CALCULUS

Solutions Manual for

Salas/Hille

SOLUTIONS MANUAL FOR SALAS/HILLE

CALCULUS

ONE AND SEVERAL VARIABLES

Prepared by

S. L. Salas

and

Dagmar Noll

XEROX COLLEGE PUBLISHING

WALTHAM, MASSACHUSETTS | TORONTO

ISBN: 0-536-00651-2
PRINTED IN THE UNITED STATES OF AMERICA.

SECTION 1.3

1. $x=2$
2. $x=\frac{5}{3}$
3. $x=a$ or $x=b$
4. $(1,3)$
5. $[1,3]$
6. $(-\infty,1)\cup(3,\infty)$
7. $(-\infty,\frac{1}{3})$
8. $(-\frac{1}{8},\infty)$
9. $(-1,1)$
10. $(-\infty,\infty)$
11. $[2,\infty)$
12. $[0,\infty)$
13. $(-\infty,0)$
14. $(-\infty,0)\cup(0,1)\cup(2,\infty)$
15. $(1,\infty)$
16. $[-2,2]$
17. $(-1,0)\cup(1,\infty)$
18. $(0,\infty)$
19. $(-\infty,0]\cup(5,\infty)$
20. $(\frac{5}{4},\frac{7}{4})$
21. $(-\infty,1)\cup(2,\infty)$

SECTION 1.4

1. dom $(f)=(-\infty,\infty)$; ran $(f)=[0,\infty)$
2. dom $(g)=(-\infty,\infty)$; ran $(g)=[-1,\infty)$
3. dom $(F)=(-\infty,\infty)$; ran $(F)=(-\infty,\infty)$
4. dom $(G)=[0,\infty)$; ran $(G)=[-1,\infty)$
5. dom $(f)=(-\infty,0)\cup(0,\infty)$; ran $(f)=(-\infty,0)\cup(0,\infty)$
6. dom $(g)=[-1,1]$; ran $(g)=[-1,0]$
7. dom $(F)=(-1,1)$; ran $(F)=[1,\infty)$
8. dom $(H)=[0,\infty)$; ran $(H)=[1,\infty)$
9. one-to-one
10. not one-to-one
11. one-to-one
12. one-to-one [$g(x)=x^2$ if $x\geq 0$ and $g(x)=-x^2$ if $x\leq 0$]
13. one-to-one

14. not one-to-one

15. $\sqrt{x^2+5}$ 16. $\dfrac{1}{(x-1)^2}$ 17. $\dfrac{1}{x}-1$

18. $|x^2(x-1)^2-1|$ 19. $\dfrac{1}{x^2+1}$

20. $\dfrac{1}{\frac{1}{x^2}} - \dfrac{1}{\frac{1}{x^2}+1}$, or more simply, $x^2 - \dfrac{x^2}{1+x^2}$ with $x \neq 0$

21. $y^{5/3}$ 22. $(y-1)^{1/5}$ 23. $(1-y^{1/5})^{1/3}$

24. $(1-y)^3+2$ 25. $\dfrac{1}{y}$ 26. $1-\dfrac{1}{y}$

27. $\left(\dfrac{1-y}{y}\right)^{1/3}$ 28. $1-\dfrac{1}{y+1}$

29. odd 30. even 31. neither 32. odd

33. even 34. odd 35. even 36. even

37. odd

38. symmetric with respect to the y-axis

39. " " " " " origin

SECTION 2.2

1. 2 2. -5 3. 7 4. 3

5. -1 6. 1 7. 0 8. 1

9. 0 10. no limit 11. 1 12. no limit

13. 2 14. no limit 15. 0 16. 0

17. 1 18. 4 19. no limit 20. 0

21. take $\delta = \epsilon$ 22. take $\delta = \dfrac{\epsilon}{3}$

23. take $\delta = \dfrac{\epsilon}{5}$ 24. take $\delta = \dfrac{\epsilon}{6}$

25. take $\delta = \varepsilon$

26. $$\lim_{x \to 9} \sqrt{x} = 3$$

Proof: Let $\varepsilon > 0$. δ can be chosen to be any positive number which is less than 9 and less than 3ε. For suppose that

$$0 < |x-9| < \delta.$$

Since $\delta < 9$, $x > 0$ and we can form $\sqrt{x}$ and write

$$|\sqrt{x}+3||\sqrt{x}-3| = |x-9|.$$

This means that we have

$$|\sqrt{x}+3||\sqrt{x}-3| < \delta.$$

Since

$$3 < |\sqrt{x}+3|,$$

we have

$$3|\sqrt{x}-3| < \delta$$

and thus

$$|\sqrt{x}-3| < \frac{\delta}{3} < \frac{3\varepsilon}{3} = \varepsilon. \quad \blacksquare$$

27. take δ less than 1 and less than ε

28. $$\lim_{x \to 10} \sqrt{x-1} = 3$$

Proof: Let $\varepsilon > 0$ and note that

$$|x-10| = |(x-1)-9|.$$

δ can be chosen to be any positive number which is less than 9 and less than 3ε. For suppose that

$$0 < |x-10| < \delta.$$

Since $\delta < 9$, $x > 1$ and we can form $\sqrt{x-1}$ and write

$$|\sqrt{x-1}+3|\,|\sqrt{x-1}-3| = |(x-1)-9| = |x-10|.$$

This means that we have

$$|\sqrt{x-1}+3|\,|\sqrt{x-1}-3| < \delta.$$

Since

$$3 < |\sqrt{x-1}+3|,$$

we have

$$3\,|\sqrt{x-1}-3| < \delta$$

and thus

$$|\sqrt{x-1}-3| < \frac{\delta}{3} < \frac{3\varepsilon}{3} = \varepsilon. \quad \blacksquare$$

29. $$\lim_{x\to 2} x^2 = 4$$

Proof: Let $\varepsilon > 0$. Take δ to be a positive number which is less than 1 and less than $\frac{\varepsilon}{5}$. (Other choices of δ are also possible.) Suppose now that

$$0 < |x-2| < \delta.$$

From the fact that $\delta < 1$ we know that

$$1 < x < 3$$

and thus

$$* \qquad |x+2| < 5.$$

From the fact that $\delta < \frac{\varepsilon}{5}$ we know that

$$** \qquad |x-2| < \frac{\varepsilon}{5}.$$

Combining * and **, we have

$$|x^2-4| = |x+2||x-2| < 5\left(\frac{\varepsilon}{5}\right) = \varepsilon. \blacksquare$$

30. $$\lim_{x \to 3} x^2 = 9$$

Proof: (similar to the proof of 29.)

Let $\varepsilon > 0$. Take δ to be a positive number which is less than 1 and less than $\frac{\varepsilon}{7}$.

Suppose now that

$$0 < |x-3| < \delta.$$

From the fact that $\delta < 1$ we know that

$$2 < x < 4$$

and thus

$$* \qquad |x+3| < 7.$$

From the fact that $\delta < \frac{\varepsilon}{7}$ we know that

$$** \qquad |x-3| < \frac{\varepsilon}{7}.$$

Combining * and **, we have

$$|x^2-9| = |x+3||x-3| < 7\left(\frac{\varepsilon}{7}\right) = \varepsilon. \blacksquare$$

SECTION 2.3

1. (a) $\frac{7}{8}$ (b) $\frac{1}{4}$ (c) 4 (d) $7-\sqrt{2}$ (e) no limit
   (f) 0 (g) $-\frac{23}{20}$ (h) 1
2. (a) $\frac{5}{8}$ (b) 2 (c) no limit (d) 0
3. (a) 3 (b) 4 (c) -2 (d) 0 (e) no limit
   (f) $\frac{1}{3}$
4. (a) 27 (b) no limit (c) 0 (d) 3

5. Proof: Suppose on the contrary that the limit exists and is some number ℓ:

$$\lim_{x \to c} f(x) = \ell.$$

Since

$$\lim_{x \to c} g(x) = 0$$

we have (by the product rule)

$$* \quad \lim_{x \to c} [f(x) g(x)] = \ell \cdot 0 = 0.$$

But since

$$f(x) g(x) = 1 \quad \text{for all real } x,$$

we know that

$$** \quad \lim_{x \to c} [f(x) g(x)] = 1.$$

Combining $*$ with $**$, we have

$$0 = 1,$$

a contradiction. ■

SECTION 2.4

1. (a) 0 (b) 1 (c) 0 (d) 10 (e) 0

(f) 9 (g) 1 (h) no limit (i) no limit (j) −1

(k) no limit (l) 1 (m) no limit

2. Proof: use the inequality

$$\big|\,|f(x)| - |\ell|\,\big| \le |f(x) - \ell|.$$

3. (a) no limit (b) 1

4. For each $c > 0$

$$\lim_{x\to c} \sqrt{x} = \sqrt{c}.$$

Proof: From the hint, for $x \geq 0$ we have

$$0 \le |\sqrt{x} - \sqrt{c}| \le \frac{1}{\sqrt{c}}|x - c|.$$

Since

$$\lim_{x\to c} 0 = 0 \quad \text{and} \quad \lim_{x\to c} \frac{1}{\sqrt{c}}|x-c| = 0,$$

the pinching theorem tells us that

$$\lim_{x\to c} |\sqrt{x} - \sqrt{c}| = 0.$$

It follows then by (2.4.2) that

$$\lim_{x\to c} \sqrt{x} = \sqrt{c}. \quad \blacksquare$$

5. (a) $$\max\{f(x), g(x)\} = \frac{1}{2}\{[f(x)+g(x)] + |f(x)-g(x)|\}$$

Proof: If $f(x) \geq g(x)$, then

$$\max\{f(x), g(x)\} = f(x)$$

and, since

$$|f(x) - g(x)| = f(x) - g(x),$$

$$\frac{1}{2}\{[f(x)+g(x)] + |f(x)-g(x)|\} = \frac{1}{2}\{[f(x)+g(x)] + [f(x)-g(x)]\}$$
$$= \frac{1}{2}\{2f(x)\}$$
$$= f(x).$$

If $f(x) \leq g(x)$, then

$$\max\{f(x), g(x)\} = g(x),$$

and, since

$$|f(x)-g(x)| = g(x)-f(x),$$

$$\frac{1}{2}\{[f(x)+g(x)] + |f(x)-g(x)|\} = \frac{1}{2}\{[f(x)+g(x)] + [g(x)-f(x)]\}$$
$$= \frac{1}{2}\{2g(x)\}$$
$$= g(x). \quad \blacksquare$$

(b) $\min\{f(x), g(x)\} = \frac{1}{2}\{[f(x)+g(x)] - |f(x)-g(x)|\}$

6. From exercise 5(b), we know that

$$h(x) = \frac{1}{2}\{[f(x)+g(x)] - |f(x)-g(x)|\}.$$

If

$$\lim_{x\to c} f(x) = \ell \quad \text{and} \quad \lim_{x\to c} g(x) = \ell,$$

then

$$\lim_{x\to c} \frac{1}{2}[f(x)+g(x)] = \frac{1}{2}[\ell+\ell] = \ell$$

and

$$\lim_{x\to c} |f(x)-g(x)| = 0,$$

so that

$$\lim_{x\to c} h(x) = \ell + 0 = \ell.$$

the other limit can be handled in a similar manner. $\blacksquare$

7. In 2.4.2 statement (i) is equivalent to statement (iv).

Proof: Let $\varepsilon > 0$. If

$$\lim_{x \to c} f(x) = \ell,$$

then there must exist $\delta > 0$ such that

* if $0 < |x - c| < \delta$ then $|f(x) - \ell| < \varepsilon$.

Suppose now that

$$0 < |h| < \delta.$$

Then

$$0 < |(c+h) - c| < \delta$$

and thus by *

$$|f(c+h) - \ell| < \varepsilon.$$

This proves that

if $\lim_{x \to c} f(x) = \ell$ then $\lim_{h \to 0} f(c+h) = \ell$.

If, on the other hand,

$$\lim_{h \to 0} f(c+h) = \ell,$$

then there must exist $\delta > 0$ such that

** if $0 < |h| < \delta$ then $|f(c+h) - \ell| < \varepsilon$.

Suppose now that

$$0 < |x - c| < \delta.$$

Then by **

$$|f((x-c)+c)-\ell| < \varepsilon.$$

More simply stated,

$$|f(x)-\ell| < \varepsilon.$$

This proves that

$$\text{if} \quad \lim_{h\to 0} f(c+h) = \ell \quad \text{then} \quad \lim_{x\to c} f(x) = \ell. \quad \blacksquare$$

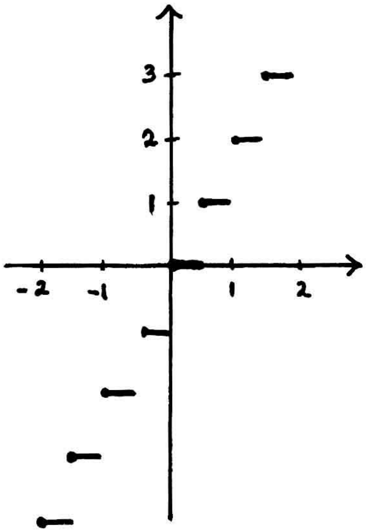
SECTION 2.5

1. (a) 1 (b) 1 (c) −1 (d) 1 (e) no limit

(f) 1 (g) 0 (h) 0 (i) 1 (j) 0

(k) −1 (ℓ) 1 (m) no limit (n) 0

2. (a)



$[2x]$

(b) (i) 0 (ii) 1 (iii) 1 (iv) 1

(v) no limit (vi) 1

3. Proof of (2.5.2):

Let $\epsilon > 0$. We begin by supposing that

$$\lim_{x \to c} f(x) = \ell.$$

Then there must exist $\delta > 0$ such that

$$\text{if} \quad 0 < |x-c| < \delta \quad \text{then} \quad |f(x) - \ell| < \epsilon.$$

If

$$c - \delta < x < c$$

then

$$0 < |x-c| < \delta$$

and thus

$$|f(x) - \ell| < \epsilon.$$

This shows that

$$\lim_{x \uparrow c} f(x) = \ell.$$

If

$$c < x < c + \delta$$

then again

$$0 < |x-c| < \delta$$

and thus

$$|f(x) - \ell| < \epsilon.$$

This shows that

$$\lim_{x \downarrow c} f(x) = \ell.$$

We now suppose that

$$\lim_{x \uparrow c} f(x) = \ell \quad \text{and} \quad \lim_{x \downarrow c} f(x) = \ell$$

and show that

$$\lim_{x \to c} f(x) = \ell.$$

Once again we let ε be positive and arbitrary. From the left-hand limit condition, we know that there exists $\delta_1 > 0$ such that

$$\text{if} \quad c - \delta_1 < x < c \quad \text{then} \quad |f(x) - \ell| < \varepsilon.$$

From the right-hand limit condition, we know that there exists $\delta_2 > 0$ such that

$$\text{if} \quad c < x < c + \delta_2 \quad \text{then} \quad |f(x) - \ell| < \varepsilon.$$

Now we choose

$$\delta = \min\{\delta_1, \delta_2\}.$$

If

$$0 < |x - c| < \delta$$

then

$$\text{either} \quad c - \delta_1 < x < c \quad \text{or} \quad c < x < c + \delta_2$$

but in any case

$$|f(x) - \ell| < \varepsilon.$$

This shows that

$$\lim_{x \to c} f(x) = \ell. \qquad \blacksquare$$

4. (a) We begin by assuming that

$$\lim_{x \uparrow c} f(x) = \ell$$

and showing that

$$\lim_{h \to 0} f(c - |h|) = \ell.$$

Let $\varepsilon > 0$. Since

$$\lim_{x \uparrow c} f(x) = \ell,$$

we know that there exists $\delta > 0$ such that

$$* \quad \text{if} \quad c - \delta < x < c \quad \text{then} \quad |f(x) - \ell| < \varepsilon.$$

Suppose now that

$$0 < |h| < \delta.$$

Then

$$-\delta < -|h| < 0$$

so that

$$c - \delta < c - |h| < c$$

and, by $*$,

$$|f(c - |h|) - \ell| < \varepsilon.$$

This shows that

$$\lim_{h \to 0} f(c - |h|) = \ell.$$

Conversely we now assume that

$$\lim_{h \to 0} f(c - |h|) = \ell.$$

We know then that for $\varepsilon > 0$ there exists $\delta > 0$ such that.

** if $0 < |h| < \delta$ then $|f(c - |h|) - \ell| < \varepsilon$.

Suppose now that

$$c - \delta < x < c.$$

Then

$$-\delta < x - c < 0$$

and

$$0 < c - x < \delta$$

so that, by **,

$$|f(c - (c-x)) - \ell| < \varepsilon.$$

More simply we have

$$|f(x) - \ell| < \varepsilon.$$

It follows that

$$\lim_{x \uparrow c} f(x) = \ell. \qquad \blacksquare$$

(b) This case is similar to (a).

5. (a) 0 (b) 0 (c) 1 (d) 0

SECTION 2.6

1. (a) continuous at 2 (b) continuous at 2

(c) continuous at 2