MECHANICS OF SOLIDS

The Rodney Hill 60th Anniversary Volume

Editors

H. G. HOPKINS and M. J. SEWELL

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RODNEY HILL

Editors' Preface

RODNEY HILL celebrates his sixtieth birthday on the 11th June 1981. This Volume is presented to him by the contributors, as their glad tribute to a man whose dedication of purpose and high level of achievement have permanently influenced developments in large and basic areas of the mechanics of solids, especially, although not exclusively, in the theory and applications of plasticity in metals.

We are confident that the Volume will be accompanied by good wishes to him, on this anniversary and for the future, from numerous scientists of international standing in many countries. As Editors, we decided at the outset that the occasion called for the planning of a substantial Volume having some balanced structure, in the hope that it might achieve a degree of permanence in the literature of the subject. The eighteen articles contained herein therefore provide a mosaic of selected subject areas. The choice was made from topics which Rodney Hill himself has sought to illuminate at one time or another, or from certain topics immediately adjacent thereto in either physical or mathematical or engineering terms. This policy of a restricted number of substantial articles implied, regretfully, that we were not able to invite papers from all who we are sure would have wished to contribute.

We regard it as a privilege to have had the opportunity to edit the Volume. It is a pleasure to record the alacrity with which contributors gave their support, and the ready co-operation from many people which has helped the project to move ahead smoothly. We make no attempt here to give a technical assessment of the work of Rodney Hill, which happily is still continuing strongly. However, we have felt that the inclusion of a general biographical sketch, and also of a complete current bibliography, would be widely appreciated. In the preparation of these we have been greatly assisted by a number of people. We express our thanks to Rodney himself, for his ready response to our requests for autobiographical source material. We are also grateful to several associates from the different periods of Rodney's career for being willing to exercise their memories for us and provide recollections of events sometimes many years ago. The photograph was taken by Edward Leigh of King's Parade, Cambridge.

Particular thanks are due to the Publishers for their most helpful collaboration with us, and for their agreement to publish the Volume given when it was yet unwritten, but existed simply as a proposal in the form of a list of authors and subject areas; and also to Mr. Robert Maxwell himself for his encouraging personal interest.

H. G. HOPKINS, *Manchester* M. J. SEWELL, *Reading*
30 *April* 1980

Rodney Hill

Biographical Note

RODNEY HILL was born on the 11th June 1921 at Stourton, near Leeds, in Yorkshire. He comes from a family with deep roots in the practical and cultural traditions of the West Riding, although with no known mathematical ability in an earlier generation. Rodney's father, Harold Harrison Hill, had been an only child and he was educated at the University of Leeds, gaining an M.A. for postgraduate work in history. He also took an external London degree in economics. After wartime service in the Royal Navy he became a schoolmaster, and was eventually Senior History Master at Leeds Boys' Modern School. Rodney's mother had been a student at Leeds School of Art. Rodney himself was also an only child, in an immediate home background which encouraged scholarship and self-sufficiency.

Rodney entered Leeds Grammar School with a scholarship in 1932, and there gave regular prize-winning evidence of all-round intellectual ability not only in mathematics, but equally in art, English literature, and other Arts subjects. During this period he taught himself to play the piano, and became proficient at chess in which he was later to represent Cambridge University and town. Thus were developing the powers of accurate observation and analysis to be brought to bear on the mathematics and physics which became his formal specialism from the age of 15. The customary large-team games did not attract him at school, but Rodney enjoyed the one-to-one sports of squash, fencing, and golf. He left school as Head of House, and in December 1938 he was awarded an Open Major Scholarship at Pembroke College, Cambridge. However, it needed the State and County Scholarships gained in the preceding summer to make a financially independent undergraduate.

Hill went up to Cambridge to read Mathematics in October 1939, against a background of external events which must have seemed the least auspicious since the very founding of the University. Major Scholars were expected to take Part II of the Tripos in two years instead of three by omitting all first-year courses. This imposed a heavy workload, to be carried under spartan conditions created by wartime restrictions such as blackout and rationing combined with antique College plumbing. For example, there was no running hot water, the nearest bath was two courts away, and the winter allocation of one sack of coal per week fuelled a fire in one's room only in the evenings. Hill was not deflected by the adverse general situation from his aim of a first-class honours degree, and he became a Wrangler in June 1941. This entitled him to take Part III of the Mathematical Tripos, in the applied mathematical part of which quantum mechanics figured prominently at that time. However, he felt obliged to volunteer for war-work, and so lost the opportunity for advanced training which those lecture courses would have provided.

The Mathematical Laboratory at Cambridge University had become established by the beginning of the war, with Professor J. E. Lennard-Jones, F.R.S., as its Director, and it soon became closely allied to the External Ballistics Department of the Ordnance Board under the control of the Ministry of Supply. Hill was directed to work in the Laboratory in the late summer of 1941. He was assigned the task of calculating the trajectories of shells for the compilation of range tables, by small-arc iteration on desk machines such as the Brunsviga. After some months in that activity Hill began helping to operate the Bush differential analyser for solving the internal ballistics equations ("internal" indicates that the shell is still travelling down the gun barrel). In this context he was able to show the existence of connected families of solutions, so reducing the need for separate computations, and thus wrote his first research paper.

The developing war effort in 1942 saw the appointment of Lennard-Jones as Chief Superintendent of Armament Research, with a staff of ultimately over 3000 under his authority in various parts of the country, in an Armament Research Department reconstituted from the old Research Department at Woolwich Arsenal. Lennard-Jones made his headquarters at Fort Halstead, an isolated but attractive location on a spur of the North Downs near Sevenoaks in Kent. Among his innovations was the setting up of new research groups headed by various scientists of distinction brought in for the purpose. Professor Nevill Mott, F.R.S., became the first superintendent of a group for theoretical research in armaments, which became a permanent feature at Fort Halstead. In the spring of 1943 Mott assembled a team of able young scientists from Cambridge, Woolwich, and elsewhere, many of whom subsequently went on to influential and distinguished careers in universities or government service. They included J. W. Maccoll (Mott's deputy), J. Corner, D. C. Pack, A. F. Devonshire, L. Howarth, E. H. Lee, I. N. Sneddon, C. K. Thornhill, and J. H. Wilkinson. Rodney Hill joined the team in May 1943 and he stayed for three years.

It can be seen that this was a pivotal date in the career not only of Hill, but also of many other now well-known scientists. Their attention was brought at a formative age to a range of practical and urgent problems, and their contribution deserves to be widely recognized. The atmosphere in the Theoretical Research Branch was stimulating, with Mott frequently sketching the line of approach to a problem almost on the back of a postcard, and expecting his recruits to fill in the details. The Branch was responsible for advising military designers about both basic theory and day-to-day problems concerning the performance of explosives, fragmentation bombs, internal and external and terminal ballistics, attack and defence of tanks and other armoured vehicles, and so on. For example, they were involved in reconstructing the German V-2 rocket from intelligence reports and estimating its range.

The physical background to this activity was utterly different from Cambridge. Fort Halstead was a vast establishment not only of civilian but also of military personnel, with war equipment much in evidence. A balloon barrage was strung out overhead along the North Downs. V-1 flying bombs and V-2 rockets descended all around and in the areas like Petts Wood nearer London where staff were billeted in private houses. In fact Fort Halstead lay within what became known as "flying bomb alley", and one flying bomb landed in the Establishment, blasting windows in the Theoretical Research Branch. Fortunately this was at night, otherwise the Branch could have suffered heavy casualties.

Problems brought to the Theoretical Research Branch were distributed initially according to the specialisms of the more senior members, some of whom had acquired relevant experience at Woolwich Arsenal. Those problems which were quite new in context tended to go to the young inexperienced graduates newly arrived from university. This was indeed a baptism of fire for them, but it was a test which was to reveal Hill's true métier. One of his initial assignments was the deep penetration of very thick armour by Munroe jets and by high-velocity shells with tungsten–carbide cores. This required a mechanics of plastic deformation with unlimited magnitude, and thus was aroused Hill's interest in the field in which he later became perhaps the foremost world authority. At this stage, however, he had no prior knowledge of the physics and metallurgy of plasticity, and little of stress, strain or the tensors which the mathematics would eventually require. There was no useful textbook, but G. I. Taylor had written one or two helpful reports on shaped charges and Munroe jets. Nevertheless, working at first with Mott and Pack, Hill was soon able to show, for example, that penetration by a tungsten–carbide core with pure ogival head would be seriously degraded if too much of the tip were ground conical (the British practice for manufacturing convenience). The demonstration was achieved not only theoretically, but also in field trials planned by Hill in collaboration with an experimental group under Dr. Charles Sykes, F.R.S.

In such ways Hill acquired a training in pertinent science in the prime of his youth, and the influence of it may be recognized repeatedly in his later work. The problems at Fort Halstead called for simple but effective mathematics guided by physical intuition and a willingness to communicate with others, including non-mathematicians and experimentalists. There was not time for complicated mathematics, there were no electronic computers to assist it, and the experimental data were usually too crude to warrant it anyway. He acquired a lasting taste for a pragmatic blend of rigour, elegance, and simple realism in the application of mathematics.

The sense of purpose discovered at this time was noticed by colleagues as a cheerful and sparkling earnestness. Popular relaxations among the group at Cambridge had included music, books, and lightning chess. At Fort Halstead ballroom dancing was added as a consuming passion for some, and Hill was not slow to find that he had medal-winning ability in this new enthusiasm. He met his future wife, Jeanne Wickens, early in 1945. She had been transferrred to work at Fort Halstead from the bombing range at Shoeburyness. Previously she had trained as a dancer and teacher of ballet, but the war cut short a promising career. They were married in Cambridge in 1946, and they have one daughter, born in 1955. The strength of his wife's support could already be detected in the Preface to Hill's first book, and the passage of years has happily reinforced this bond.

In April 1946 Hill was seconded by the Ministry of Supply to work with a group of metal physicists under Dr. E. Orowan, F.R.S., at the Cavendish Laboratory in Cambridge. By this time the applied mechanics of both solids and fluids was being forced to push the boat out onto a sea of nonlinear problems, and away from the haven of linearity in which much pre-war work had lingered. The trend was evident not only in England, of course, but in other countries too. Hill found himself in demand as the sole adviser on continuum plasticity in England, not only concerning problems arising from the interests at Fort Halstead, but also for new theories of metal-working processes needed by engineers in the steel industry. He obtained a Cambridge Ph.D. in 1948 for a Thesis entitled "Theoretical studies of the plastic deformation of metals". In February 1949 he moved to Sheffield, having accepted an invitation to head a new Section in the Metal Flow Research Laboratory of the British Iron and Steel Research Association.

From the Ph.D. Thesis grew a much more extensive monograph on *The Mathematical Theory of Plasticity*, published at the Clarendon Press, Oxford, in 1950. This very rapidly established Hill as an international authority. The final draft was written in his spare time, i.e. in the evenings and weekends left to him after discharging his responsibilities at B.I.S.R.A. He was then still only in his 28th year, and it is timely to recall a remark from the review of the book in *Engineering*: "The author has done his work so well that it is difficult to see how it could be bettered. The book should rank for many years as an authoritative source of reference." This prognostication was fully borne out. The book was in print at Oxford for 21 years, Japanese and Russian translations have been made, and total sales currently approach 13 000.

The work in Sheffield produced new opportunities for collaboration with practical laboratory work, for example on rolling and wire drawing. Once again the physical environment was very different from Cambridge. This was before the days of Clean Air Acts, and the Laboratory was situated in a smoky valley, one of the grimiest and most derelict areas of the city. The theoreticians themselves were housed in a temporary hut while main buildings were being constructed. In compensation, however, Hill enjoyed walking with his wife on the moors outside the city at weekends. At the suggestion of Professor H. W. Swift, Hill gave courses on slipline field analysis in the Mechanical Engineering Department at Sheffield University. Swift was very helpful in promoting these new ideas within the Iron and Steel Institute and in the industry. At this time, too, Hill was joined by A. P. Green and shortly afterwards by J. F. W. Bishop, who thus became the first of a later sequence of postgraduate students wishing to learn the new subject of mathematical plasticity from him at first hand.

The next move was to Bristol University in September 1950. A three-year Research Fellowship was created for Hill, jointly in the Departments of Physics and Theoretical Mechanics. As his list of publications reveals, joint papers were written here from the standpoints both of physics (relating single crystal behaviour to that of a polycrystalline aggregate—with Bishop who had moved with him) and of engineering (bars and tubes under combined loading—with M. P. L. Siebel and B. Crossland). For the latter, advantage was taken of the laboratory facilities in the Mechanical Engineering Department, with the encouragement of Professor J. L. M. Morrison. There was also the opportunity for more teaching and related reading. A new family relaxation taken up at this time was the study of field botany and mycology. Ever since, and in many parts of the English countryside, this has proved an absorbing diversion for the mind from the exigencies of a sustained research programme.

The *Journal of Mechanics and Physics of Solids* was launched with the encouragement of the infant Pergamon Press in 1952. Hill suggested the title and the general aim of a forum for effective applied mathematics, linked with experimentation, in engineering science. Any new journal must begin energetically if it is to stamp out an identity for itself in the international community, and Hill took the lead by contributing 10 articles (some jointly) to the total of 55 which appeared in the first two Volumes. From then onwards the *Journal* has been regarded as among the foremost in its general field, and unique in flavour. Hill served as Editor-in-Chief until handing over in 1968 to H. G. Hopkins.

The University of Nottingham had received its Charter, and independence from London, in 1948, and was shortly to embark on two decades of substantial expansion. Professor H. R. Pitt was appointed in 1950 to head the existing Mathematics Department, and he was soon instrumental in securing the creation of a new Chair of Applied Mathematics. Rodney Hill applied, and was offered the post in 1953 while still only 31. It was his responsibility to modernize the teaching of applied mathematics. Hill took over some existing courses himself, and instigated new ones with the aim of encouraging research students. His undergraduate lectures were characterized by conciseness and a tendency for brevity. He would never exceed the time limit. But those students who took the trouble to write down what he said, in addition to what was written on the blackboard, found after reflection that they had a first-class and substantial set of notes.

Funds gradually became available for essential new staff in applied mathematics, and J. E. Adkins was appointed. He was followed later by two of Hill's Ph.D. students,

R. J. Knops and M. J. Sewell. Another major innovation which he sought was the creation, in 1960, of a Department of Theoretical Mechanics to teach and do research within the Faculty of Engineering. This was headed first by Adkins as Professor until his untimely death in 1964, and since by A. J. M. Spencer, and is known as a flourishing and influential Department. Hill was also consulted by the Mining Engineering Department on subsidence problems. These were evident in many parts of the city, which is built over part of the Nottinghamshire coalfield. He proposed a theory of ground movement based on the idea that extracting a seam of coal by the longwall method resulted in an edge dislocation of the rock strata. The idea was developed by D. S. Berry, who led a stress-analysis section recruited for the purpose in the Mining Department.

During this period Hill undertook the administrative tasks customarily associated with an established Chair, and without previous experience of such work. If the quantity of his personal research decreased because of this, there was also a change to work of a broader and more general scope. It may only have been a coincidence that the emergence of interest in the so-called rational continuum mechanics was taking place in some American and British universities at this time. Hill's writings demonstrate an independent view of these developments, and no taste at all for axiomatics. He was beginning to lay down the basis of general studies of non-uniqueness and instability in continua which were to prove highly influential over the next two decades, and which in due course brought further students and able collaborators.

The University of Cambridge conferred the degree of Sc.D. upon Rodney Hill in 1959. The highest honour to which any British scientist aspires followed in 1961, when he was elected a Fellow of the Royal Society. This gave much pleasure to his colleagues at Nottingham and to his friends elsewhere.

By this time Nottingham University was widely recognized as a centre of activity in applied mathematics, and in the mechanics of solids in particular, and Professor Hill could justifiably feel that his various efforts to that end had borne fruit. He resigned from his Chair as from October 1962, and the University conferred on him an Honorary Professorship while he remained in Nottingham for the ensuing academic year. During this time Hill found considerable satisfaction in the writing of an undergraduate textbook on *Principles of Dynamics*. This was the product of 10 years spent re-thinking and teaching classical dynamics. It is a subject which, as the book's Preface shows, Hill regards as an exemplar of scientific theory and method, which is indispensable for students of structural engineering, applied mathematics and theoretical physics alike. The book does not shirk conceptual difficulties, and it leads the reader to think deeply and afresh about the subject.

In 1963 Hill was elected to a Berkeley Bye-Fellowship (for research) at Gonville and Caius College, Cambridge. This he held for 6 years until the University conferred a personal Readership in the Mechanics of Solids. Thus he became a member of the teaching staff of the Department of Applied Mathematics and Theoretical Physics, and in 1972 a personal Professorship was conferred. During this Cambridge period the foundations laid at Nottingham for the unambiguous formulation of incremental boundary value problems at large strain were built upon in various directions. Properties of heterogeneous media (including fibre composites), single crystals, continuum plasticity, and an independent reformulation of rubber elasticity were explored. A sequence of new research students found fruitful employment in these or older fields. They included I. F. Collins, D. J. F. Ewing, J. P. Miles, R. W. Ogden, G. P. Parry, L. J. Walpole, and N. J. B. Young. During the seventies Hill was gratified to be able to collaborate with several distinguished scholars from abroad, who chose to come to Cambridge for their sabbatical leave in order to work with him.

This is not the place to offer an assessment of Rodney Hill's impact on the theoretical and applied mechanics of solids. His list of scientific articles speaks for itself. Currently (February 1980) there are 2 books and 132 articles, 44 of the latter being written jointly with a total of 20 collaborators. He has himself contributed to *Festschriften* for G. I. Taylor, N. I. Muskhelishvili, W. Prager, F. K. G. Odqvist, V. V. Novozhilov, and A. A. Ilyushin. His style of writing is spare and concise. Every word is weighed, and must be weighed again by the reader. Mathematics has frequently to be read with paper and pencil to hand—they are essential when reading Hill. He does not offer an easy ride—he would not think one worth while. A comment that this or that paper was not easy to read may draw the swift reply that it was not easy to write. But the reader who persists finds a rich vein of understanding, and fresh thinking at every turn.

His standards of scholarship and intellectual honesty are the highest. He is ready in his appreciation of the good work of others; and he has been sharp in candid criticism of misguided thinking or slack presentation (especially by those mature enough to know better) if he thought the subject-matter would be best served thereby—as some celebrated footnotes and book reviews testify.

The outward character of the man is not unlike his papers: physically tall and slim, with the long fingers of a pianist, and having a quiet but compelling presence. His unusually deep reserve has meant that casual social gatherings and conferences have held less interest and been less rewarding for him than for others. Some lack of robustness in health has militated against unnecessary travel. Thus, his General Lectures given on such occasions as the 9th International Congress of Theoretical and Applied Mechanics at Brussels in 1956, or the 3rd British Theoretical Mechanics Colloquium at Newcastle upon Tyne in 1961, have been rare events.

On the other hand, in circumstances where his contribution could be telling his effort has been unstinting. His students testify to the very thorough and considerate way in which he supervised their research work, with a just and effective balance of encouragement and the measured push, whichever was currently deserved. His colleagues, if they would have preferred to have known him better, have respected his desire for privacy and understood his wish to optimize the unique contribution which he has made.

The honorary degree of Doctor of Science was conferred upon Rodney Hill by the University of Manchester in 1976, and by the University of Bath in 1978. Also in 1978 he was awarded the Theodore von Kármán Medal, instituted in 1960 for annual award by the American Society of Civil Engineers for "distinguished achievement in engineering mechanics". The only previous British recipient of this medal was G. I. Taylor.

In 1979 he resigned his Chair, but was elected to another Fellowship at Gonville and Caius College, Cambridge. His flow of papers shows no sign of diminishing, and his friends will hope that that short legend "by R. Hill" will appear in the scientific literature many more times yet.

GEOFFREY HOPKINS MICHAEL SEWELL

List of Publications by Rodney Hill

A. Scientific Papers

1943a (With J. Crank.) Families of solutions of the internal ballistics equation, U.K. Ministry of Defence (Procurement Executive), unpublished report.

b (With J. Crank.) Effect of various types of band resistance on gun ballistics, U.K. Ministry of Defence (Procurement Executive), unpublished report.

1944a (With N. F. Mott and D. C. Pack.) Penetration by Munroe jets, U.K. Ministry of Defence (Procurement Executive), unpublished report.

b (With D. C. Pack and N. F. Mott.) Penetration of armour by high-velocity projectiles and Munroe jets, U.K. Ministry of Defence (Procurement Executive), unpublished report.

c (With D. C. Pack.) Penetration into wood, U.K. Ministry of Defence (Procurement Executive), unpublished report.

d Cavitation phenomena in ductile materials and the dynamic term in the resistance to penetration, U.K. Ministry of Defence (Procurement Executive), unpublished report.

1945a The analysis of projectile penetration of non-ferrous ductile materials, U.K. Ministry of Defence (Procurement Executive), unpublished report.

b The theory of armour penetration by tungsten carbide shot and the effect of headshape on performance at high velocities, U.K. Ministry of Defence (Procurement Executive), unpublished report.

c (With E. H. Lee.) The weakening of a gun barrel due to the presence of a hole normal to the axis, U.K. Ministry of Defence (Procurement Executive), unpublished report.

d (With N. F. Mott and R. F. Bishop.) The theory of indentation and hardness tests. *Proc. Phys. Soc.* **57,** 147–159.

1947a Critical survey of soil mechanics with suggestions for future research, U.K. Ministry of Defence (Procurement Executive), unpublished report.

b (With E. H. Lee and S. J. Tupper.) The theory of wedge indentation of ductile materials, *Proc. R. Soc. London* A **188,** 273–289.

c (With E. H. Lee and S. J. Tupper.) The theory of combined plastic and elastic deformation with particular reference to a thick tube under internal pressure, *Proc. R. Soc. London* A **191,** 278–303.

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e A theory of yielding and plastic flow of anisotropic metals, *Proc. R. Soc. London* A **193,** 281–297.
f A variational principle of maximum plastic work in classical plasticity, *Q. Jl Mech. Appl. Math.* **1,** 18–28.
g (With S. J. Tupper.) Theory of plastic deformation in wire drawing, *J. Iron Steel Inst.* **159,** 353–359.
h Special problems of indentation and compression, *Proceedings of the Seventh International Congress for Applied Mechanics, London, 5–11 September 1948,* **1,** 365–377, published by the Organizing Committee.
1949a The theory of plane plastic strain for anisotropic metals, *Proc. R. Soc. London* A **198,** 428–437.
b The calculation of stresses in the ironing of metal cups, *J. Iron Steel Inst.* **161,** 41–44.
c General features of plastic–elastic problems as exemplified by some particular solutions, *J. Appl. Mech.* **16,** 295–300.
d Plastic distortion of non-uniform sheets, *Phil. Mag.* **40,** 971–983.
e Plastic yielding of notched bars, *Q. Jl Mech. Appl. Math.* **2,** 40–52.
1950a A comparative study of some variational principles in the theory of plasticity, *J. Appl. Mech.* **17,** 64–66.
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d On the state of stress in a plastic–rigid body at the yield point, *Phil. Mag.* **42,** 868–875.
e (With J. F. W. Bishop.) A theory of the plastic distortion of a polycrystalline aggregate under combined stresses, *Phil. Mag.* **42,** 414–427.
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