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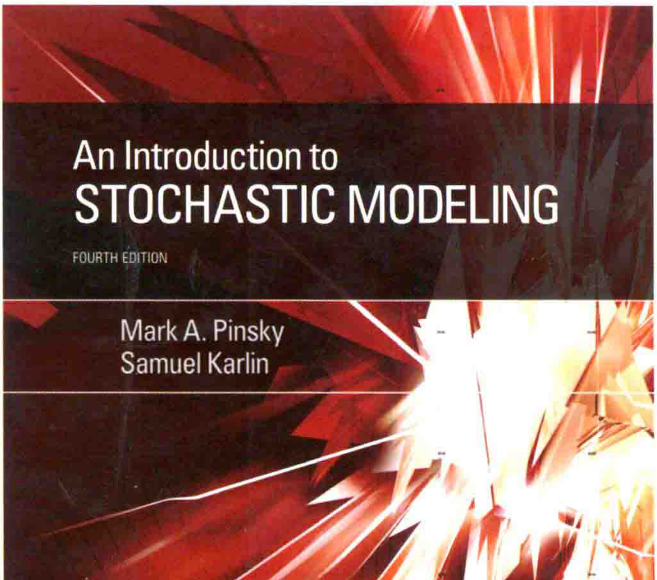

（美）Mark A. Pinsky 西北大学　Samuel Karlin 斯坦福大学 著

机械工业出版社
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An Introduction to Stochastic Modeling

(Fourth Edition)

（美）Mark A. Pinsky Samuel Karlin 著
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图书在版编目（CIP）数据

随机模型概论（英文版·第4版）/（美）品斯基（Pinsky, M. A.），（美）卡尔林（Karlin, S.）著．—北京：机械工业出版社，2013.1
（华章数学原版精品系列）
书名原文：An Introduction to Stochastic Modeling, Fourth Edition

ISBN 978-7-111-41231-1

I. 随…　II. ① 品…　② 卡…　III. 随机过程－数学模型－教材－英文　IV. O226

中国版本图书馆CIP数据核字（2013）第012567号

本书版权登记号：图字：01-2012-7893

An Introduction to Stochastic Modeling, Fourth Edition
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ISBN: 978-0-12-381416-6

Elsevier (Singapore) Pte Ltd.
3 Killiney Road
#08-01 Winsland House I
Singapore 239519
Tel: (65) 6349-0200
Fax: (65) 6733-1817
First Published 2013

机械工业出版社（北京市西城区百万庄大街22号　邮政编码　100037）
责任编辑：迟振春
北京京师印务有限公司印刷
2013年2月第1版第1次印刷
147mm × 210mm · 18.125印张
标准书号：ISBN 978-7-111-41231-1
定价：69.00元

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Preface to the Fourth Edition

Since the publication of the third edition in 1998, some new developments have occurred. Samuel Karlin died in 2007, leaving a gap at the authorship level and the new designation of authors.

In the fourth edition, we have added two new chapters: Chapter 10 on random evolution and Chapter 11 on characteristic functions. *Random Evolution* denotes a set of stochastic models, which describe continuous motion with piecewise linear sample functions. Explicit formulas are available in the simplest cases. In the general case, one has a central limit theorem, which is pursued more generally in Chapter 11, "Characteristic Functions and Their Applications." Here the necessary tools from Fourier Analysis are developed and applied when necessary. Many theorems are proved in full detail, while other proofs are sketched—in the spirit of the earlier Chapters 1–9. Complete proofs may be found by consulting the intermediate textbooks listed in the section on further reading. Instructors who have taught from the third edition may be reassured that Chapters 1–9 of the new edition are identical to the corresponding chapters of the new book.

We express our thanks to Michael Perlman of the University of Washington and Russell Lyons of Indiana University for sharing their lists of errata from the third edition. We would also like to thank Craig Evans for useful advice on partial differential equations.

Biographical Note

Samuel Karlin earned his undergraduate degree from the Illinois Institute of Technology and his doctorate from Princeton University in 1947 at age 22. He served on the faculty of Caltech from 1948–1956 before joining the faculty of Stanford University, where he spent the remainder of his career. Karlin made fundamental contributions to mathematical economics, bioinformatics, game theory, evolutionary theory, biomolecular sequence analysis, mathematical population genetics, and total positivity.

Karlin authored 10 books and more than 450 articles. He was a member of the American Academy of Arts and Sciences and the National Academy of Sciences. In 1989, he received the National Medal of Science for his broad and remarkable researches in mathematical analysis, probability theory, and mathematical statistics and in the application of these ideas to mathematical economics, mechanics, and genetics. He died on December 18, 2007.

Mark A. Pinsky
Department of Mathematics
Northwestern University
Evanston, IL 60208-2730

Preface to the Third Edition

The purposes, level, and style of this new edition conform to the tenets set forth in the original preface. We continue with our objective of introducing some theory and applications of stochastic processes to students having a solid foundation in calculus and in calculus-level probability, but who are not conversant with the "epsilon–delta" definitions of mathematical analysis. We hope to entice students toward the deeper study of mathematics that is prerequisite to further work in stochastic processes by showing the myriad and interesting ways in which stochastic models can help us understand the real world.

We have removed some topics and added others. We added a small section on martingales that includes an example suggesting the martingale concept as appropriate for modeling the prices of assets traded in a perfect market. A new chapter introduces the Brownian motion process and includes several applications of it and its variants in financial modeling. In this chapter the Black–Scholes formula for option pricing is evaluated and compared with some reported prices of options. A Poisson process whose intensity is itself a stochastic process is described in another new section.

Some treatments have been updated. The law of rare events is presented via an inequality that measures the accuracy of a Poisson approximation for the distribution of the sum of independent, not necessarily identically distributed, Bernoulli random variables. We have added the shot noise model and related it to a random sum.

The text contains more than 250 exercises and 350 problems. Exercises are elementary drills intended to promote active learning and to develop familiarity with concepts through use. They often simply involve the substitution of numbers into given formulas or reasoning one or two steps away from a definition. They are the kinds of simple questions that we, as instructors, hope that students would pose and answer for themselves as they read a text. Answers to the exercises are given at the end of the book so that students may gauge their understanding as they go along.

Problems are more difficult. Some involve extensive algebraic or calculus manipulation. Many are "word problems" wherein the student is asked, in effect, to model some described scenario. As in formulating a model, the first step in the solution of a word problem is often a sentence of the form "Let $x = \ldots$." A manual containing the solutions to the problems is available from the publisher.

A reasonable strategy on the part of the teacher might be to hold students responsible for all of the exercises, but to require submitted solutions only to selected problems. Every student should attempt a representative selection of the problems in order to develop his or her ability to carry out stochastic modeling in his or her area of interest.

A small number of problems are labeled "Computer Challenges." These call for more than pencil and paper for their analyses, and either simulation, numerical exploration, or symbol manipulation may prove helpful. Computer Challenges are meant to be open-ended, intended to explore what constitutes an answer in today's world of computing power. They might be appropriate as part of an honors requirement.

Because our focus is on stochastic modeling, in some instances, we have omitted a proof and contented ourselves with a precise statement of a result and examples of its application. All such omitted proofs may be found in *A First Course in Stochastic Processes*, by the present authors. In this more advanced text, the ambitious student will also find additional material on martingales, Brownian motion, and renewal processes, and presentations of several other classes of stochastic processes.

Preface to the First Edition

Stochastic processes are ways of quantifying the dynamic relationships of sequences of random events. Stochastic models play an important role in elucidating many areas of the natural and engineering sciences. They can be used to analyze the variability inherent in biological and medical processes, to deal with uncertainties affecting managerial decisions and with the complexities of psychological and social interactions, and to provide new perspectives, methodology, models, and intuition to aid in other mathematical and statistical studies.

This book is intended as a beginning text in stochastic processes for students familiar with elementary probability calculus. Its aim is to bridge the gap between basic probability know-how and an intermediate-level course in stochastic processes—for example, *A First Course in Stochastic Processes*, by the present authors.

The objectives of this book are as follows: (1) to introduce students to the standard concepts and methods of stochastic modeling; (2) to illustrate the rich diversity of applications of stochastic processes in the sciences; and (3) to provide exercises in the application of simple stochastic analysis to appropriate problems.

The chapters are organized around several prototype classes of stochastic processes featuring Markov chains in discrete and continuous time, Poisson processes and renewal theory, the evolution of branching events, and queueing models. After the concluding Chapter 9, we provide a list of books that incorporate more advanced discussions of several of the models set forth in this text.

To the Instructor

If possible, we recommend having students skim the first two chapters, referring as necessary to the probability review material, and starting the course with Chapter 3, on Markov chains. A one-quarter course adapted to the junior–senior level could consist of a cursory (1-week) review of Chapters 1 and 2, followed in order by Chapters 3 through 6. For interested students, Chapters 7, 8, and 9 discuss other currently active areas of stochastic modeling. Starred sections contain material of a more advanced or specialized nature.

Acknowledgments

Many people helped to bring this text into being. We gratefully acknowledge the help of Anna Karlin, Shelley Stevens, Karen Larsen, and Laurieann Shoemaker. Chapter 9 was enriched by a series of lectures on queueing networks given by Ralph Disney at The Johns Hopkins University in 1982. Alan Karr, Ivan Johnstone, Luke Tierney, Bob Vanderbei, and others besides ourselves have taught from the text, and we have profited from their criticisms. Finally, we are grateful for improvements suggested by the several generations of students who have used the book over the past few years and have given us their reactions and suggestions.

Contents

1 Introduction

1.1 Stochastic Modeling

A quantitative description of a natural phenomenon is called a mathematical model of that phenomenon. Examples abound, from the simple equation $S = \frac{1}{2}gt^2$ describing the distance S traveled in time t by a falling object starting at rest to a complex computer program that simulates a biological population or a large industrial system.

In the final analysis, a model is judged using a single, quite pragmatic, factor, the model's *usefulness*. Some models are useful as detailed quantitative prescriptions of behavior, e.g., an inventory model that is used to determine the optimal number of units to stock. Another model in a different context may provide only general qualitative information about the relationships among and relative importance of several factors influencing an event. Such a model is useful in an equally important but quite different way. Examples of diverse types of stochastic models are spread throughout this book.

Such often mentioned attributes, such as realism, elegance, validity, and reproducibility, are important in evaluating a model only insofar as they bear on that model's ultimate usefulness. For instance, it is both unrealistic and quite inelegant to view the sprawling city of Los Angeles as a geometrical point, a mathematical object of no size or dimension. Yet, it is quite useful to do exactly that when using spherical geometry to derive a minimum-distance great circle air route from New York City, another "point."

There is no such thing as the best model for a given phenomenon. The pragmatic criterion of usefulness often allows the existence of two or more models for the same event, but serving distinct purposes. Consider light. The wave form model, in which light is viewed as a continuous flow, is entirely adequate for designing eyeglass and telescope lenses. In contrast, for understanding the impact of light on the retina of the eye, the photon model, which views light as tiny discrete bundles of energy, is preferred. Neither model supersedes the other; both are relevant and useful.

The word "stochastic" derives from a Greek word ($\sigma\tau o\chi \acute{\alpha}\zeta\epsilon\sigma\theta\alpha\iota$: to aim, to guess) and means "random" or "chance." The antonym is "sure," "deterministic," or "certain." A deterministic model predicts a single outcome from a given set of circumstances. A stochastic model predicts a set of possible outcomes weighted by their likelihoods or probabilities. A coin flipped into the air will surely return to earth somewhere. Whether it lands heads or tails is random. For a "fair" coin, we consider these alternatives equally likely and assign to each the probability $\frac{1}{2}$.

However, phenomena are not in and of themselves inherently stochastic or deterministic. Rather, to model a phenomenon as stochastic or deterministic is the choice of the observer. The choice depends on the observer's purpose; the criterion for judging the choice is usefulness. Most often the proper choice is quite clear, but controversial

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situations do arise. If the coin once fallen is quickly covered by a book so that the outcome "heads" or "tails" remains unknown, two participants may still usefully employ probability concepts to evaluate what is a fair bet between them; i.e., they may usefully view the coin as random, even though most people would consider the outcome now to be fixed or deterministic. As a less mundane example of the converse situation, changes in the level of a large population are often usefully modeled deterministically, in spite of the general agreement among observers that many chance events contribute to their fluctuations.

Scientific modeling has three components: (1) a natural phenomenon under study, (2) a logical system for deducing implications about the phenomenon, and (3) a connection linking the elements of the natural system under study to the logical system used to model it. If we think of these three components in terms of the great-circle air route problem, the natural system is the earth with airports at Los Angeles and New York; the logical system is the mathematical subject of spherical geometry; and the two are connected by viewing the airports in the physical system as points in the logical system.

The modern approach to stochastic modeling is in a similar spirit. Nature does not dictate a unique definition of "probability," in the same way that there is no nature-imposed definition of "point" in geometry. "Probability" and "point" are terms in pure mathematics, defined only through the properties invested in them by their respective sets of axioms. (See Section 1.2.8 for a review of axiomatic probability theory.) There are, however, three general principles that are often useful in relating or connecting the abstract elements of mathematical probability theory to a real or natural phenomenon that is to be modeled. These are (1) the principle of equally likely outcomes, (2) the principle of long run relative frequency, and (3) the principle of odds making or subjective probabilities. Historically, these three concepts arose out of largely unsuccessful attempts to define probability in terms of physical experiences. Today, they are relevant as guidelines for the assignment of probability values in a model, and for the interpretation of the conclusions of a model in terms of the phenomenon under study.

We illustrate the distinctions between these principles with a long experiment. We will pretend that we are part of a group of people who decide to toss a coin and observe the event that the coin will fall heads up. This event is denoted by H, and the event of tails, by T.

Initially, everyone in the group agrees that $\Pr\{H\} = \frac{1}{2}$. When asked why, people give two reasons: Upon checking the coin construction, they believe that the two possible outcomes, heads and tails, are equally likely; and extrapolating from past experience, they also believe that if the coin is tossed many times, the fraction of times that heads is observed will be close to one-half.

The equally likely interpretation of probability surfaced in the works of Laplace in 1812, where the attempt was made to define the probability of an event A as the ratio of the total number of ways that A could occur to the total number of possible outcomes of the experiment. The equally likely approach is often used today to assign probabilities that reflect some notion of a total lack of knowledge about the outcome of a chance phenomenon. The principle requires judicious application if it is to be useful, however.