore a concept from different perspectives so that you can develop an appreciation for the diversity of problems that can be solved from a single concept.

Math Matters and Excursions are two features we have incorporated in the text. Math Matters are vignettes of interesting applications of the topic being discussed. Each section of the text ends with an Excursion, which is an extension of one of the topics of that section.

The exercise sets of *Mathematical Excursions* have been carefully selected to reinforce and extend the concepts developed in each section. The exercises range from drill and practice to interesting challenges. Some of the exercise sets include outlines for further explorations, suggestions for essays, critical thinking, and cooperative learning activities. In all cases, the exercises were chosen to illustrate the many facets of the topic under discussion.

The purpose of this book is to be a brief excursion into the castle of mathematics with all its myriad of rooms. Although we assume that the reader has a intermediate algebra background, each topic is carefully developed and appropriate material reviewed whenever necessary. When deciding on the depth of coverage, our singular criteria was to make mathematics accessible.

Chapter Opening Features

Chapter Opening Photos

Each chapter begins with photos and captions that are related to an Excursion in the chapter.

Web Icon

The web icon on this opening page lets students know of additional online resources at **math.college.hmco.com/students.**

Roller coasters have been using increasingly more sophisticated methods of acceleration, such as electromagnets, to reach higher speeds. The **Excursion** on **page 345** expands the concept of velocity to include negative velocity. You will be asked to interpret distance-time graphs in which the slopes represent either positive or negative motion.

CHAPTER 6

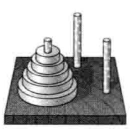
Applications of Functions

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at math.college.hmco.com/students.

page 322

Chapter Opener Subject Matter

The second page of each chapter opener presents a motivational topic, an application from the chapter, or a new mathematical concept.



The Tower of Hanoi is a puzzle that has the following form. Three pegs are placed on a board. A number of disks, graded in size, are stacked on one of the pegs with the largest disk on the bottom and successively smaller disks placed on top. The disks are moved according to the following rules:

1. Only one disk at a time may be moved.
2. A larger disk may not be placed on top of a smaller disk.

The object of the game is to transfer all of the disks, one at a time, from one peg to one of the other two pegs. If initially there is only one disk, then only one move is required. With two disks, three moves are required; with three disks, seven moves are required.

The chart below shows the minimum number of moves required for a given number of disks. The increase in the number of moves required for each additional disk is also given.

Number of disks	1	2	3	4	5	6	7
Minimum number of moves	1	3	7	15	31	63	127
Increase in number of moves		3−1=2	7−3=4	15−7=8	31−15=16	63−31=32	127−63=64

point of interest

If recent estimates of the age of the universe are accurate, our solar system is about 4.5 billion years old. The first known life forms occurred about 3 billion years ago.

For the list of numbers in the bottom row of the table, each successive number can be found by multiplying the preceding number by a constant (in this case 2). This list of numbers can be represented by the equation $f(n) = 2^n$, which is an example of an *exponential function*, one of the topics of this chapter.

The formula for the minimum number of moves is given by $M = 2^n - 1$, which contains the exponential expression 2^n. In this formula, M is the minimum number of moves required to transfer n disks to one of the other pegs.

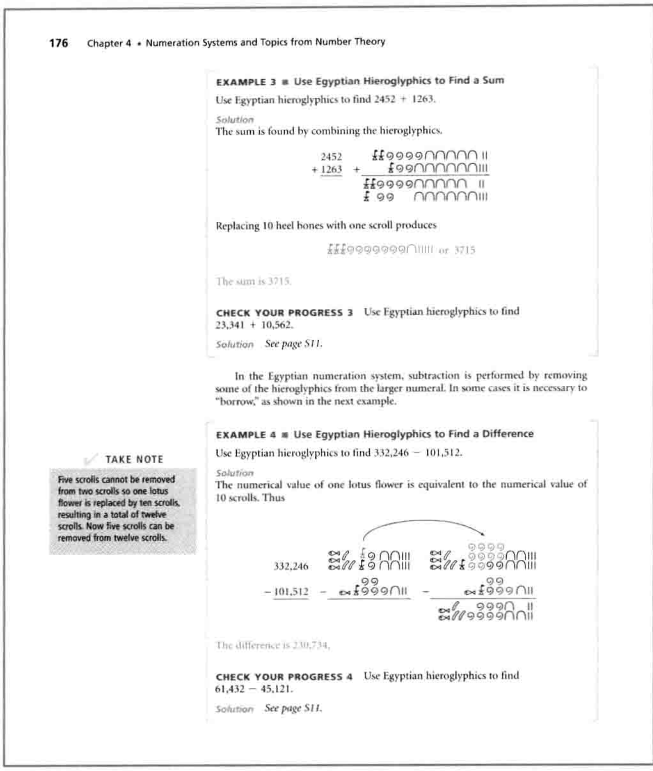
There is an ancient myth involving the Tower of Hanoi puzzle and the lifetime of the universe. In this myth, three priests sit in the center of the universe with 3 diamond needles and 64 golden disks on one of the needles. The only job of the priests is to transfer the golden disks to one of the other needles using the rules of the Tower of Hanoi puzzle. The priests can transfer one disk to another needle every second. According to the myth, the universe will cease to exist at the precise moment the priests have completed the transfer of all 64 disks to one of the other needles.

Thus according to the myth, the lifetime of the universe is given by $2^{64} - 1$ seconds. Use a calculator to show that this amounts to approximately 585 billion years! Even if the priests started the transfer of the disks 12 billion years ago (when astronomers estimate our universe began), the myth indicates that the universe will continue to exist for another 573 billion years.

323

page 323

Interactive Method


176 Chapter 4 • Numeration Systems and Topics from Number Theory

EXAMPLE 3 ■ Use Egyptian Hieroglyphics to Find a Sum

Use Egyptian hieroglyphics to find 2452 + 1263.

Solution

The sum is found by combining the hieroglyphics.

2452
+ 1263

Replacing 10 heel bones with one scroll produces

or 3715

The sum is 3715.

CHECK YOUR PROGRESS 3 Use Egyptian hieroglyphics to find 23,341 + 10,562.

Solution *See page S11.*

In the Egyptian numeration system, subtraction is performed by removing some of the hieroglyphics from the larger numeral. In some cases it is necessary to "borrow," as shown in the next example.

EXAMPLE 4 ■ Use Egyptian Hieroglyphics to Find a Difference

Use Egyptian hieroglyphics to find 332,246 − 101,512.

Solution

The numerical value of one lotus flower is equivalent to the numerical value of 10 scrolls. Thus

332,246
− 101,512

The difference is 230,734.

CHECK YOUR PROGRESS 4 Use Egyptian hieroglyphics to find 61,432 − 45,121.

Solution *See page S11.*

TAKE NOTE

Five scrolls cannot be removed from two scrolls so one lotus flower is replaced by ten scrolls, resulting in a total of twelve scrolls. Now five scrolls can be removed from twelve scrolls.

page 176

An Interactive Approach

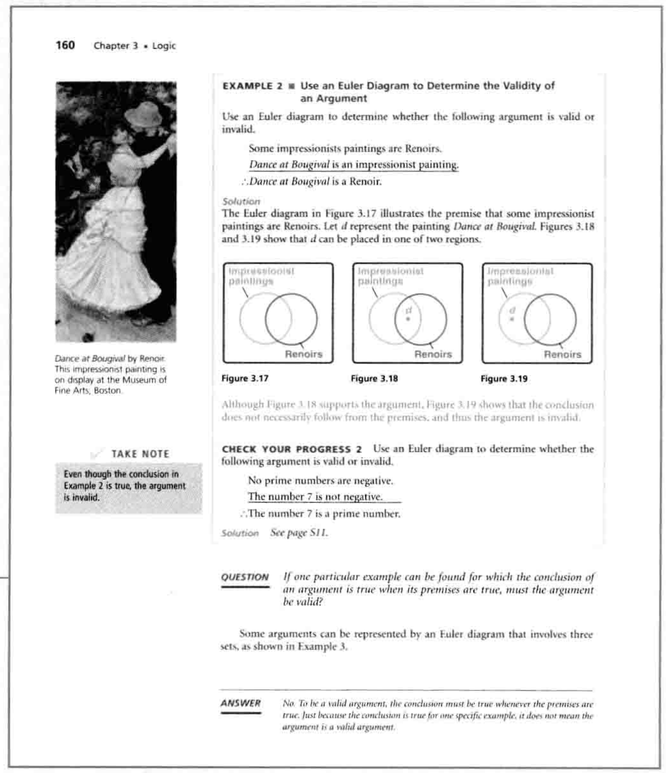
Mathematical Excursions is written in a style that encourages the student to interact with the textbook. Each section contains a variety of worked examples. Each example is given a title so that the student can see at a glance the type of problem that is being solved. Most examples include annotations that assist the student in moving from step to step, and the final answer is in color in order to be readily identifiable.

Check Your Progress Exercises

Following each worked example is a Check Your Progress exercise for the student to work. By solving this exercise, the student actively practices concepts as they are presented in the text. For each Check Your Progress exercise, there is a detailed solution in the Solutions appendix.

Question/Answer Feature

At various places throughout the text, a Question is posed about the topic that is being developed. This question encourages students to pause, think about the current discussion, and answer the question. Students can immediately check their understanding by referring to the Answer to the question provided in a footnote on the same page. This feature creates another opportunity for the student to interact with the textbook.


160 Chapter 3 • Logic

Dance at Bougival by Renoir. This impressionist painting is on display at the Museum of Fine Arts, Boston.

EXAMPLE 2 ■ Use an Euler Diagram to Determine the Validity of an Argument

Use an Euler diagram to determine whether the following argument is valid or invalid.

Some impressionists paintings are Renoirs.
Dance at Bougival is an impressionist painting.
∴ *Dance at Bougival* is a Renoir.

Solution

The Euler diagram in Figure 3.17 illustrates the premise that some impressionist paintings are Renoirs. Let *d* represent the painting *Dance at Bougival*. Figures 3.18 and 3.19 show that *d* can be placed in one of two regions.

Figure 3.17 Figure 3.18 Figure 3.19

Although Figure 3.18 supports the argument, Figure 3.19 shows that the conclusion does not necessarily follow from the premises, and thus the argument is invalid.

TAKE NOTE

Even though the conclusion in Example 2 is true, the argument is invalid.

CHECK YOUR PROGRESS 2 Use an Euler diagram to determine whether the following argument is valid or invalid.

No prime numbers are negative.
The number 7 is not negative.
∴ The number 7 is a prime number.

Solution *See page S11.*

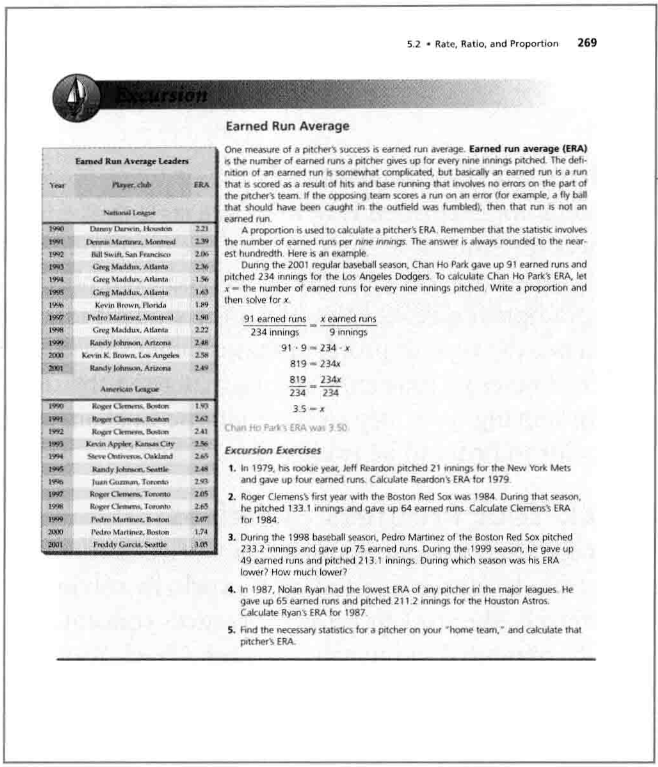
QUESTION *If one particular example can be found for which the conclusion of an argument is true when its premises are true, must the argument be valid?*

Some arguments can be represented by an Euler diagram that involves three sets, as shown in Example 3.

ANSWER *No. To be a valid argument, the conclusion must be true whenever the premises are true. Just because the conclusion is true for one specific example, it does not mean the argument is a valid argument.*

page 160

Interactive Method, *continued*



5.2 • Rate, Ratio, and Proportion 269

Excursion

Earned Run Average

Earned Run Average Leaders		
Year	Player, club	ERA
	National League	
1990	Danny Darwin, Houston	2.21
1991	Dennis Martinez, Montreal	2.39
1992	Bill Swift, San Francisco	2.06
1993	Greg Maddux, Atlanta	2.36
1994	Greg Maddux, Atlanta	1.56
1995	Greg Maddux, Atlanta	1.63
1996	Kevin Brown, Florida	1.89
1997	Pedro Martinez, Montreal	1.90
1998	Greg Maddux, Atlanta	2.22
1999	Randy Johnson, Arizona	2.48
2000	Kevin K. Brown, Los Angeles	2.58
2001	Randy Johnson, Arizona	2.49
	American League	
1990	Roger Clemens, Boston	1.93
1991	Roger Clemens, Boston	2.62
1992	Roger Clemens, Boston	2.41
1993	Kevin Appier, Kansas City	2.56
1994	Steve Ontiveros, Oakland	2.65
1995	Randy Johnson, Seattle	2.48
1996	Juan Guzman, Toronto	2.93
1997	Roger Clemens, Toronto	2.05
1998	Roger Clemens, Toronto	2.65
1999	Pedro Martinez, Boston	2.07
2000	Pedro Martinez, Boston	1.74
2001	Freddy Garcia, Seattle	3.05

One measure of a pitcher's success is earned run average. **Earned run average (ERA)** is the number of earned runs a pitcher gives up for every nine innings pitched. The definition of an earned run is somewhat complicated, but basically an earned run is a run that is scored as a result of hits and base running that involves no errors on the part of the pitcher's team. If the opposing team scores a run on an error (for example, a fly ball that should have been caught in the outfield was fumbled), then that run is not an earned run.

A proportion is used to calculate a pitcher's ERA. Remember that the statistic involves the number of earned runs per *nine innings*. The answer is always rounded to the nearest hundredth. Here is an example.

During the 2001 regular baseball season, Chan Ho Park gave up 91 earned runs and pitched 234 innings for the Los Angeles Dodgers. To calculate Chan Ho Park's ERA, let x = the number of earned runs for every nine innings pitched. Write a proportion and then solve for x.

$$\frac{91 \text{ earned runs}}{234 \text{ innings}} = \frac{x \text{ earned runs}}{9 \text{ innings}}$$
$$91 \cdot 9 = 234 \cdot x$$
$$819 = 234x$$
$$\frac{819}{234} = \frac{234x}{234}$$
$$3.5 = x$$

Chan Ho Park's ERA was 3.50.

Excursion Exercises

1. In 1979, his rookie year, Jeff Reardon pitched 21 innings for the New York Mets and gave up four earned runs. Calculate Reardon's ERA for 1979.
2. Roger Clemens's first year with the Boston Red Sox was 1984. During that season, he pitched 133.1 innings and gave up 64 earned runs. Calculate Clemens's ERA for 1984.
3. During the 1998 baseball season, Pedro Martinez of the Boston Red Sox pitched 233.2 innings and gave up 75 earned runs. During the 1999 season, he gave up 49 earned runs and pitched 213.1 innings. During which season was his ERA lower? How much lower?
4. In 1987, Nolan Ryan had the lowest ERA of any pitcher in the major leagues. He gave up 65 earned runs and pitched 211.2 innings for the Houston Astros. Calculate Ryan's ERA for 1987.
5. Find the necessary statistics for a pitcher on your "home team," and calculate that pitcher's ERA.

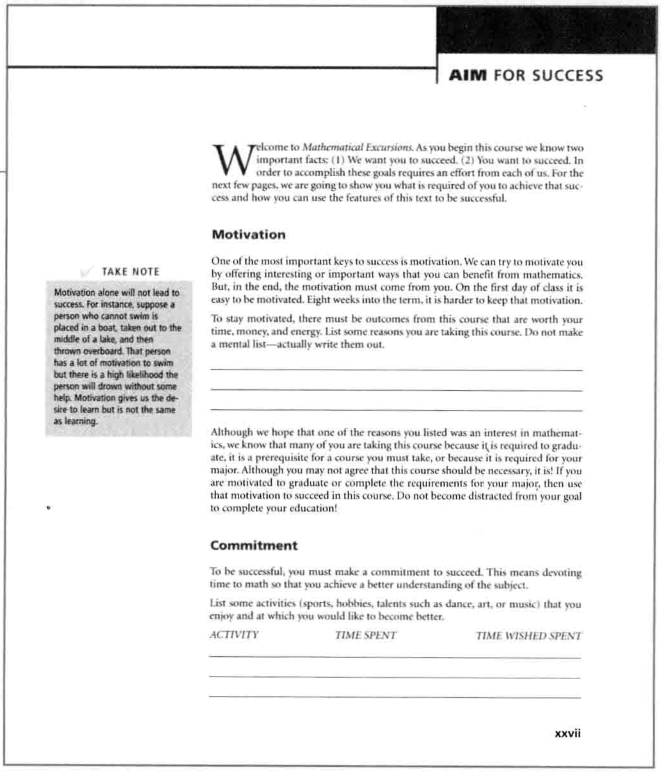
page 269

Excursions

Each section ends with an Excursion along with corresponding Excursion Exercises. These activities engage students in the mathematics of the section. Some Excursions are designed as in-class cooperative learning activities that lend themselves to a hands-on approach. They can also be assigned as projects or extra credit assignments. The Excursions are a unique and important feature of this text. They provide opportunities for students to take an active role in the learning process. The photos on the first page of a chapter opener relate to one of the Excursions in that chapter.

AIM for Success Student Preface

This 'how to use this text' preface explains what is required of a student to be successful and how this text has been designed to foster student success. AIM for Success can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills.



AIM FOR SUCCESS

Welcome to *Mathematical Excursions.* As you begin this course we know two important facts: (1) We want you to succeed. (2) You want to succeed. In order to accomplish these goals requires an effort from each of us. For the next few pages, we are going to show you what is required of you to achieve that success and how you can use the features of this text to be successful.

Motivation

TAKE NOTE

Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation to swim but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning.

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways that you can benefit from mathematics. But, in the end, the motivation must come from you. On the first day of class it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

To stay motivated, there must be outcomes from this course that are worth your time, money, and energy. List some reasons you are taking this course. Do not make a mental list—actually write them out.

Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate, it is a prerequisite for a course you must take, or because it is required for your major. Although you may not agree that this course should be necessary, it is! If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal to complete your education!

Commitment

To be successful, you must make a commitment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better.

ACTIVITY *TIME SPENT* *TIME WISHED SPENT*

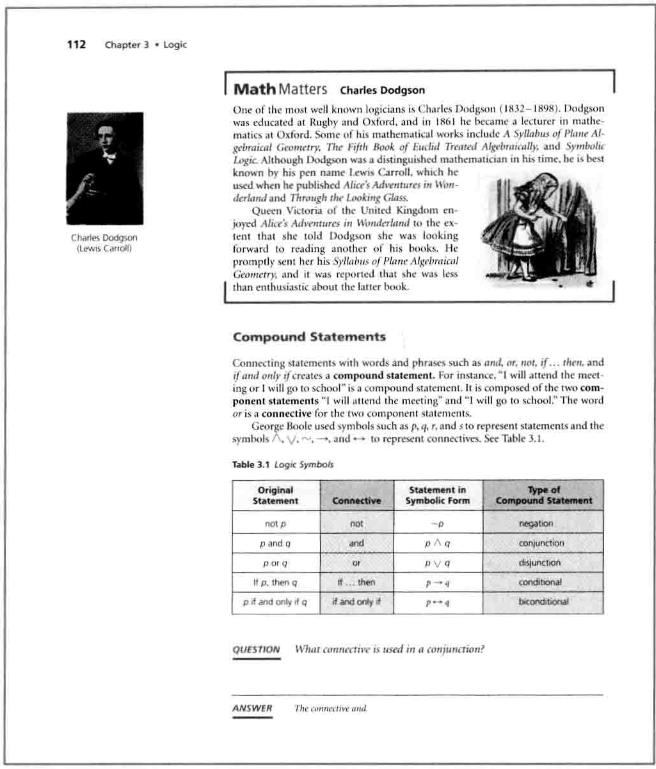
xxvii

page xxvii

Math Matters and Margin Notes

Math Matters

This feature of the text typically contains an interesting sidelight about mathematics, its history, or its applications.



112 Chapter 3 • Logic

Math Matters **Charles Dodgson**

One of the most well known logicians is Charles Dodgson (1832–1898). Dodgson was educated at Rugby and Oxford, and in 1861 he became a lecturer in mathematics at Oxford. Some of his mathematical works include *A Syllabus of Plane Algebraical Geometry, The Fifth Book of Euclid Treated Algebraically,* and *Symbolic Logic.* Although Dodgson was a distinguished mathematician in his time, he is best known by his pen name Lewis Carroll, which he used when he published *Alice's Adventures in Wonderland* and *Through the Looking Glass.*

Queen Victoria of the United Kingdom enjoyed *Alice's Adventures in Wonderland* to the extent that she told Dodgson she was looking forward to reading another of his books. He promptly sent her his *Syllabus of Plane Algebraical Geometry,* and it was reported that she was less than enthusiastic about the latter book.

Charles Dodgson (Lewis Carroll)

Compound Statements

Connecting statements with words and phrases such as *and, or, not, if … then,* and *if and only if* creates a **compound statement.** For instance, "I will attend the meeting or I will go to school" is a compound statement. It is composed of the two **component statements** "I will attend the meeting" and "I will go to school." The word *or* is a **connective** for the two component statements.

George Boole used symbols such as *p, q, r,* and *s* to represent statements and the symbols $\wedge$, $\vee$, $\sim$, $\rightarrow$, and $\leftrightarrow$ to represent connectives. See Table 3.1.

Table 3.1 *Logic Symbols*

Original Statement	Connective	Statement in Symbolic Form	Type of Compound Statement
not p	not	$\sim p$	negation
p and q	and	$p \wedge q$	conjunction
p or q	or	$p \vee q$	disjunction
If p, then q	If … then	$p \rightarrow q$	conditional
p if and only if q	if and only if	$p \leftrightarrow q$	biconditional

QUESTION *What connective is used in a conjunction?*

ANSWER *The connective and.*

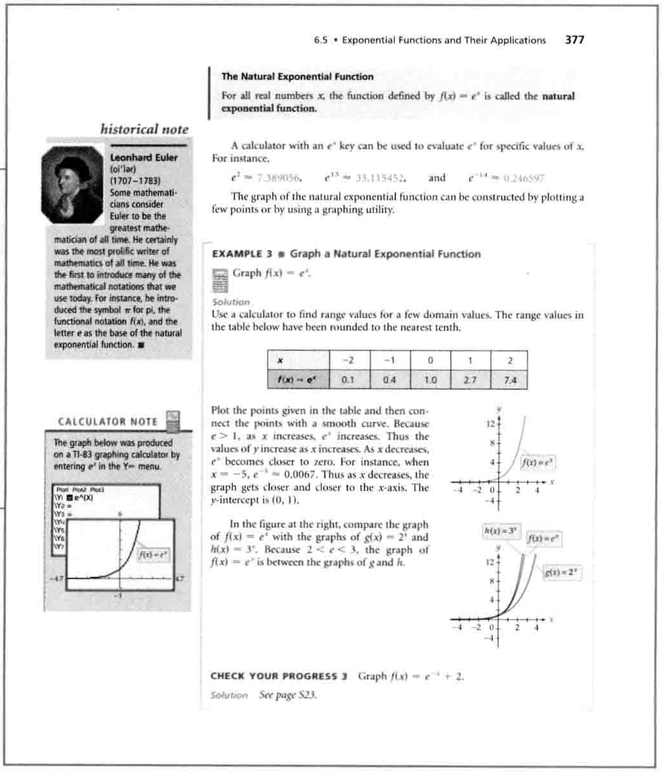
page 112

Historical Note

These margin notes provide historical background information related to the concept under discussion or vignettes of individuals who were responsible for major advancements in their fields of expertise.

Calculator Note

These notes provide information about how to use the various features of a calculator.



6.5 • Exponential Functions and Their Applications 377

The Natural Exponential Function

For all real numbers x, the function defined by $f(x) = e^x$ is called the **natural exponential function.**

historical note

Leonhard Euler (oi'lər) (1707–1783) Some mathematicians consider Euler to be the greatest mathematician of all time. He certainly was the most prolific writer of mathematics of all time. He was the first to introduce many of the mathematical notations that we use today. For instance, he introduced the symbol π for pi, the functional notation $f(x)$, and the letter e as the base of the natural exponential function. ■

A calculator with an e^x key can be used to evaluate e^x for specific values of x. For instance,

$$e^2 \approx 7.389056, \quad e^{3.5} \approx 33.115452, \quad \text{and} \quad e^{-1.4} \approx 0.246597$$

The graph of the natural exponential function can be constructed by plotting a few points or by using a graphing utility.

EXAMPLE 3 ■ Graph a Natural Exponential Function

Graph $f(x) = e^x$.

Solution

Use a calculator to find range values for a few domain values. The range values in the table below have been rounded to the nearest tenth.

x	−2	−1	0	1	2
$f(x) = e^x$	0.1	0.4	1.0	2.7	7.4

Plot the points given in the table and then connect the points with a smooth curve. Because $e > 1$, as x increases, e^x increases. Thus the values of y increase as x increases. As x decreases, e^x becomes closer to zero. For instance, when $x = -5$, $e^{-5} \approx 0.0067$. Thus as x decreases, the graph gets closer and closer to the x-axis. The y-intercept is $(0, 1)$.

In the figure at the right, compare the graph of $f(x) = e^x$ with the graphs of $g(x) = 2^x$ and $h(x) = 3^x$. Because $2 < e < 3$, the graph of $f(x) = e^x$ is between the graphs of g and h.

CALCULATOR NOTE

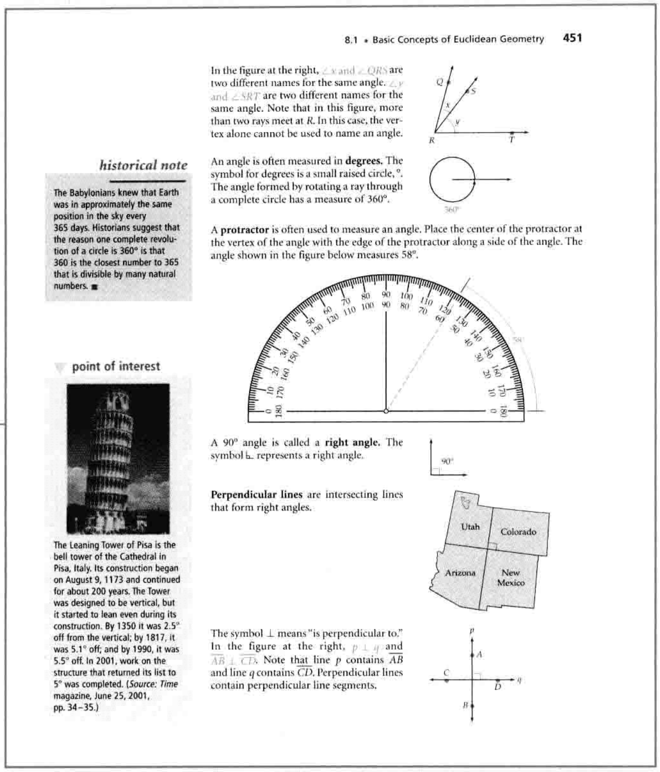
The graph below was produced on a TI-83 graphing calculator by entering e^x in the Y= menu.

CHECK YOUR PROGRESS 3 Graph $f(x) = e^{-x} + 2$.

Solution *See page S23.*

page 377

Margin Notes, *continued*



8.1 • Basic Concepts of Euclidean Geometry **451**

historical note

The Babylonians knew that Earth was in approximately the same position in the sky every 365 days. Historians suggest that the reason one complete revolution of a circle is 360° is that 360 is the closest number to 365 that is divisible by many natural numbers. ■

In the figure at the right, $\angle x$ and $\angle QRS$ are two different names for the same angle. $\angle y$ and $\angle SRT$ are two different names for the same angle. Note that in this figure, more than two rays meet at R. In this case, the vertex alone cannot be used to name an angle.

An angle is often measured in **degrees.** The symbol for degrees is a small raised circle, °. The angle formed by rotating a ray through a complete circle has a measure of 360°.

A **protractor** is often used to measure an angle. Place the center of the protractor at the vertex of the angle with the edge of the protractor along a side of the angle. The angle shown in the figure below measures 58°.

point of interest

The Leaning Tower of Pisa is the bell tower of the Cathedral in Pisa, Italy. Its construction began on August 9, 1173 and continued for about 200 years. The Tower was designed to be vertical, but it started to lean even during its construction. By 1350 it was 2.5° off from the vertical; by 1817, it was 5.1° off; and by 1990, it was 5.5° off. In 2001, work on the structure that returned its list to 5° was completed. (*Source: Time* magazine, June 25, 2001, pp. 34–35.)

A 90° angle is called a **right angle.** The symbol ∟ represents a right angle.

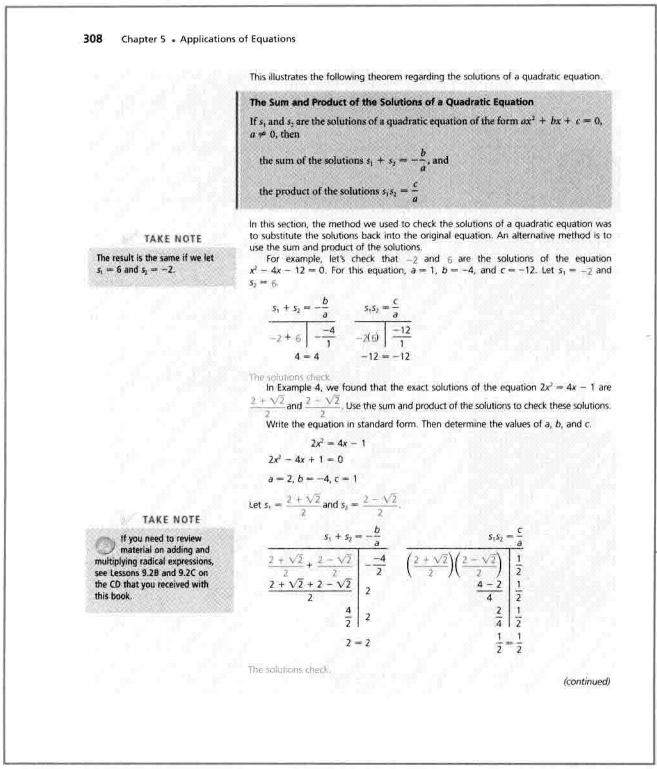
Perpendicular lines are intersecting lines that form right angles.

The symbol ⊥ means "is perpendicular to." In the figure at the right, $p \perp q$ and $\overline{AB} \perp \overline{CD}$. Note that line p contains $\overline{AB}$ and line q contains $\overline{CD}$. Perpendicular lines contain perpendicular line segments.

page 451

Point of Interest

These notes provide interesting information related to the topics under discussion. Many of these are of a contemporary nature and, as such, they provide students with the needed motivation for studying concepts that may at first seem abstract and obscure without this information.



308 Chapter 5 • Applications of Equations

This illustrates the following theorem regarding the solutions of a quadratic equation.

The Sum and Product of the Solutions of a Quadratic Equation

If s_1 and s_2 are the solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, then

the sum of the solutions $s_1 + s_2 = -\frac{b}{a}$, and

the product of the solutions $s_1 s_2 = \frac{c}{a}$

TAKE NOTE

The result is the same if we let $s_1 = 6$ and $s_2 = -2$.

In this section, the method we used to check the solutions of a quadratic equation was to substitute the solutions back into the original equation. An alternative method is to use the sum and product of the solutions.

For example, let's check that -2 and 6 are the solutions of the equation $x^2 - 4x - 12 = 0$. For this equation, $a = 1$, $b = -4$, and $c = -12$. Let $s_1 = -2$ and $s_2 = 6$.

$$s_1 + s_2 = -\frac{b}{a} \qquad s_1 s_2 = \frac{c}{a}$$

$$-2 + 6 \;\Big|\; -\frac{-4}{1} \qquad -2(6) \;\Big|\; \frac{-12}{1}$$

$$4 = 4 \qquad -12 = -12$$

The solutions check.

In Example 4, we found that the exact solutions of the equation $2x^2 = 4x - 1$ are $\frac{2 + \sqrt{2}}{2}$ and $\frac{2 - \sqrt{2}}{2}$. Use the sum and product of the solutions to check these solutions.

Write the equation in standard form. Then determine the values of a, b, and c.

$$2x^2 = 4x - 1$$
$$2x^2 - 4x + 1 = 0$$
$$a = 2,\ b = -4,\ c = 1$$

Let $s_1 = \frac{2 + \sqrt{2}}{2}$ and $s_2 = \frac{2 - \sqrt{2}}{2}$.

TAKE NOTE

If you need to review material on adding and multiplying radical expressions, see Lessons 9.2B and 9.2C on the CD that you received with this book.

$$s_1 + s_2 = -\frac{b}{a} \qquad s_1 s_2 = \frac{c}{a}$$

$$\frac{2 + \sqrt{2}}{2} + \frac{2 - \sqrt{2}}{2} \;\Big|\; -\frac{-4}{2} \qquad \left(\frac{2 + \sqrt{2}}{2}\right)\left(\frac{2 - \sqrt{2}}{2}\right) \;\Big|\; \frac{1}{2}$$

$$\frac{2 + \sqrt{2} + 2 - \sqrt{2}}{2} \;\Big|\; 2 \qquad \frac{4 - 2}{4} \;\Big|\; \frac{1}{2}$$

$$\frac{4}{2} \;\Big|\; 2 \qquad \frac{2}{4} \;\Big|\; \frac{1}{2}$$

$$2 = 2 \qquad \frac{1}{2} = \frac{1}{2}$$

The solutions check.

(continued)

page 308

Take Note

These margin notes alert students to a point requiring special attention or are used to amplify the concepts that are currently being developed. Some Take Notes, identified by [CD icon], reference the student CD. A student who needs to review a prerequisite skill or concept can find the needed material on this CD.

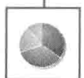
Exercises

Exercise Sets

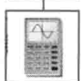
The exercise sets of *Mathematical Excursions* were carefully written to provide a wide variety of exercises that range from drill and practice to interesting challenges. Exercise sets emphasize skill building, skill maintenance, concepts, and applications, when they are appropriate. Icons are used to identify various types of exercise.

Writing exercises

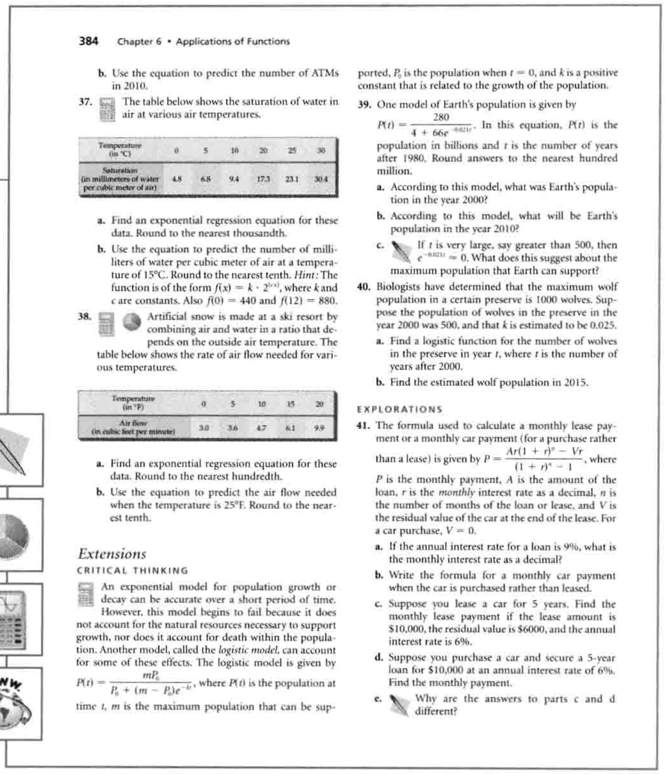
Data analysis exercises



Graphing calculator exercises



Internet exercises



384 Chapter 6 • Applications of Functions

b. Use the equation to predict the number of ATMs in 2010.

37. The table below shows the saturation of water in air at various air temperatures.

Temperature (in °C)	0	5	10	20	25	30
Saturation (in millimeters of water per cubic meter of air)	4.8	6.8	9.4	17.3	23.1	30.4

a. Find an exponential regression equation for these data. Round to the nearest thousandth.

b. Use the equation to predict the number of milliliters of water per cubic meter of air at a temperature of 15°C. Round to the nearest tenth. *Hint:* The function is of the form $f(x) = k \cdot 2^{(cx)}$, where k and c are constants. Also $f(0) = 440$ and $f(12) = 880$.

38. Artificial snow is made at a ski resort by combining air and water in a ratio that depends on the outside air temperature. The table below shows the rate of air flow needed for various temperatures.

Temperature (in °F)	0	5	10	15	20
Air flow (in cubic feet per minute)	3.0	3.6	4.7	6.1	9.9

a. Find an exponential regression equation for these data. Round to the nearest hundredth.

b. Use the equation to predict the air flow needed when the temperature is 25°F. Round to the nearest tenth.

Extensions

CRITICAL THINKING

An exponential model for population growth or decay can be accurate over a short period of time. However, this model begins to fail because it does not account for the natural resources necessary to support growth, nor does it account for death within the population. Another model, called the *logistic model*, can account for some of these effects. The logistic model is given by $P(t) = \frac{mP_0}{P_0 + (m - P_0)e^{-kt}}$, where $P(t)$ is the population at time t, m is the maximum population that can be supported, P_0 is the population when $t = 0$, and k is a positive constant that is related to the growth of the population.

39. One model of Earth's population is given by $P(t) = \frac{280}{4 + 66e^{-0.021t}}$. In this equation, $P(t)$ is the population in billions and t is the number of years after 1980. Round answers to the nearest hundred million.

a. According to this model, what was Earth's population in the year 2000?

b. According to this model, what will be Earth's population in the year 2010?

c. If t is very large, say greater than 500, then $e^{-0.021t} \approx 0$. What does this suggest about the maximum population that Earth can support?

40. Biologists have determined that the maximum wolf population in a certain preserve is 1000 wolves. Suppose the population of wolves in the preserve in the year 2000 was 500, and that k is estimated to be 0.025.

a. Find a logistic function for the number of wolves in the preserve in year t, where t is the number of years after 2000.

b. Find the estimated wolf population in 2015.

EXPLORATIONS

41. The formula used to calculate a monthly lease payment or a monthly car payment (for a purchase rather than a lease) is given by $P = \frac{Ar(1 + r)^n - Vr}{(1 + r)^n - 1}$, where P is the monthly payment, A is the amount of the loan, r is the *monthly* interest rate as a decimal, n is the number of months of the loan or lease, and V is the residual value of the car at the end of the lease. For a car purchase, $V = 0$.

a. If the annual interest rate for a loan is 9%, what is the monthly interest rate as a decimal?

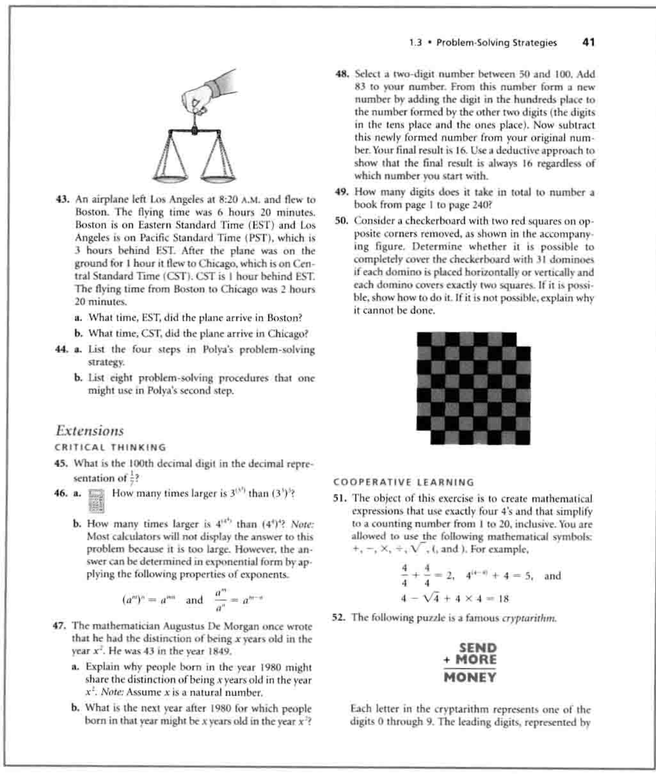
b. Write the formula for a monthly car payment when the car is purchased rather than leased.

c. Suppose you lease a car for 5 years. Find the monthly lease payment if the lease amount is $10,000, the residual value is $6000, and the annual interest rate is 6%.

d. Suppose you purchase a car and secure a 5-year loan for $10,000 at an annual interest rate of 6%. Find the monthly payment.

e. Why are the answers to parts c and d different?

page 384



1.3 • Problem-Solving Strategies 41

43. An airplane left Los Angeles at 8:20 A.M. and flew to Boston. The flying time was 6 hours 20 minutes. Boston is on Eastern Standard Time (EST) and Los Angeles is on Pacific Standard Time (PST), which is 3 hours behind EST. After the plane was on the ground for 1 hour it flew to Chicago, which is on Central Standard Time (CST). CST is 1 hour behind EST. The flying time from Boston to Chicago was 2 hours 20 minutes.

a. What time, EST, did the plane arrive in Boston?

b. What time, CST, did the plane arrive in Chicago?

44. a. List the four steps in Polya's problem-solving strategy.

b. List eight problem-solving procedures that one might use in Polya's second step.

Extensions

CRITICAL THINKING

45. What is the 100th decimal digit in the decimal representation of $\frac{1}{7}$?

46. a. How many times larger is $3^{(3^3)}$ than $(3^3)^3$?

b. How many times larger is $4^{(4^4)}$ than $(4^4)^4$? *Note:* Most calculators will not display the answer to this problem because it is too large. However, the answer can be determined in exponential form by applying the following properties of exponents.

$$(a^m)^n = a^{mn} \quad \text{and} \quad \frac{a^m}{a^n} = a^{m-n}$$

47. The mathematician Augustus De Morgan once wrote that he had the distinction of being x years old in the year x^2. He was 43 in the year 1849.

a. Explain why people born in the year 1980 might share the distinction of being x years old in the year x^2. *Note:* Assume x is a natural number.

b. What is the next year after 1980 for which people born in that year might be x years old in the year x^2?

48. Select a two-digit number between 50 and 100. Add 83 to your number. From this number form a new number by adding the digit in the hundreds place to the number formed by the other two digits (the digits in the tens place and the ones place). Now subtract this newly formed number from your original number. Your final result is 16. Use a deductive approach to show that the final result is always 16 regardless of which number you start with.

49. How many digits does it take in total to number a book from page 1 to page 240?

50. Consider a checkerboard with two red squares on opposite corners removed, as shown in the accompanying figure. Determine whether it is possible to completely cover the checkerboard with 31 dominoes if each domino is placed horizontally or vertically and each domino covers exactly two squares. If it is possible, show how to do it. If it is not possible, explain why it cannot be done.

COOPERATIVE LEARNING

51. The object of this exercise is to create mathematical expressions that use exactly four 4's and that simplify to a counting number from 1 to 20, inclusive. You are allowed to use the following mathematical symbols: +, −, ×, ÷, √, (, and). For example,

$$\frac{4}{4} + \frac{4}{4} = 2, \quad 4^{(4-4)} + 4 = 5, \quad \text{and}$$
$$4 - \sqrt{4} + 4 \times 4 = 18$$

52. The following puzzle is a famous *cryptarithm*.

SEND
+ MORE
MONEY

Each letter in the cryptarithm represents one of the digits 0 through 9. The leading digits, represented by

page 41

Extensions

Extension exercises are placed at the end of each exercise set. As the name implies, these exercises are designed to extend concepts. In most cases these exercises are more challenging and require more time and effort than the preceding exercises. The Extension exercises always include at least two of the following types of exercises:

Critical Thinking
Cooperative Learning
Explorations

Some Critical Thinking exercises require the application of two or more procedures or concepts.

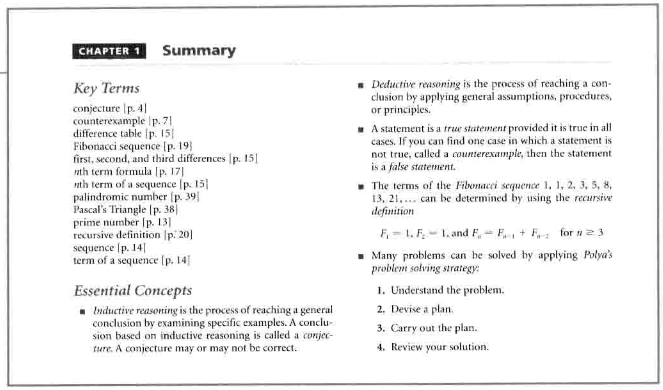
The Cooperative Learning exercises are designed for small groups of 2 to 4 students.

Many of the Exploration exercises require students to search on the Internet or through reference materials in a library.

End of Chapter

Chapter Summary

At the end of each chapter there is a Chapter Summary that includes *Key Terms* and *Essential Concepts* that were covered in the chapter. These chapter summaries provide a single point of reference as the student prepares for an examination. Each key word references the page number of the chapter where the word was first introduced.



CHAPTER 1 Summary

Key Terms

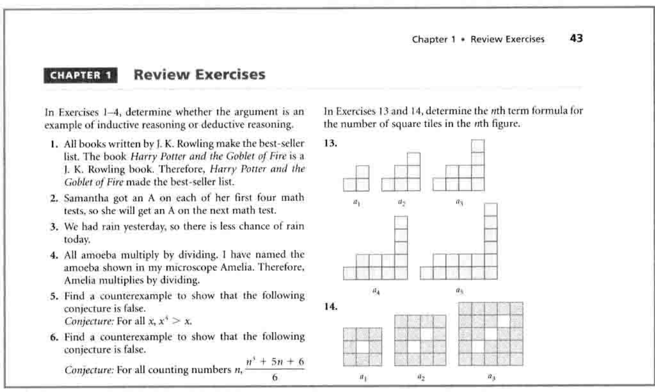
conjecture [p. 4]
counterexample [p. 7]
difference table [p. 15]
Fibonacci sequence [p. 19]
first, second, and third differences [p. 15]
*n*th term formula [p. 17]
*n*th term of a sequence [p. 15]
palindromic number [p. 39]
Pascal's Triangle [p. 38]
prime number [p. 13]
recursive definition [p. 20]
sequence [p. 14]
term of a sequence [p. 14]

Essential Concepts

- *Inductive reasoning* is the process of reaching a general conclusion by examining specific examples. A conclusion based on inductive reasoning is called a *conjecture*. A conjecture may or may not be correct.
- *Deductive reasoning* is the process of reaching a conclusion by applying general assumptions, procedures, or principles.
- A statement is a *true statement* provided it is true in all cases. If you can find one case in which a statement is not true, called a *counterexample*, then the statement is a *false statement*.
- The terms of the *Fibonacci sequence* 1, 1, 2, 3, 5, 8, 13, 21, ... can be determined by using the *recursive definition*

  $F_1 = 1, F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$
- Many problems can be solved by applying *Polya's problem solving strategy:*
  1. Understand the problem.
  2. Devise a plan.
  3. Carry out the plan.
  4. Review your solution.

page 42



Chapter 1 • Review Exercises 43

CHAPTER 1 Review Exercises

In Exercises 1–4, determine whether the argument is an example of inductive reasoning or deductive reasoning.

1. All books written by J. K. Rowling make the best-seller list. The book *Harry Potter and the Goblet of Fire* is a J. K. Rowling book. Therefore, *Harry Potter and the Goblet of Fire* made the best-seller list.
2. Samantha got an A on each of her first four math tests, so she will get an A on the next math test.
3. We had rain yesterday, so there is less chance of rain today.
4. All amoeba multiply by dividing. I have named the amoeba shown in my microscope Amelia. Therefore, Amelia multiplies by dividing.
5. Find a counterexample to show that the following conjecture is false.
   Conjecture: For all x, $x^4 > x$.
6. Find a counterexample to show that the following conjecture is false.
   Conjecture: For all counting numbers n, $\frac{n^3 + 5n + 6}{6}$

In Exercises 13 and 14, determine the *n*th term formula for the number of square tiles in the *n*th figure.

13.

14.

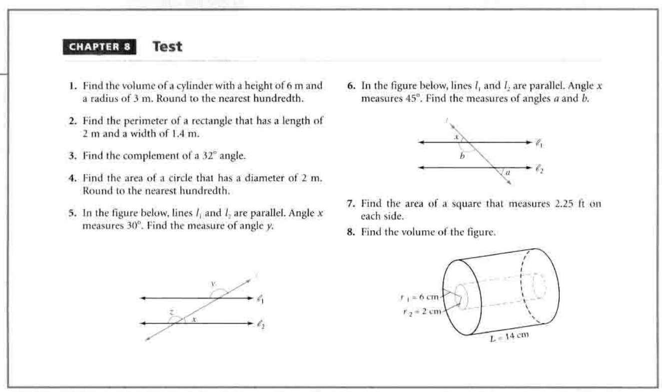
page 43

Chapter Review

Review exercises are found near the end of each chapter. These exercises were selected to help the student integrate the major topics presented in the chapter. The answers to all of the Chapter Review exercises appear in the answer section along with a section reference for each exercise. These section references indicate the section or sections where a student can locate the concepts needed to solve each exercise.

Chapter Test

The Chapter Test exercises are designed to simulate a possible test of the material in the chapter. The answers to all of the Chapter Test exercises appear in the answer section along with a section reference for each exercise. The section references indicate the section or sections where a student can locate the concepts needed to solve each exercise.



CHAPTER 8 Test

1. Find the volume of a cylinder with a height of 6 m and a radius of 3 m. Round to the nearest hundredth.
2. Find the perimeter of a rectangle that has a length of 2 m and a width of 1.4 m.
3. Find the complement of a 32° angle.
4. Find the area of a circle that has a diameter of 2 m. Round to the nearest hundredth.
5. In the figure below, lines l_1 and l_2 are parallel. Angle x measures 30°. Find the measure of angle y.
6. In the figure below, lines l_1 and l_2 are parallel. Angle x measures 45°. Find the measures of angles a and b.
7. Find the area of a square that measures 2.25 ft on each side.
8. Find the volume of the figure.

page 543

Supplements for the Instructor

Mathematical Excursions has an extensive support package for the instructor that includes:

Instructor's Annotated Edition (IAE): The *Instructor's Annotated Edition* is an exact replica of the student textbook with the following additional text-specific items for the instructor: answers to *all* of the end-of-section and end-of-chapter exercises, answers to *all* Excursion and Exploration exercises, Instructor Notes, Suggested Assignments, and icons denoting tables and art that appear in PowerPoint® slides. (The slides are available on the Instructor ClassPrep with HM Testing 6.0 CD-ROM and/or the files can be downloaded from our web site at math.college.hmco.com/instructors).

Instructor's Resource Manual: The *Instructor's Resource Manual* offers worked-out solutions to *all* of the exercises in each exercise set as well as answers to the Excursion and Exploration exercises. The manual contains a variety of ready-to-use printed Chapter Tests (two formats: free response and multiple choice). These tests can also be downloaded from our web site at math.college.hmco.com/instructors. In addition to the ready-to-use Chapter Tests, a *Printed Test Bank* is also available in the manual. The *Printed Test Bank* provides a printout of one example of each of the algorithmic items in the HM Testing 6.0 (See the description of HM Testing 6.0 below, under HM ClassPrep with HM Testing 6.0 CD-ROM). Also included in the manual are suggested syllabi that provide instructors with options for sequencing the course.

HM ClassPrep with HM Testing CD-ROM: This CD-ROM is a combination of two course management tools.

- HM Testing 6.0 computerized testing software provides instructors with an array of algorithmic test items, allowing for the creation of an unlimited number of tests for each chapter, including cumulative tests and final exams. HM Testing also offers online testing via a Local Area Network (LAN) or the Internet, as well as a grade book function.
- HM ClassPrep features supplements and text-specific resources, such as: suggested syllabi that provide instructors with options for sequencing the course, as well as an Index of Applications, Chapter Tests, PowerPoint® slides, Microsoft® Excel spreadsheets, Graphing Calculator Guide, and an Excel Guide.

Instructor Text-Specific Web Site: The companion web site provides additional teaching resources such as: suggested syllabi that provide instructors with options for sequencing the course as well as an Index of Applications, Chapter Tests, Excursions from the text, PowerPoint® slides, Excel spreadsheets, Graphing Calculator Guide, and an Excel Guide. Visit math.college.hmco.com/instructors and choose *Mathematical Excursions* from the list provided on the site. Appropriate items will be password-protected. Instructors have access to the student web site as well.

eduSpace®: eduSpace® is a text-specific online learning environment that combines algorithmic tutorials with homework capabilities and classroom management functions. Please contact your Houghton Mifflin sales representative for detailed information about the course content available for this text.

Two levels of service are provided for instructors.

- **Electronic grading** allows the instructor to complete their grades electronically and record students' results on the quizzes provided on eduSpace®.

- **Course Management** allows the instructor to manage the course on a lecture-basis or manage a distance-learning course online.

Supplements for the Student

Mathematical Excursions has an extensive support package for the student that includes:

Student Solutions Manual: The *Student Solutions Manual* contains complete, worked-out solutions to *all* odd-numbered exercises and *all* of the solutions to the Chapter Reviews and Chapter Tests in the text.

CLAST Preparation Student Guide: The CLAST Preparation Student Guide is a competency-based study guide that reviews and offers preparatory material for the CLAST (College Level Academic Skills Test) objectives required by the State of Florida for mathematics. The guide includes a correlation of the CLAST objectives to the *Mathematical Excursions* text, worked-out examples, practice examples, cumulative reviews, and sample diagnostic tests with grading sheets.

HMmathSpace™ Tutorial CD ROM: . This new tutorial CD ROM allows students to practice skills and review concepts as many times as necessary by using algorithmically generated exercises and step-by-step solutions for practice. Among the many features of the CD-ROM, there is a Prerequisite Algebra Review, Graphing Calculator Guide, Excel Guide, and Excel spreadsheets that are referred to in the text.

SMARTTHINKING™ Live, On-line Tutoring: Houghton Mifflin has partnered with SMARTTHINKING™ to provide an easy-to-use, effective, online tutorial service. Through state-of-the-art tools and a two-way whiteboard, students communicate in real-time with qualified e-structors who can help the students understand difficult concepts and guide them through the problem-solving process while studying or completing homework.

Four levels of service are offered to the students.

- **Live, online tutoring support** is available Sunday–Thursday 2pm–5pm and 9pm–1pm EST (hours are subject to change).
- **Question submission** allows students to submit questions to the tutor outside the scheduled hours and receive a response within 24 hours.
- **Pre-scheduled time** allows students to schedule tutoring with an e-structor in advance.
- **Review past online sessions** allows students to access and review their progress from previous sessions on a personal academic home page.

Houghton Mifflin Instructional Videos/DVD's: These text-specific Videos and DVD's, professionally produced by Dana Mosely, provide explanations of key concepts, examples, and exercises in a lecture-based format. They offer students a valuable resource for further instruction and review.

Student Text-Specific Web Site: This textbook has a companion web site that provides additional learning resources. Visit math.college.hmco.com/students and choose *Mathematical Excursions* from the list provided on the site.

eduSpace®: eduSpace® is a text-specific online learning environment that combines algorithmic tutorials with homework capabilities. Text-specific content is available to help you understand the mathematics covered in this textbook.

Four levels of service are offered to the students.

- **Tutorials** help the student to review concepts that he or she may miss because of an absence from class. The students can also use the tutorials to review material for upcoming quizzes and tests.
- **Practice exercises** allow the student to reinforce skills and concepts, not yet mastered, by completing different types of exercises.
- **Homework assignments** can be accessed, completed, and submitted online if the instructor assigns these assignments.
- **Quizzes** can be used for practice or taken for a grade if your instructor assigns the quizzes.

Acknowledgments

The authors would like to thank the people who have reviewed this manuscript and provided many valuable suggestions.

Randall Allbritton
Daytona Beach Community College

Isali Alsina
Kean University

Bernadette Antkoviak
Harrisburg Area Community College

Charles N. Baker
West Liberty State College

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Robert B. Sackett
Erie Community College

Mary Lee Seitz
Erie Community College—City Campus

Aaron Keith Trautwein
Carthage College

Susan Williford
Columbia State Community College

The authors would also like to give special thanks to Delaney Carrier, Tim Hempleman, Gina Sanders, and Lauri Semarne for their extra help with the preparation of this manuscript.

Welcome to *Mathematical Excursions.* As you begin this course we know two important facts: (1) We want you to succeed. (2) You want to succeed. In order to accomplish these goals, an effort is required from each of us. For the next few pages, we are going to show you what is required of you to achieve that success and how you can use the features of this text to be successful.

Motivation

> **TAKE NOTE**
>
> Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation to swim but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning.

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways that you can benefit from mathematics. But, in the end, the motivation must come from you. On the first day of class it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

To stay motivated, there must be outcomes from this course that are worth your time, money, and energy. List some reasons you are taking this course. Do not make a mental list—actually write them out.

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____

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Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate, it is a prerequisite for a course you must take, or because it is required for your major. Although you may not agree that this course should be necessary, it is! If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal to complete your education!

Commitment

To be successful, you must make a commitment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better.

ACTIVITY	*TIME SPENT*	*TIME WISHED SPENT*

Thinking about these activities, put the number of hours that you spend each week practicing these activities next to the activity. Next to that number, indicate the number of hours a week you would like to spend on these activities.

Whether you listed surfing or sailing, aerobics or restoring cars, or any other activity you enjoy, note how many hours a week you spend on each activity. To succeed in math, you must be willing to commit the same amount of time. Success requires some sacrifice.

The "I Can't Do Math" Syndrome

There may be things you cannot do, for instance, lift a two-ton boulder. You can, however, do math. It is much easier than lifting the two-ton boulder. When you first learned the activities you listed above, you probably could not do them well. With practice, you got better. With practice, you will be better at math. Stay focused, motivated, and committed to success.

It is difficult for us to emphasize how important it is to overcome the "I Can't Do Math Syndrome." If you listen to interviews of very successful athletes after a particularly bad performance, you will note that they focus on the positive aspect of what they did, not the negative. Sports psychologists encourage athletes to always be positive—to have a "Can Do" attitude. You need to develop this attitude toward math.

Strategies for Success

Know the Course Requirements To do your best in this course, you must know exactly what your instructor requires. Course requirements may be stated in a *syllabus,* which is a printed outline of the main topics of the course, or they may be presented orally. When they are listed in a syllabus or on other printed pages, keep them in a safe place. When they are presented orally, make sure to take complete notes. In either case, it is important that you understand them completely and follow them exactly. Be sure you know the answer to each of the following questions.

1. What is your instructor's name?
2. Where is your instructor's office?
3. At what times does your instructor hold office hours?
4. Besides the textbook, what other materials does your instructor require?
5. What is your instructor's attendance policy?
6. If you must be absent from a class meeting, what should you do before returning to class? What should you do when you return to class?
7. What is the instructor's policy regarding collection or grading of homework assignments?
8. What options are available if you are having difficulty with an assignment? Is there a math tutoring center?
9. If there is a math lab at your school, where is it located? What hours is it open?
10. What is the instructor's policy if you miss a quiz?