INFINITE-DIMENSIONAL DYNAMICAL SYSTEMS IN ATMOSPHERIC AND OCEANIC SCIENCE

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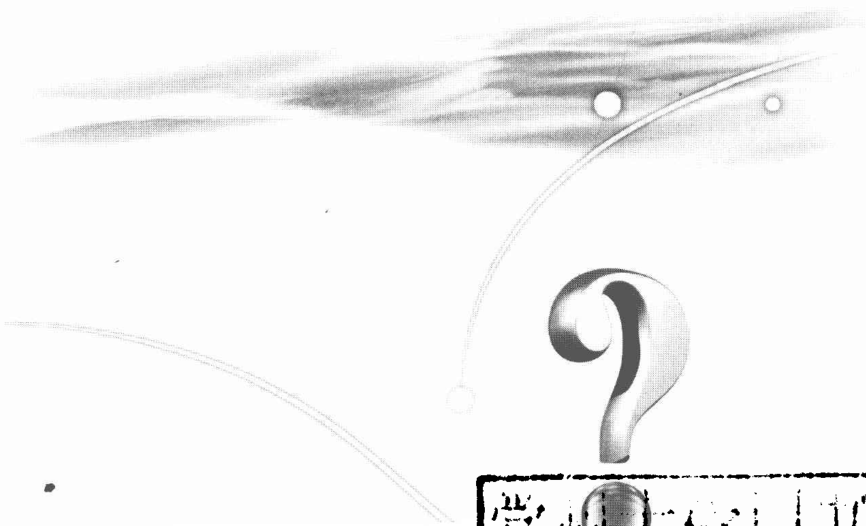

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Preface

The intent of the book is to introduce some results in the study of partial differential equations and infinite-dimensional dynamical systems in geophysical fluid dynamics, which has been mainly focused on the dynamics of large-scale phenomena in the atmosphere and the oceans. In the past several decades, there are many research works in the field. In 1979, Zeng Qingcun made some pioneering research on the theories of some mathematical models governing the atmospheric and oceanic motions. His works have aroused many mathematicians′ interest in the study of the theories about partial differential equations of the atmosphere and the oceans. From the 1980s, Chou Jifan and his collaborators made many works on the global analysis theory about the dissipative primitive equations of the atmosphere. In 1992, Jacques Louis Lions, Roger Temam and Wang Shouhong introduced a new formulation of the dissipative primitive equations of the atmosphere, and proved the global existence of weak solutions of these equations. Later, they also made many works on partial differential equations in the atmospheric and oceanic dynamics. Peter Constantin, Andrew Majda and Esteban G. Tabak studied the formation of strong fronts in the surface quasi-geostrophic equations. Andrew Majda and his collaborators contribute many theoretical and numerical works on PDEs and waves for the atmosphere and oceans. Mu Mu and Li Jianping both studied extensively on PDEs in the atmospheric and oceanic dynamics. In 2005, Cao Chongsheng and Edriss S. Titi proved the global well-posedness for the 3D viscous primitive equations. Recently, Zhou Xiuji presented the necessity and great meaning of the atmospheric random dynamics research. From 2006, the authors obtained some results about the primitive equations and some stochastic PDEs in the atmospheric and oceanic science.

This book consists of five chapters. In Chapter 1, we briefly recall some partial differential equations of the atmosphere and oceans. In Chapter 2, the quasi-geostrophic models of the atmospheric and the oceanic motions are introduced. In Chapter 3, we consider the initial boundary value problem for the three-dimensional viscous primitive equations of the large-scale moist atmosphere and oceans. In Chapter 4, we consider some stochastic models in the atmospheric and oceanic science. Chapter 5 is reserved for stability and instability theory of waves for the atmosphere and oceans.

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Guo Boling
Beijing
December, 2013

Contents

Chapter 1

Nonlinear Equations of the Atmospheric and the Oceanic Motions

There are usually two methods for predicting long-term weather and climate. First, by statistical methods, we can use the current climate, the historical record and numerical analysis to predict the future climate and the possible global climatic changes. Second, because air is compressible, and seawater is incompressible, by dynamical methods, we consider that the future status of climate is a consequence determined by the current status and the physical principles dominating these changes, thus we study equations and models describing the atmospheric and oceanic motions. Regarding weather prediction as an initial-boundary value problem in mathematical physics, we can establish numerical weather prediction models based on mathematical physical equation.

Numerical weather prediction is an outstanding applied research achievement of atmospheric science in the 20th century, of which theoretical foundation is the atmospheric dynamics. In 1922, Richardson introduced the concept of numerical weather prediction for the first time ([183]). His idea is that through solving the complete primitive equations governing the atmosphere motions numerically, one can simulate the evolution process of atmosphere, thus may predict weather quantitatively. Due to the weak calculation ability at that time, the dream of numerical weather prediction did not exist. Applying the long-wave theory and the scale-analysis theory established by Rossby and others, Charney set up a two-dimensional geostrophic model. With this model, he and his collaborators successfully made true 24-hour numerical weather prediction on the ENIAV computer of the Institute for Advanced Studies in Princeton for the first time. Along with the boom of atmosphere science and the enhancing of data dealing ability and numerical calculation ability of computer, researchers turn to numerical weather prediction by the primitive equation models from 1960s

([112,147,181,218]), greatly extend the time-range of numerical weather prediction. Afterward researchers started to make long-term numerical weather prediction, climate forecasting and numerical simulation of atmospheric circulation by some primitive equation models of the atmosphere and oceans.

To actualize long-term numerical weather prediction, climate prediction and numerical simulation of atmospheric circulation based on physical methods, the first thing is to establish some atmospheric and oceanic dynamical models, which are the nonlinear partial differential equations with initial-boundary value conditions which govern the atmospheric and oceanic motion. In this chapter, we mainly present basic and primitive equations and their boundary conditions which govern the atmospheric and oceanic motion. For more detail see [220], and also [84,145,162,205,211].

1.1 Basic Equations of the Atmospheric and the Oceanic Motions

1.1.1 *Basic Equations of the Atmosphere*

Regarding air and seawater as continuous media, one can use the Euler method to describe the atmospheric and oceanic motions. In the inertial coordinate frame (the coordinate axis is fixed with respect to the stellar), according to the Newton second law, the momentum conservation equation of the atmosphere is given by

$$\frac{\mathrm{d}_I \boldsymbol{V}_I}{\mathrm{d}t} = -\frac{1}{\rho}\mathrm{grad}_3 p + g_I + D,$$

where $\boldsymbol{V}_I$ is the absolute velocity of the atmosphere (velocity in the inertial coordinate frame), $\dfrac{\mathrm{d}_I \boldsymbol{V}_I}{\mathrm{d}t} = \dfrac{\partial \boldsymbol{V}_I}{\partial t} + (\boldsymbol{V}_I \cdot \boldsymbol{\nabla}_3)\boldsymbol{V}_I$ is the absolute acceleration (acceleration in the inertial coordinate frame), ρ is the density of air, p is the atmospheric pressure, $-\dfrac{1}{\rho}\mathrm{grad}_3 p$ is the pressure-gradient force, g_I is the gravity, and D is a molecular viscous force (molecular friction force, dissipative force), which is a dissipative force caused by air internal friction or turbulent momentum transmission.

In general, researchers are concerned with the relative motions of the atmosphere to the earth. So taking a coordinate frame rotating together with the earth as a reference frame, researchers can observe atmospheric relative motions. Suppose that the angular velocity of rotation in the rotating coordinate frame is $\boldsymbol{\Omega}$ (that is the rotational angular velocity of the earth), $\boldsymbol{V}$ is the atmospheric relative velocity, $\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}$ is the atmospheric relative acceleration in the rotating coordinate frame, then

$$\boldsymbol{V}_I = \boldsymbol{V} + \boldsymbol{\Omega} \times \mathbf{r},$$

$$\frac{\mathrm{d}_I \boldsymbol{V}_I}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{V}_I}{\mathrm{d}t} + \boldsymbol{\Omega} \times \boldsymbol{V}_I,$$

where $\mathbf{r}$ is the radius vector. The proof of the second equation above appears in section 1.5 in [172]. According to the previous three equations, we get in the rotating coordinate frame **the atmospheric momentum conservation equation**

$$\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = -\frac{1}{\rho}\mathrm{grad}_3 p + g - 2\boldsymbol{\Omega} \times \boldsymbol{V} + D, \tag{1.1.1}$$

where $g = g_I + \Omega^2 \mathbf{r}$ is commonly referred to gravity (Ω is the value of the earth rotation angular velocity), $-2\boldsymbol{\Omega} \times \boldsymbol{V}$ is the Coriolis force, $\Omega^2 \mathbf{r}$ is the inertial centrifugal force,

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{V} \cdot \boldsymbol{\nabla}_3$$

is the substantial derivative (often called the total derivative).

According to the mass conservation law, **the continuity equation** is given by

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \mathrm{div}_3 \boldsymbol{V} = 0. \tag{1.1.2}$$

In general, when describing large-scale motions of the troposphere and the stratosphere, one may consider dry air as ideal gas, and can get the **atmospheric state equation**

$$p = R\rho T, \tag{1.1.3}$$

where the vaporation in the atmosphere is negligible, T means the temperature absolute term of the atmosphere, and $R = 287\ \mathrm{J \cdot kg^{-1} K^{-1}}$ is a gas constant of dry air.

According to the first law of thermodynamics, the **atmospheric thermodynamic equation** is given by

$$c_v \frac{\mathrm{d}T}{\mathrm{d}t} + p \frac{\mathrm{d}\frac{1}{\rho}}{\mathrm{d}t} = \frac{\mathrm{d}Q}{\mathrm{d}t},$$

where $c_v = 718$ J·kg^{-1}K^{-1}, and $\frac{dQ}{dt}$ is the quantity of heat per unit mass of air obtained from external environment per unit time. Applying (1.1.3), we have

$$R\frac{dT}{dt} = \frac{d\frac{p}{\rho}}{dt} = \frac{1}{\rho}\frac{dp}{dt} + p\frac{d\frac{1}{\rho}}{dt} = \frac{RT}{p}\frac{dp}{dt} + p\frac{d\frac{1}{\rho}}{dt}.$$

Combining the above two equations together, we get

$$c_p\frac{dT}{dt} - \frac{RT}{p}\frac{dp}{dt} = \frac{dQ}{dt}, \tag{1.1.4}$$

where $c_p = c_v + R$ is specific heat at constant pressure.

Equations (1.1.1)-(1.1.4) are called the **fundamental equations of dry air**, where the unknown functions are $\boldsymbol{V}$, ρ, p, and T in these equations. If D and $\frac{dQ}{dt}$ are fixed, equations (1.1.1)-(1.1.4) are self-closed.

When one has to consider vaporation in the air, the moist air state equation is

$$p = R\rho T(1 + cq), \tag{1.1.5}$$

where $q = \frac{\rho_1}{\rho}$ is the mixing ratio of water vapor in the air, and ρ_1 is the density of water vapor in the air. Here, c represents positive constant varying with context. $c = 0.618$ in (1.1.5). The thermodynamic equation of the moist atmosphere is

$$c_p\frac{dT}{dt} - \frac{RT(1+cq)}{p}\frac{dp}{dt} = \frac{dQ}{dt}, \tag{1.1.6}$$

the conservation equation of the water vapor in the air is

$$\frac{dq}{dt} = \frac{1}{\rho}W_1 + W_2, \tag{1.1.7}$$

where W_1 is the condensation ratio of steam per unit volume, and W_2 is the volume change ratio of unit mass steam due to horizontal and vertical diffusions. Equations (1.1.1), (1.1.2) and (1.1.5)-(1.1.7) are called the **equations of the moist atmospheric**.

1.1.2 *Basic Equations of the Oceans*

Suppose that there are massless source-sinks within the oceans. In the rotating coordinate frame, the equations of oceans consist of the following equations:

the momentum conservation equation

$$\rho \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = -\mathrm{grad}_3 p + \rho g - 2\rho \boldsymbol{\Omega} \times \boldsymbol{V} + D,$$

the continuity equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \mathrm{div}_3 \boldsymbol{V} = 0,$$

the state equation

$$\rho = f(T, S, p),$$

the thermodynamic equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = Q_1,$$

and the salinity conservation equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = Q_2,$$

where S is salinity, Q_1 is the heat source per unit mass seawater derive from the external environment in unit time, and Q_2 is the salt source per unit mass seawater derive from the external environment in unit time.

Since the equations above are too complex, one has to do some simplification. Generally, one takes **Boussinesq approximation**, that is, consider ρ in ρg and the state equation as unknown function, but ρ in other position as constant ρ_0. Moreover, we use the following approximation equation to replace the above state equation

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)],$$

where β_T and β_S are positive constants, and T_0, S_0 are the reference values of temperature and salinity, respectively. Thus, we get the equations of oceans as

$$\rho_0 \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = -\mathrm{grad}_3 p + \rho g - 2\rho_0 \boldsymbol{\Omega} \times \boldsymbol{V} + D, \tag{1.1.8}$$

$$\mathrm{div}_3 \boldsymbol{V} = 0, \tag{1.1.9}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)], \tag{1.1.10}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = Q_1, \tag{1.1.11}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = Q_2. \tag{1.1.12}$$

Remark 1.1.1. State equation (1.1.10) is an empirical equation, which appears in [212]. The more general form is

$$\rho = \rho_0 \left[1 - \beta_T(T - T_0) + \beta_S(S - S_0) + \frac{p}{\rho_0 c_s^2}\right],$$

where c_s is a positive constant, and this equation appears in section 2.4.1 of [205].