Microcontinuum Field Theories

II: FLUENT MEDIA

A. Cemal Eringen

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With 67 Figures

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To Meva,
Source for Daddy's encouragement and time

Preface

This book is concerned with the exposition of microcontinuum field theories (micropolar, microstretch, and micromorphic, briefly, 3M continua) for fluent media. These are structured fluid media as exemplified by anisotropic fluids, liquid crystals, polymeric melts, suspensions, slurries, biological fluids, dusty gases, etc. It is assumed that the media under consideration can be modeled by continuum theories. Thus, the discrete nature of the fluent substances is taken into account by only their continuum substructures as described in Volume I of these treatises.

The foundation of these theories is established fully in Volume I. However, for the sake of easy reference, basic equations are developed for each chapter. Nevertheless, the reader is encouraged to study the first three chapters of Volume I.

Microcontinuum field theories constitute extensions of classical field theories (elasticity, fluid dynamics, and electromagnetism) in that, against the classical notions, here each material point of continuum carries directors that may be deformable. In micropolar media, directors are considered rigid. In microstretch media they can stretch and contract; and in micromorphic media, they are deformable. The presence of directors brings three extra degrees of freedom, over classical fluids, in micropolar continuum: four extra degrees of freedom in microstretch continuum; and nine extra degrees of freedom in micromorphic media.

Microcontinuum field theories make it possible to discuss many different physical properties of substances that fall outside the coverage of the classical field theories. For example, liquid crystals that fall outside the scope of the Navier–Stokes fluids, now find natural mechanisms within the theory of micropolar continua.

This book consists of four parts:

1. Theory of micropolar fluids (Chapters 9-11).

2. Liquid crystals (Chapters 12-14).

3. Microstretch fluids (Chapters 15 and 16).

4. Micromorphic fluids (Chapter 17).

Chapters 9-16 include the discussion of electromagnetic interactions as well.

The main purpose of this volume is the establishment of the fundamental equations of 3M continuum theories. Ample examples of solutions are provided for the demonstration of the predictions of these theories. However, attention is focused on the new and significant physical phenomena predicted by these theories over the classical fluid mechanics. Consequently, number of problems selected are limited.

Excluding the solutions of some problems treated in Chapter 9, the present volume is based entirely on the author's work over several decades. Historically, the first paper that originated in this field is that of the author, Eringen [1964]. Since then, the literature has become so large that the coverage of this book had to be curtailed. I have, therefore, placed more weight on the foundations, which are crucial to future research. Moreover, much of the contents of Chapters 10 through 17 did not appear in the publications prior to this book.

It is pleasing to observe the harmony in the fundamental equations developed, and comforting to see that predictions of the theories are reasonably in accord with existing experimental observations. Nevertheless, treading on new ground, one must be cautious. On this, I ask the readers' indulgence and understanding.

It is my hope that much of the theories presented in this book will provide a basis for the resolution of some outstanding problems (e.g. suspensions, turbulence) and give impetus for research in new directions. Of course, the final verdict for this remains with future generations.

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Contents

9 Micropolar Fluid Dynamics[1]

9.0 Scope

Micropolar fluid dynamics is concerned with the motion of fluids whose material points possess orientations. It is distinguished from classical fluid dynamics (which is also known as Newtonian fluid dynamics or the Navier–Stokes (N–S) theory) in that classical fluid dynamics is assumed not to possess oriented material points. Thus, against the three translational degrees of freedom of the classical theory, micropolar fluids possess six degrees of freedom: three translational degrees and three rotational degrees. The rotational degrees of freedom bring into play nonsymmetric stress tensors and couple stresses, which are missing from the classical theory.

In Section 9.1, I introduce the basic notions of material particles which possess inner structures represented by deformable directors. The motions are defined. The places of micropolar and classical fluid dynamics are sketched in the hierarchy of micromorphic fluids whose material points are endowed with deformable directors. Section 9.2 summarizes the strain and rotation measures, the deformation tensors, the axiom of objectivity, compatibility conditions for micromorphic, microstretch, and micropolar continua, which I also call 3M continua. An account on the subject was more thoroughly discussed in Volume I. The balance laws of micropolar continua are copied from Section 2.2 into Section 9.3.

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[1]The theory of micropolar fluid dynamics was introduced by Eringen [1966]. It is the special case of micromorphic theory of Eringen [1964].

Constitutive equations are given in Section 9.4, where I also discuss the thermodynamics stability, which places restrictions on the micropolar viscosities. Section 9.5 presents the field equations' boundary and initial conditions. The theory of micropolar fluids is now ready for applications.

Section 9.6 shows how one may obtain classical theory from the present theory. It gives a norm for the velocity field of micropolar theory to converge to the velocity field of the (N–S) theory.

The solution of a pipe flow is presented in Section 9.7, where we see the emergence of internal characteristic lengths that are missing from the N–S theory.

Similarity parameters of micropolar theory are many, as shown in Section 9.8.

Micropolar theory naturally gives rise to the notion of vortices as a fundamental concept. Section 9.9 discusses the change of shape and decay of vortices in space and time.

In Section 9.10, we describe how the turbulence can be discussed with the mechanisms of micropolar fluid dynamics. To this end, the Reynolds stresses are calculated and turbulence energy is given for spherically symmetric motions. Microrotations induced by sudden disturbance is elaborated. Kampé de Fériet and Taylor-type solutions are presented in Section 9.11.

In Section 9.12, we present the solution of the flow problem in a rheometer, where two parallel plates of the rheometer containing micropolar fluid are rotating about two noncoincident axes. Section 9.13 presents a theory of lubrication.

In Section 9.14 are obtained fundamental solutions for slow motions. These are the solutions due to prescribed body forces and couples in fluids. The Stokes flow is obtained. The following section (Section 9.15) presents the Stokes flow about a sphere, indicating that the drag on a sphere is reduced as compared to N–S fluids.

Stagnation flow is discussed in Section 9.16.

The thermal instability of a layer of fluid heated from below or above (the so-called classical problem of Benard) is the subject of Section 9.17.

Boundary layer flow (classically known as Blasius flow) is presented in Section 9.18, where the boundary layers and sublayers are discussed. In Section 9.19, the problem of mixed convection in a vertical flow (relevant to nuclear reactors and heat exchangers) is solved. Section 9.20 discusses invariant solutions of plane micropolar fluids. The method presented here allows us to obtain an infinite number of time-independent solutions from any steady-state solution.

Micropolar fluid dynamics is still in the fast development state. It possesses the basic mechanisms relevant to turbulence, suspensions, and is the key to many other physical phenomena (e.g., liquid crystals, anisotropic fluids, see Chapters 11 and 12). Once these aspects become widely known, perhaps the theory will open other dimensions of research in the future.

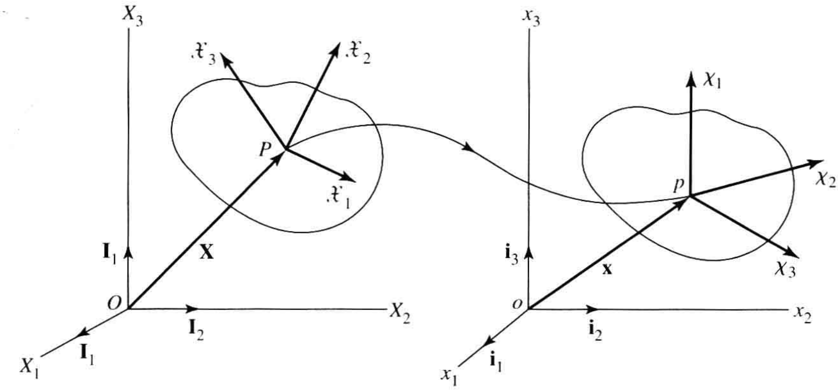

FIGURE 9.1.1. Deformable directors.

9.1 Motion, Micromotion

As discussed in Section 1.2, in microfluids, the material points of the fluid are considered to be small deformable particles. The motion and micromotion of the material particles are expressed by

$$x_k = \hat{x}_k(\mathbf{X}, t), \qquad k = 1, 2, 3, \tag{9.1.1}$$

$$\xi_k = \chi_{kK}(\mathbf{X}, t)\Xi_K, \qquad K = 1, 2, 3, \tag{9.1.2}$$

where summation over repeated indices is understood throughout.

Here (9.1.1) maps the center of mass of the particle $\mathbf{X}$ in the body at the reference frame K_C, at time $t = 0$, to a spatial place $\mathbf{x}$ at time t (Figure 9.1.1). This represents the **macromotion** (or simply the **motion**). The mapping (9.1.2) is the expression of the **micromotion**, which is equivalent to the rotation and the microdeformations of a particle. Since the material particles are considered to be geometrical points with mass and inertia, $\chi_{kK}(\mathbf{X}, t)$ here represents the three deformable directors attached to the material particle so that the microdeformation is none other than the time evolution of these deformable directors.

It is assumed that both (9.1.1) and (9.1.2) are invertible uniquely so that the single-valued functions $X_K(\mathbf{x}, t)$ and $\mathfrak{X}_{Kk}(\mathbf{x}, t)$, the inverse of $\chi_{kK}(\mathbf{X}, t)$, can be solved from (9.1.1) and (9.1.2), i.e.,

$$X_K = \hat{X}_K(\mathbf{x}, t), \tag{9.1.3}$$

$$\chi_{kK}\mathfrak{X}_{Kl} = \delta_{kl}, \qquad \chi_{kK}\mathfrak{X}_{Lk} = \delta_{KL}. \tag{9.1.4}$$

These are guaranteed by the implicite function theorem which requires that (9.1.1) possesses continuous partial derivatives with respect to X_K and the Jacobians J and j are positive:

$$J \equiv \det x_{k,K} > 0, \qquad j \equiv \det \chi_{kK} > 0, \tag{9.1.5}$$