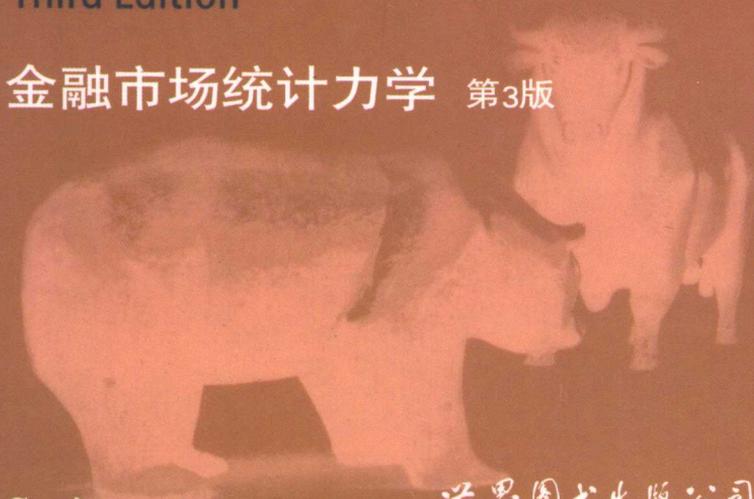


J. Voit

# The Statistical Mechanics of Financial Markets

Third Edition

金融市场统计力学 第3版



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## Preface to the Third Edition

One must act on what has not happened yet.

Lao Zi

The present third edition of *The Statistical Mechanics of Finance* was published only four years after the first edition. The success of the first edition highlights the interest in a summary of the broad research activities on the application of statistical physics to financial markets. I am very grateful to readers and reviewers for their positive reception and comments. Why then prepare a new edition instead of only reprinting and correcting the second edition?

The new edition has been significantly expanded, giving it a more pronounced twist towards banking. The most important extensions are due to my practical experience as a risk manager in the German Savings Banks' Association (BSGV). Two new chapters on risk management and on the closely related topic of economic and regulatory capital for financial institutions, respectively, have been added. The chapter on risk management contains both the basics as well as advanced topics, e.g. coherent risk measures, which have not yet reached the statistical physics community interested in financial markets. Similarly, it is surprising how little research by academic physicists has appeared on topics relating to Basel II. Basel II is the new capital adequacy framework which will set the standards in risk management in many countries for the years to come. Basel II is responsible for many job openings in banks for which physicists are extremely well qualified. For these reasons, an outline of Basel II takes a major part of the chapter on capital.

Feedback from readers, in particular Guido Montagna and Gian May, has led to new sections on American-style options and the application of path-integral methods for their pricing and hedging, and on volatility indices, respectively. To make them consistent, sections on sensitivities of options to changes in model parameters and variables ("the Greeks") and on the synthetic replication of options have been added, too. Chin-Kun Hu and Bernd Kaiber have stimulated extensions of the discussion of cross-correlations in financial markets. Finally, new research results on the description and prediction of financial crashes have been incorporated.

Some layout and data processing work was done in the Institute of Mathematical Physics at the University of WÜRZBURG. I am very grateful to Wolfgang Wonnoberger and Ferdinand Lehmann for their kind hospitality and generous

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## VIII Preface to the Third Edition

support there. The University of Ulm and Academia Sinica, Taipei, provided opportunities for testing some of the material in courses.

My wife, Jinping Shen, and my daughter, Jiayi Sun, encouraged and supported me whenever I was in doubt about this project, and I would like to thank them very much.

Finally, I wish You, Dear Reader, a good time with and inspiration from this book.

Berlin, July 2005

*Johannes Voit*

## Preface to the First Edition

This book grew out of a course entitled “Physikalische Modelle in der Finanzwirtschaft” which I have taught at the University of Freiburg during the winter term 1998/1999, building on a similar course a year before at the University of Bayreuth. It was an experiment.

My interest in the statistical mechanics of capital markets goes back to a public lecture on self-organized criticality, given at the University of Bayreuth in early 1994. Bak, Tang, and Wiesenfeld, in the first longer paper on their theory of self-organized criticality [Phys. Rev. A **38**, 364 (1988)] mention Mandelbrot’s 1963 paper [J. Business **36**, 394 (1963)] on power-law scaling in commodity markets, and speculate on economic systems being described by their theory. Starting from about 1995, papers appeared with increasing frequency on the Los Alamos preprint server, and in the physics literature, showing that physicists found the idea of applying methods of statistical physics to problems of economy exciting and that they produced interesting results. I also was tempted to start work in this new field.

However, there was one major problem: my traditional field of research is the theory of strongly correlated quasi-one-dimensional electrons, conducting polymers, quantum wires and organic superconductors, and I had no prior education in the advanced methods of either stochastics and quantitative finance. This is how the idea of proposing a course to our students was born: learn by teaching! Very recently, we have also started research on financial markets and economic systems, but these results have not yet made it into this book (the latest research papers can be downloaded from my homepage <http://www.phy.uni-bayreuth.de/btp314/>).

This book, and the underlying course, deliberately concentrate on the main facts and ideas in those physical models and methods which have applications in finance, and the most important background information on the relevant areas of finance. They lie at the interface between physics and finance, not in one field alone. The presentation often just scratches the surface of a topic, avoids details, and certainly does not give complete information. However, based on this book, readers who wish to go deeper into some subjects should have no trouble in going to the more specialized original references cited in the bibliography.

Despite these shortcomings, I hope that the reader will share the fun I had in getting involved with this exciting topic, and in preparing and, most of all, actually teaching the course and writing the book.

Such a project cannot be realized without the support of many people and institutions. They are too many to name individually. A few persons and institutions, however, stand out and I wish to use this opportunity to express my deep gratitude to them: Mr. Ralf-Dieter Brunowski (editor in chief, Capital – Das Wirtschaftsmagazin), Ms. Margit Reif (Consorts Discount Broker AG), and Dr. Christof Kreuter (Deutsche Bank Research), who provided important information; L. A. N. Amaral, M. Ausloos, W. Breyman, H. Büttner, R. Cont, S. Dresel, H. Eißfeller, R. Friedrich, S. Ghashghaie, S. Hügler, Ch. Jelitto, Th. Lux, D. Obert, J. Peinke, D. Sornette, H. E. Stanley, D. Stauffer, and N. Vandewalle provided material and challenged me in stimulating discussions. Specifically, D. Stauffer's pertinent criticism and many suggestions significantly improved this work. S. Hügler designed part of the graphics. The University of Freiburg gave me the opportunity to elaborate this course during a visiting professorship. My students there contributed much critical feedback. Apart from the year in Freiburg, I am a Heisenberg fellow of Deutsche Forschungsgemeinschaft and based at Bayreuth University. The final correction were done during a sabbatical at Science & Finance, the research division of Capital Fund Management, Levallois (France), and I would like to thank the company for its hospitality. I also would like to thank the staff of Springer-Verlag for all the work they invested on the way from my typo-congested L<sup>A</sup>T<sub>E</sub>X files to this first edition of the book.

However, without the continuous support, understanding, and encouragement of my wife Jinping Shen and our daughter Jiayi, this work would not have got its present shape. I thank them all.

Bayreuth,  
August 2000

*Johannes Voit*

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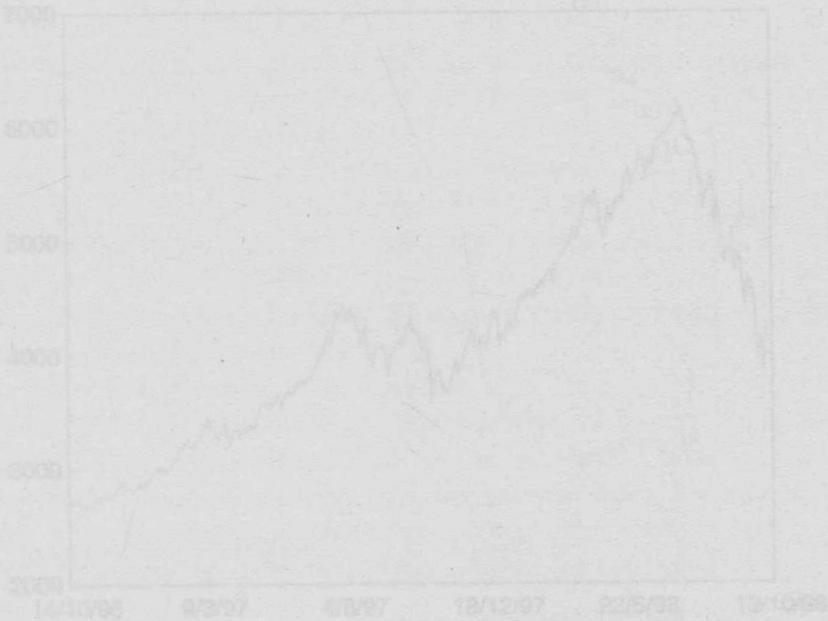


Fig. 1.1. Evolution of the DAX German stock index from October 14, 1996 to October 13, 1998. Data provided by Deutsche Bank Research

# 1. Introduction

## 1.1 Motivation

The public interest in traded securities has continuously grown over the past few years, with an especially strong growth in Germany and other European countries at the end of the 1990s. Consequently, events influencing stock prices, opinions and speculations on such events and their consequences, and even the daily stock quotes, receive much attention and media coverage. A few reasons for this interest are clearly visible in Fig. 1.1 which shows the evolution of the German stock index DAX [1] over the two years from October 1996 to October 1998. Other major stock indices, such as the US Dow Jones Industrial Average, the S&P500, or the French CAC40, etc., behaved in a similar manner in that interval of time. We notice three important features: (i) the continuous rise of the index over the first almost one and a half years which

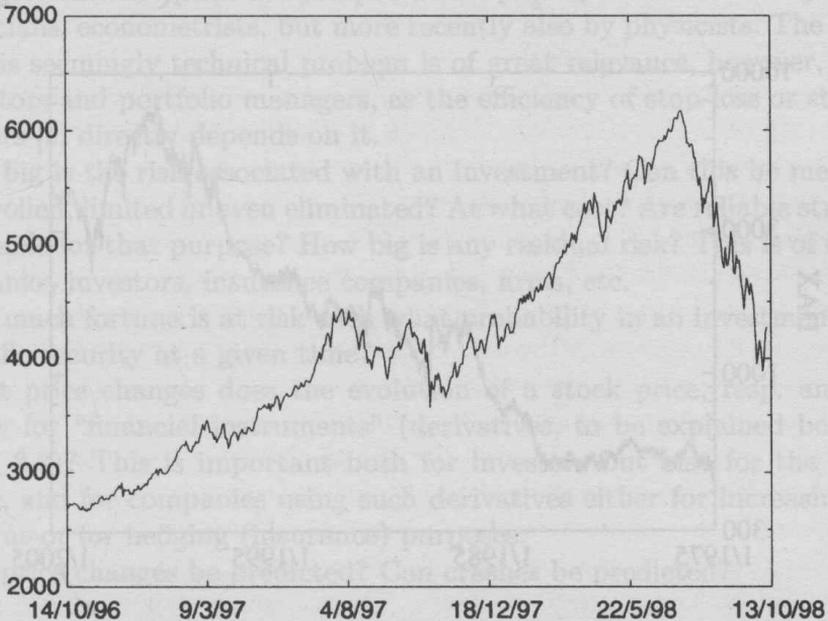


Fig. 1.1. Evolution of the DAX German stock index from October 14, 1996 to October 13, 1998. Data provided by Deutsche Bank Research

was interrupted only for very short periods; (ii) the crash on the “second black Monday”, October 27, 1997 (the “Asian crisis”, the reaction of stock markets to the collapse of a bank in Japan, preceded by rumors about huge amounts of foul credits and derivative exposures of Japanese banks, and a period of devaluation of Asian currencies). (iii) the very strong drawdown of quotes between July and October 1998 (the “Russian debt crisis”, following the announcement by Russia of a moratorium on its debt reimbursements, and a devaluation of the Russian rouble), and the collapse of the Long Term Capital Management hedge fund.

While the long-term rise of the index until 2000 seemed to offer investors attractive, high-return opportunities for making money, enormous fortunes of billions or trillions of dollars were annihilated in very short times, perhaps less than a day, in crashes or periods of extended drawdowns. Such events – the catastrophic crashes perhaps more than the long-term rise – exercise a strong fascination.

To place these events in a broader context, Fig. 1.2 shows the evolution of the DAX index from 1975 to 2005. Several different regimes can be distinguished. In the initial period 1975–1983, the returns on stock investments were extremely low, about 2.6% per year. Returns of 200 DAX points, or 12%, per year were generated in the second period 1983–1996. After 1996, we see a marked acceleration with growth rates of 1200 DAX points, or 33%, per year. We also notice that, during the growth periods of the stock market, the losses incurred in a sudden crash usually persist only over a short



Fig. 1.2. Long-term evolution of the DAX German stock index from January 1, 1975 to January 1, 2005. Data provided by Deutsche Bank Research supplemented by data downloaded from Yahoo, <http://de.finance.yahoo.com>