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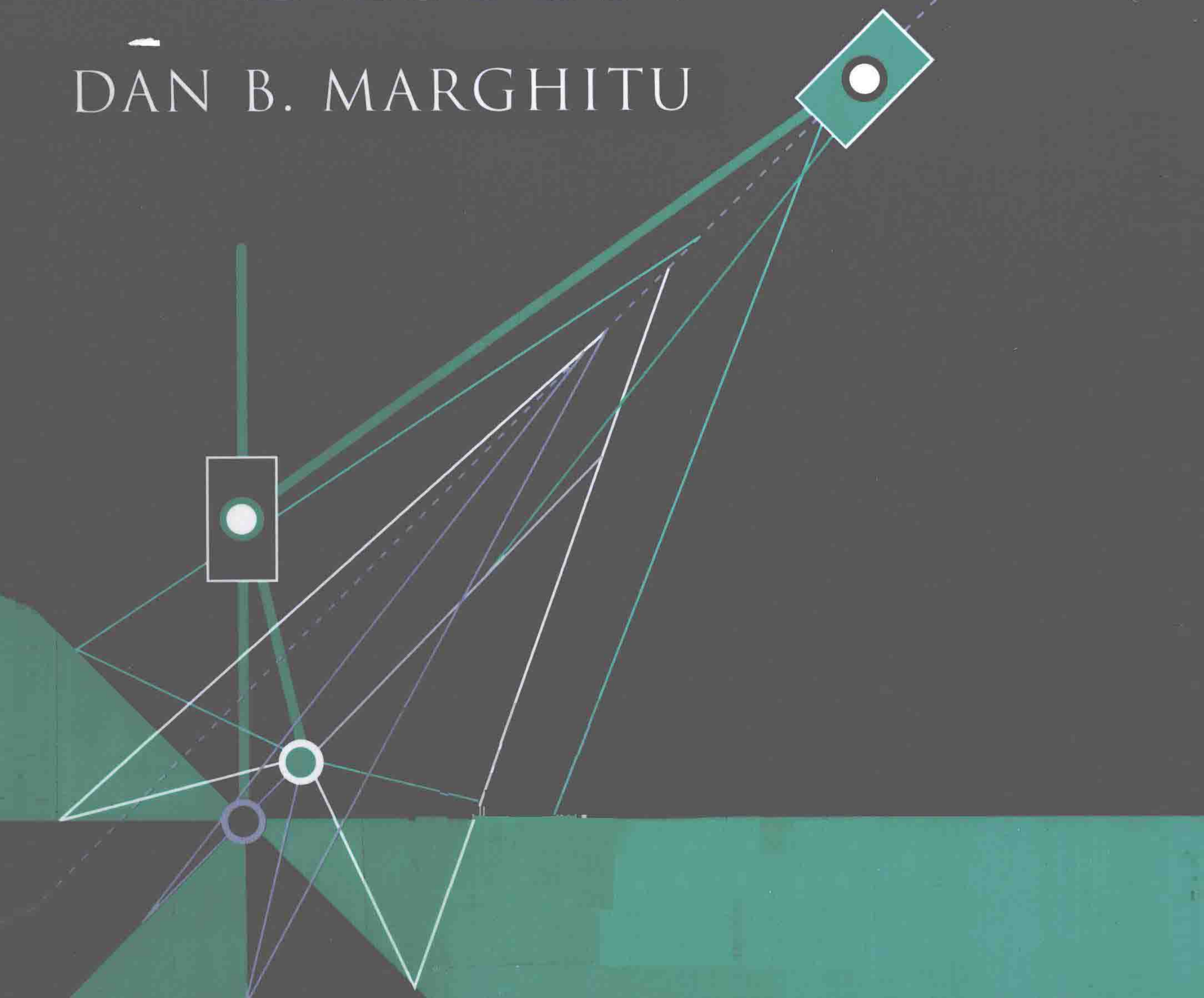
DAN B. MARGHITU



KINEMATIC CHAINS AND MACHINE COMPONENTS DESIGN

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Elsevier Academic Press
30 Corporate Drive, Suite 400, Burlington, MA 01803, USA
525 B Street, Suite 1900, San Diego, California 92101-4495, USA
84 Theobald's Road, London WC1X 8RR, UK

This book is printed on acid-free paper. ♾

Library of Congress Cataloging-in-Publication Data

Marghitu, Dan B.
Kinematic chains and machine components design / Dan Marghitu.
p. cm.
Includes bibliographical references and index.
ISBN 0-12-471352-1 (alk. paper)
1. Machinery, Kinematics of. 2. Machinery–Design and construction. I. Title.

TJ175.M243 2005
621.8′11–dc22

2004061907

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library.

ISBN: 0-12-471352-1

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Printed in the United States of America
05 06 07 08 09 10 9 8 7 6 5 4 3 2 1

Preface

A number of bodies linked by joints form a kinematic chain. On the basis of the presence of loops in a mechanical structure it can be distinguished closed kinematic chains, if there are one or more loops so that each link and each joint is contained in at least one of them. A closed kinematic chain have no open attachment point. An open kinematic chain contains no loop. Kinematic chains design is a vital component of modern machine design practice. Kinematic chains are used to transmit forces and moments and to manipulate objects. A knowledge of the kinematic and dynamic properties of these machines is crucial for their design and control. A feature of this book and its main distinction from other books is that it presents a different method for kinematic and dynamic force analysis of kinematic chains. The other important feature of the approach used here is the attention given to the solution of the problems using the symbolical software Mathematica. Methods, algorithms and software packages for the solution of classical mechanical problems are presented. The book presents texts that are teachable and computer-oriented.

The book will assist all those interested in the design of mechanisms, manipulators, building machines, textile machines, vehicles, aircraft, satellites, ships, biomechanical systems (vehicle simulators, barrier tests, human motion studies, etc.), controlled mechanical systems, mechatronical devices and many others.

This book is appropriate for use as a text for undergraduate or graduate courses in mechanical engineering dealing with the subjects of the analysis and design of mechanisms, vehicle dynamics, mechatronics and multibody systems and machine components design. A basic knowledge of mechanics and calculus is assumed. The book may also be useful for practicing engineers and researchers in the fields of machine design and dynamics, and also biomechanics and mechatronics.

About the Author

Dan Marghitu is currently a professor at AUBURN UNIVERSITY, Mechanical Engineering Department, involved in teaching and research activities.

He received a D.E.A. from Paul Sabatier University and a Ph.D. from Southern Methodist University.

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I.1.1 Vector Algebra

Vector Terminology

Scalars are mathematics quantities that can be fully defined by specifying their magnitude in suitable units of measure. The mass is a scalar and can be expressed in kilograms, the time is a scalar and can be expressed seconds, and the temperature can be expressed in degrees.

Vectors are quantities that require the specification of magnitude, orientation, and sense. The characteristics of a vector are the magnitude, the orientation, and the sense.

The *magnitude* of a vector is specified by a positive number and a unit having appropriate dimensions. No unit is stated if the dimensions are those of a pure number. The *orientation* of a vector is specified by the relationship between the vector and given reference lines and/or planes. The *sense* of a vector is specified by the order of two points on a line parallel to the vector.

Orientation and sense together determine the *direction* of a vector. The *line of action* of a vector is a hypothetical infinite straight line collinear with the vector. Displacement, velocity, and force are examples of vectors.

To distinguish vectors from scalars it is customary to denote vectors by boldface letters. Thus, the vector shown in Figure I.1.1(a) is denoted by $\mathbf{r}$ or $\mathbf{r}_{AB}$. The symbol $|\mathbf{r}| = r$ represents the magnitude (or module, or absolute value) of the vector $\mathbf{r}$. In handwritten work a distinguishing mark is used for vectors, such as an arrow over the symbol, $\vec{r}$ or $\overrightarrow{AB}$, a line over the symbol, $\bar{r}$, or an underline, $\underline{r}$.

The vectors are depicted by either straight or curved arrows. A vector represented by a straight arrow has the direction indicated by the arrow. The direction of a vector represented by a curved arrow is the same as the direction in which a right-handed screw moves when the axis of the screw is normal to the plane in which the arrow is drawn and the screw is rotated as indicated by the arrow.

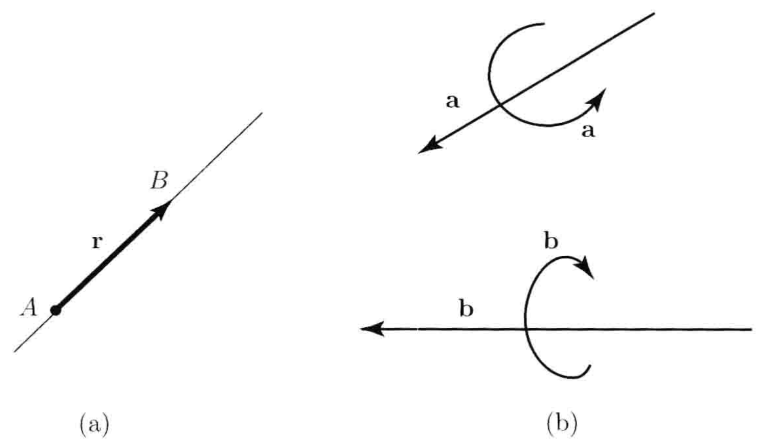

FIGURE I.1.1 *Vector representations: (a) straight arrow and (b) straight and curved arrows.*

Figure I.1.1(b) shows representations of vectors. Sometimes vectors are represented by means of a straight or curved arrow together with a measure number. In this case the vector is regarded as having the direction indicated by the arrow if the measure number is positive, and the opposite direction if it is negative.

A *bound* (or *fixed*) vector is a vector associated with a particular point P in space (Fig. I.1.2). The point P is the *point of application* of the vector, and the line passing through P and parallel to the vector is the line of action of the vector. The point of application can be represented as the tail [Fig. I.1.2(a)] or the head of the vector arrow [Fig. I.1.2(b)].

A *free* vector is not associated with a particular point or line in space. A *transmissible* (or *sliding*) vector is a vector that can be moved along its line of action without change of meaning.

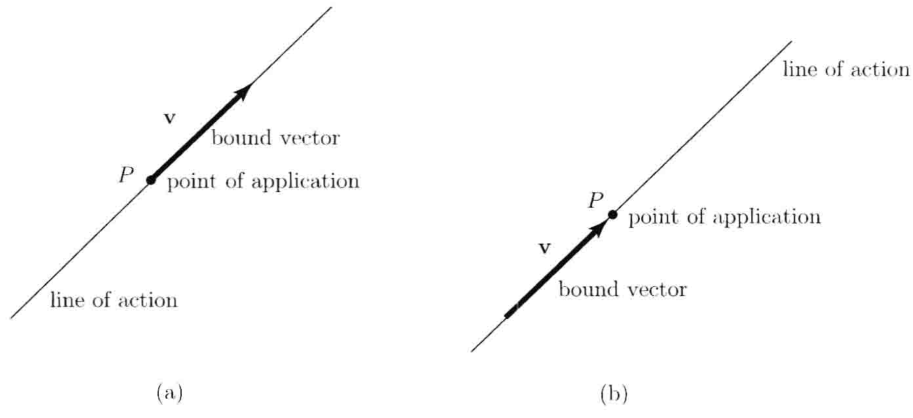

FIGURE I.1.2 *Bound or fixed vector: (a) point of application represented as the tail of the vector arrow and (b) point of application represented as the head of the vector arrow.*

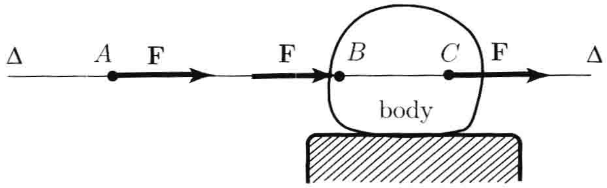

FIGURE I.1.3 *Transmissible vector: the force vector* **F** *can be applied anywhere along the line* Δ.

To move the body in Figure I.1.3 the force vector **F** can be applied anywhere along the line Δ or may be applied at specific points A, B, C. The force vector **F** is a transmissible vector because the resulting motion is the same in all cases.

The force **F** applied at B will cause a different deformation of the body than the same force **F** applied at a different point C. The points B and C are on the body. If one is interested in the deformation of the body, the force **F** positioned at C is a bound vector.

The operations of vector analysis deal only with the characteristics of vectors and apply, therefore, to both bound and free vectors. Vector analysis is a branch of mathematics that deals with quantities that have both magnitude and direction.

Vector Equality

Two vectors **a** and **b** are said to be equal to each other when they have the same characteristics

$$\mathbf{a} = \mathbf{b}.$$

Equality does not imply physical equivalence. For instance, two forces represented by equal vectors do not necessarily cause identical motions of a body on which they act.

Product of a Vector and a Scalar

Definition

The product of a vector $\mathbf{v}$ and a scalar s, $s\mathbf{v}$ or $\mathbf{v}s$, is a vector having the following characteristics:

1. Magnitude.

$$|s\mathbf{v}| \equiv |\mathbf{v}s| = |s||\mathbf{v}|,$$

   where $|s|$ denotes the absolute value (or magnitude, or module) of the scalar s.
2. Orientation. $s\mathbf{v}$ is parallel to $\mathbf{v}$. If $s = 0$, no definite orientation is attributed to $s\mathbf{v}$.
3. Sense. If $s > 0$, the sense of $s\mathbf{v}$ is the same as that of $\mathbf{v}$. If $s < 0$, the sense of $s\mathbf{v}$ is opposite to that of $\mathbf{v}$. If $s = 0$, no definite sense is attributed to $s\mathbf{v}$.

Zero Vectors

Definition

A *zero vector* is a vector that does not have a definite direction and whose magnitude is equal to zero. The symbol used to denote a zero vector is $\mathbf{0}$.

Unit Vectors

Definition

A *unit vector* (versor) is a vector with the magnitude equal to 1. Given a vector $\mathbf{v}$, a unit vector $\mathbf{u}$ having the same direction as $\mathbf{v}$ is obtained by forming the quotient of $\mathbf{v}$ and $|\mathbf{v}|$:

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}.$$

Vector Addition

The sum of a vector $\mathbf{v}_1$ and a vector $\mathbf{v}_2$: $\mathbf{v}_1 + \mathbf{v}_2$ or $\mathbf{v}_2 + \mathbf{v}_1$ is a vector whose characteristics are found by either graphical or analytical processes. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ add according to the parallelogram law: $\mathbf{v}_1 + \mathbf{v}_2$ is equal to the diagonal of a parallelogram formed by the graphical representation of the vectors [(Fig. I.1.4(a))]. The vector $\mathbf{v}_1 + \mathbf{v}_2$ is called the *resultant* of $\mathbf{v}_1$ and $\mathbf{v}_2$. The vectors can be added by moving them successively to parallel positions so that the head of one vector connects to the tail of the next vector. The resultant is the vector whose tail connects to the tail of the first vector, and whose head connects to the head of the last vector [(Fig. I.1.4(b))].

The sum $\mathbf{v}_1 + (-\mathbf{v}_2)$ is called the *difference* of $\mathbf{v}_1$ and $\mathbf{v}_2$ and is denoted by $\mathbf{v}_1 - \mathbf{v}_2$ [(Figs. I.1.4(c) and I.1.4 (d))].

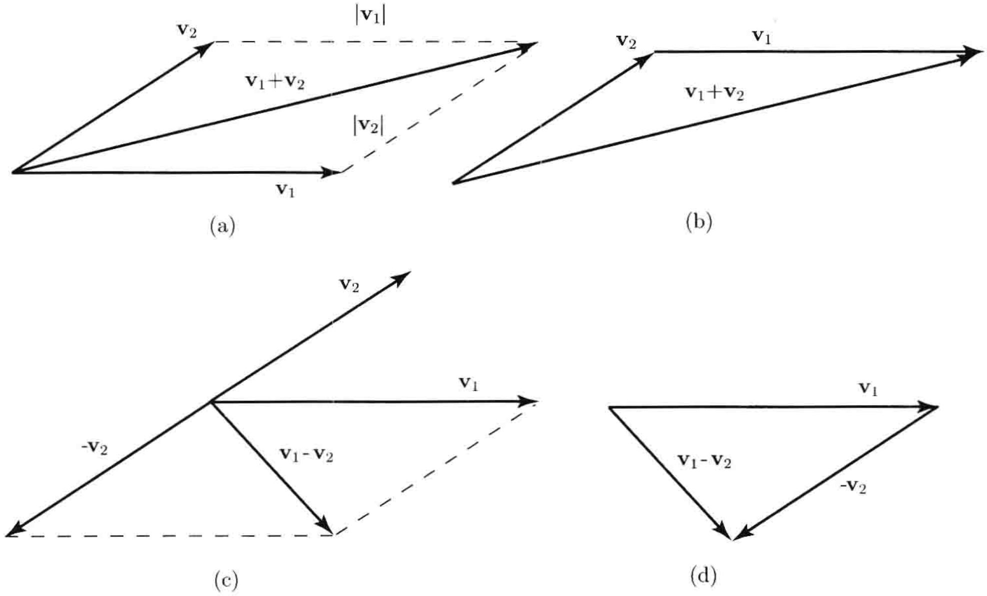

FIGURE I.1.4 *Vector addition: (a) parallelogram law, (b) moving the vectors successively to parallel positions. Vector difference: (c) parallelogram law, (d) moving the vectors successively to parallel positions.*

The sum of n vectors $\mathbf{v}_i$, $i = 1, \ldots, n$,

$$\sum_{i=1}^{n} \mathbf{v}_i \quad \text{or} \quad \mathbf{v}_1 + \mathbf{v}_2 + \cdots + \mathbf{v}_n$$

is called the *resultant* of the vectors $\mathbf{v}_i$, $i = 1, \ldots, n$.

The vector addition is:

1. Commutative. The characteristics of the resultant are independent of the order in which the vectors are added (commutativity):

$$\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_2 + \mathbf{v}_1.$$

2. Associative. The characteristics of the resultant are not affected by the manner in which the vectors are grouped (associativity):

$$\mathbf{v}_1 + (\mathbf{v}_2 + \mathbf{v}_3) = (\mathbf{v}_1 + \mathbf{v}_2) + \mathbf{v}_3.$$

3. Distributive. The vector addition obeys the following laws of distributivity:

$$\mathbf{v} \sum_{i=1}^{p} s_i = \sum_{i=1}^{p} (\mathbf{v} s_i), \quad \text{for } s_i \neq 0,\ s_i \in \mathcal{R},$$

$$s \sum_{i=1}^{n} \mathbf{v}_i = \sum_{i=1}^{n} (s \mathbf{v}_i), \quad \text{for } s \neq 0,\ s \in \mathcal{R},$$

where $\mathcal{R}$ is the set of real numbers.

Every vector can be regarded as the sum of n vectors ($n = 2, 3, \ldots$) of which all but one can be selected arbitrarily.

Resolution of Vectors and Components

Let $\mathbf{\imath}_1$, $\mathbf{\imath}_2$, $\mathbf{\imath}_3$ be any three unit vectors not parallel to the same plane (noncollinear vectors):

$$|\mathbf{\imath}_1| = |\mathbf{\imath}_2| = |\mathbf{\imath}_3| = 1$$

For a given vector $\mathbf{v}$ (Fig. I.1.5), there are three unique scalars, v_1, v_2, v_3, such that $\mathbf{v}$ can be expressed as:

$$\mathbf{v} = v_1 \mathbf{\imath}_1 + v_2 \mathbf{\imath}_2 + v_3 \mathbf{\imath}_3$$

The opposite action of addition of vectors is the *resolution* of vectors. Thus, for the given vector $\mathbf{v}$ the vectors $v_1\mathbf{\imath}_1$, $v_2\mathbf{\imath}_2$, and $v_3\mathbf{\imath}_3$ sum to the original vector. The vector $v_k\mathbf{\imath}_k$ is called the $\mathbf{\imath}_k$ *component* of $\mathbf{v}$ and v_k is called the $\mathbf{\imath}_k$ *scalar component* of $\mathbf{v}$, where $k = 1, 2, 3$. A vector is often replaced by its components since the components are equivalent to the original vector.

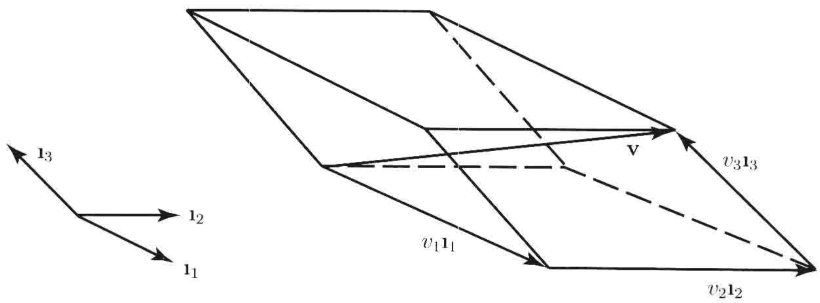

FIGURE I.1.5 *Resolution of a vector* $\mathbf{v}$ *and components.*

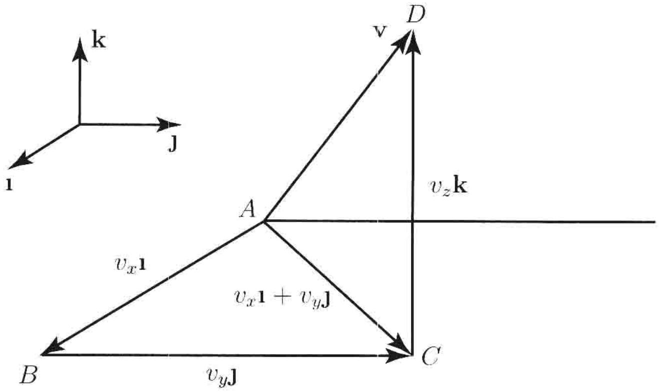

FIGURE I.1.6 *Cartesian reference frame and the orthogonal scalar components* v_x, v_y, v_z.

Every vector equation $\mathbf{v} = \mathbf{0}$, where $\mathbf{v} = v_1\mathbf{1}_1 + v_2\mathbf{1}_2 + v_3\mathbf{1}_3$, is equivalent to three scalar equations: $v_1 = 0, \quad v_2 = 0, \quad v_3 = 0$.

If the unit vectors $\mathbf{1}_1, \mathbf{1}_2, \mathbf{1}_3$ are mutually perpendicular they form a *Cartesian reference frame*. For a Cartesian reference frame the following notation is used (Fig. I.1.6):

$$\mathbf{1}_1 \equiv \mathbf{1}, \quad \mathbf{1}_2 \equiv \mathbf{J}, \quad \mathbf{1}_3 \equiv \mathbf{k},$$

and

$$\mathbf{1} \perp \mathbf{J}, \quad \mathbf{1} \perp \mathbf{k}, \quad \mathbf{J} \perp \mathbf{k}.$$

The symbol $\perp$ denotes perpendicular.

When a vector $\mathbf{v}$ is expressed in the form $\mathbf{v} = v_x\mathbf{1} + v_y\mathbf{J} + v_z\mathbf{k}$, where $\mathbf{1}, \mathbf{J}, \mathbf{k}$ are mutually perpendicular unit vectors (Cartesian reference frame or orthogonal reference frame), the magnitude of $\mathbf{v}$ is given by

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

The vectors $\mathbf{v}_x = v_x\mathbf{\imath}$, $\mathbf{v}_y = v_y\mathbf{\jmath}$, and $\mathbf{v}_z = v_z\mathbf{k}$ are the *orthogonal* or *rectangular component vectors* of the vector $\mathbf{v}$. The measures v_x, v_y, v_z are the *orthogonal* or *rectangular scalar components* of the vector $\mathbf{v}$.

If $\mathbf{v}_1 = v_{1x}\mathbf{\imath} + v_{1y}\mathbf{\jmath} + v_{1z}\mathbf{k}$ and $\mathbf{v}_2 = v_{2x}\mathbf{\imath} + v_{2y}\mathbf{\jmath} + v_{2z}\mathbf{k}$, then the sum of the vectors is

$$\mathbf{v}_1 + \mathbf{v}_2 = (v_{1x} + v_{2x})\,\mathbf{\imath} + \left(v_{1y} + v_{2y}\right)\mathbf{\jmath} + (v_{1z} + v_{2z})\, v_{1z}\mathbf{k}.$$

Angle Between Two Vectors

Two vectors **a** and **b** are considered. One can move either vector parallel to itself (leaving its sense unaltered) until their initial points (tails) coincide. The *angle* between **a** and **b** is the angle θ in Figures I.1.7(a) and I.1.7(b). The angle between **a** and **b** is denoted by the symbols $(\mathbf{a}, \mathbf{b})$ or $(\mathbf{b}, \mathbf{a})$. Figure I.1.7(c) represents the case $(\mathbf{a}, \mathbf{b}) = 0$, and Figure I.1.7(d) represents the case $(\mathbf{a}, \mathbf{b}) = 180°$.

The direction of a vector $\mathbf{v} = v_x\mathbf{\imath} + v_y\mathbf{\jmath} + v_z\mathbf{k}$ relative to a Cartesian reference, $\mathbf{\imath}$, $\mathbf{\jmath}$, $\mathbf{k}$, is given by the cosines of the angles formed by the vector and the respective unit vectors.

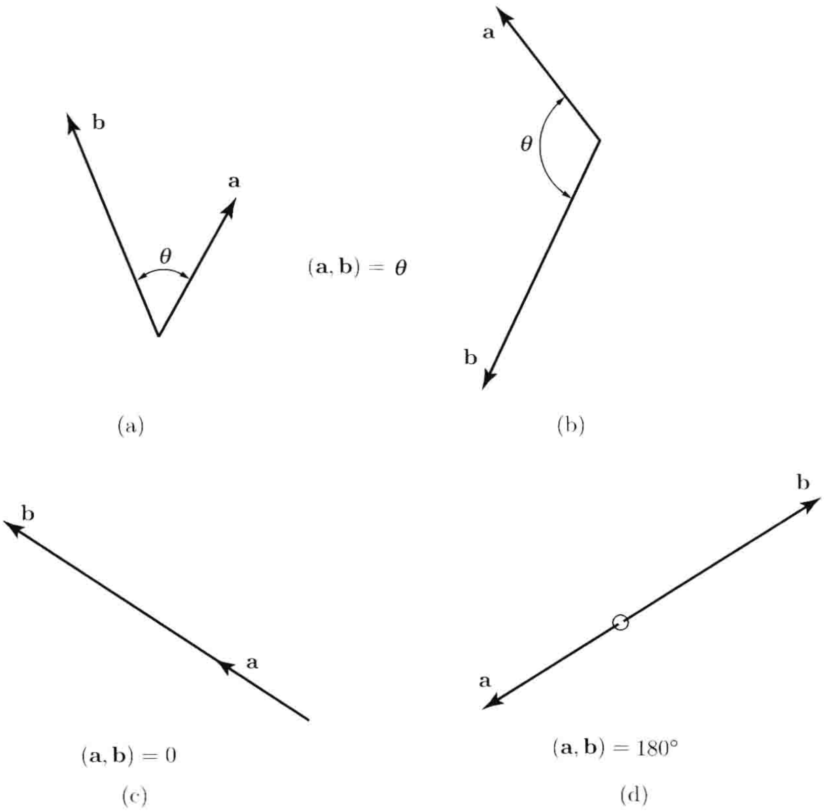

FIGURE I.1.7 *The angle θ between the vectors* **a** *and* **b**: *(a) $0 < \theta < 90°$, (b) $90° < \theta < 180°$, (c) $\theta = 0°$, and (d) $\theta = 180°$.*

These are called *direction cosines* and are denoted as (Fig. I.1.8):

$$\cos(\mathbf{v}, \mathbf{\imath}) = \cos\alpha = l; \;\; \cos(\mathbf{v}, \mathbf{\jmath}) = \cos\beta = m; \;\; \cos(\mathbf{v}, \mathbf{k}) = \cos\gamma = n.$$

The following relations exist:

$$v_x = |\mathbf{v}|\cos\alpha; \;\; v_y = |\mathbf{v}|\cos\beta; \;\; v_z = |\mathbf{v}|\cos\gamma,$$

$$l^2 + m^2 + n^2 = 1, \;\; (v_x^2 + v_y^2 + v_z^2 = v^2).$$

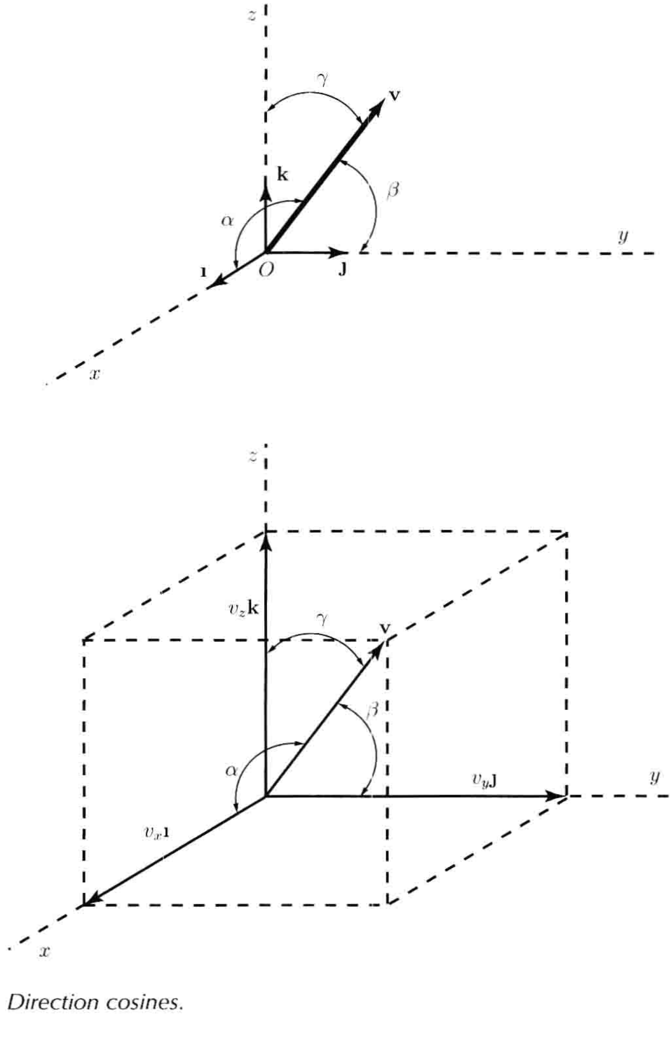

FIGURE I.1.8 *Direction cosines.*

Scalar (Dot) Product of Vectors

Definition

The scalar (dot) product of a vector **a** and a vector **b** is

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} = |\mathbf{a}|\,|\mathbf{b}| \cos(\mathbf{a}, \mathbf{b}).$$

For any two vectors **a** and **b** and any scalar s

$$(s\mathbf{a})\cdot\mathbf{b} = s(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a}\cdot(s\mathbf{b}) = s\mathbf{a} \cdot \mathbf{b}.$$

If

$$\mathbf{a} = a_x\mathbf{\imath} + a_y\mathbf{\jmath} + a_z\mathbf{k},$$

and

$$\mathbf{b} = b_x\mathbf{\imath} + b_y\mathbf{\jmath} + b_z\mathbf{k},$$

where $\mathbf{\imath}$, $\mathbf{\jmath}$, $\mathbf{k}$ are mutually perpendicular unit vectors, then

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z .$$

The following relationships exist:

$$\mathbf{\imath} \cdot \mathbf{\imath} = \mathbf{\jmath} \cdot \mathbf{\jmath} = \mathbf{k} \cdot \mathbf{k} = 1,$$
$$\mathbf{\imath} \cdot \mathbf{\jmath} = \mathbf{\jmath} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{\imath} = 0.$$

Every vector **v** can be expressed in the form

$$\mathbf{v} = \mathbf{\imath} \cdot \mathbf{v}\,\mathbf{\imath} + \mathbf{\jmath} \cdot \mathbf{v}\,\mathbf{\jmath} + \mathbf{k} \cdot \mathbf{v}\,\mathbf{k}.$$

The vector **v** can always be expressed as

$$\mathbf{v} = v_x\mathbf{\imath} + v_y\mathbf{\jmath} + v_z\mathbf{k}.$$

Dot multiply both sides by $\mathbf{\imath}$

$$\mathbf{\imath} \cdot \mathbf{v} = v_x\mathbf{\imath} \cdot \mathbf{\imath} + v_y\mathbf{\imath} \cdot \mathbf{\jmath} + v_z\mathbf{\imath} \cdot \mathbf{k}.$$

But,

$$\mathbf{\imath} \cdot \mathbf{\imath} = 1, \quad \text{and} \quad \mathbf{\imath} \cdot \mathbf{\jmath} = \mathbf{\imath} \cdot \mathbf{k} = 0.$$

Hence,

$$\mathbf{\imath} \cdot \mathbf{v} = v_x.$$