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ALGEBRA

THE LANGUAGE AND SYMBOLISM OF MATHEMATICS

James W. Hall ▪ Brian A. Mercer

Beginning and Intermediate ALGEBRA

THE LANGUAGE AND SYMBOLISM OF MATHEMATICS

James W. Hall ■ **Brian A. Mercer**

Parkland College *Parkland College*

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BEGINNING AND INTERMEDIATE ALGEBRA: THE LANGUAGE AND SYMBOLISM OF MATHEMATICS

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DEDICATION

To our families for their support and encouragement.

ABOUT THE AUTHORS

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PREFACE

The Universe is a grand book which cannot be read until one first learns to comprehend the language and become familiar with the characters in which it is composed. It is written in the language of mathematics.

—Galileo

Beginning and Intermediate Algebra: The Language and Symbolism of Mathematics covers the topics from both Beginning and Intermediate Algebra. It is fully integrated, rather than the combination of two separate texts.

Our primary goal is to implement the AMATYC standards, as outlined in *Crossroads in Mathematics,* and to give strong support to faculty members who teach this material. These standards were used as guiding principles to organize the topics. This organization is designed to work for students with a variety of learning styles and for teachers with a variety of experiences and backgrounds. Examples of this organization include an early presentation of function notation and graphing of linear equations in two variables.

The inclusion of multiple perspectives—verbal, numerical, algebraic, and graphical—has proven popular with a broad cross section of students. Calculator Perspectives help students see the relationship between mathematics and technology. The specific instructions provided in the Calculator Perspectives also eliminate the need for instructors to create separate keystroke handouts.

The Beginning Algebra portion of this text concentrates primarily on material related to linear equations. (Nonlinear material is reserved for the Intermediate Algebra portion of the text.) The review material in Chapter 1 is presented through the evaluation of algebraic expressions, the checking of solutions to equations, and other contexts. This nontraditional approach motivates students through real-life applications to review background concepts. It also gives students who have already had this material in high school a fresh approach and helps them to connect previously separated topics.

TEACHING APPROACH

Emphasis on the Rule of Four and Multiple Perspectives

The "rule of four" is a phrase that means concepts should be examined algebraically (symbolically), numerically, graphically, and verbally. Reviewers of the manuscript were very pleased that we integrated multiple perspectives throughout the book. We use multiple perspectives not only in examples and exercises, but also in definitions and exposition. Our experience leads us to believe that students who use the rule of four develop a deeper understanding of the concepts they study. They are less likely to memorize steps, they are more likely to retain the material they understand, and they are more likely to apply mathematics outside the classroom. (See AMATYC Standard P-4.)

Technology Is Built-In, Not Added-On

Topics that once were postponed until many manipulative skills had been developed can now be considered earlier by using technology to focus on concepts instead of computation. The use of a graphing calculator is demonstrated throughout the book. The students' use of technology enables them to examine realistic problems such as producing the payment schedule for a car loan. Together, realistic applications and the use of calculators facilitate the development of modeling skills by the students. Technology is woven throughout the text—it is not simply inserted into a standard presentation. (See AMATYC Standards I-2 and I-6.)

Functions in Beginning Algebra

The Beginning Algebra chapters give the student many opportunities to become familiar with function notation and with the input-output concept. This portion of the book concentrates primarily on material related to linear functions, with nonlinear material reserved for the Intermediate Algebra portion of the text. (See AMATYC Standard C-4.)

Functions in Intermediate Algebra

The Intermediate Algebra portion of the textbook contains an introduction to the definition of a function and various notations used to represent functions. This topic is intended to serve as a transition for students who are directly entering the book at this point. The use of function notation provides an opportunity to review the operations with algebraic expressions in a new context and to reexamine linear equations. Chapter 6 contains material on several families of functions, including linear, absolute value, quadratic, square root, cubic, and cube root. (See AMATYC Standard C-4.)

Mathematical Modeling and Word Equations

The residual value of mathematics—the mathematics that students can still use four or more years after taking a course—is not a collection of tricks or memorized steps. What endures is an understanding that allows students to see mathematics as useful in improving their daily lives. Most people encounter mathematics through words, either orally or in writing, not through equations that people want them to solve. Students need to model real problems in a course if we expect them to use mathematics on their own. Word equations help students bridge the gap between the statement of a word problem and the formation of an algebraic equation that models the problem. To that end, the text presents many realistic examples and exercises involving data (see the Index of Applications). (See AMATYC Standard C-2.)

Factoring

Factoring is developed gradually as multiplication is considered.

- In Section 1.6 we examine both multiplication and factoring when the distributive property is illustrated.

- In Section 5.5 we continue to stress the relationship between multiplication and factoring when the multiplication of polynomials is examined. The role of the distributive property is again emphasized.
- In Section 5.6 we use long division of polynomials to complete the factorization of a polynomial when one factor is known.
- Section 5.7 covers some special products and the corresponding factored forms.
- In Section 6.7, factoring out of the GCF (greatest common factor) of a polynomial is used to examine factoring from numerical and graphical perspectives. We then examine the relationship between linear factors of a polynomial, the zeros of a polynomial function, and the x-intercepts of a graph.
- A comprehensive algebraic approach to factoring polynomials is presented in Chapter 7.

Again, our goal is to develop connections among concepts, especially the relationship between the distributive property and multiplication and factoring. Chapter 7 also explores the relationship between the zeros of a function and the factors of a polynomial as introduced in Chapter 6.

Using Systems of Equations

Word problems that involve two unknowns are solved in Chapter 3 using two variables rather than one variable. This approach has been received well by the students who often have more trouble identifying two unknowns using one variable than using a separate variable for each unknown. This approach also received favorable feedback from teachers who class-tested the manuscript. Later in the book we examine alternate approaches that build on creating functional models and the relation of one variable to another.

Using Discrete Data

The book contains data and problems that give the students experience with discrete data. This will give students a better perspective on mathematical models, especially those who will use Intermediate Algebra as their prerequisite to an Introductory Statistics course. (See AMATYC Standards C-5 and C-6.)

Using the Language and Symbolism of Mathematics

Each exercise set starts with a few questions on using the language and symbolism of mathematics. One benefit of assigning these exercises that we have noted in our classes is that the students spend more time reading the book before starting the other exercises. (See AMATYC Standard I-5.)

NOTABLE FEATURES

A Different Kind of Chapter 1

The organization of Chapter 1, "Operations with Real Numbers," is intentionally different from that in most Beginning Algebra books. One benefit of this organization is that it stimulates the interest of students who may be reviewing this material. Many of the chapter's topics are presented either within a new context or in a nontraditional order. The material on operations with real numbers includes problems that evaluate algebraic expressions and problems that check solutions to equations. Also, the commutative, associative, and distributive properties are presented as needed within the arithmetic review rather than in an intimidating section that focuses only on terminology.

Calculator Perspectives in the Main Body of the Book

The TI-83 Plus™ is used to work sample problems in the book as concepts are developed. These Calculator Perspectives provide both students and teachers with calculator material right where it is needed without additional handouts or supplements. The TI-83 Plus™ was selected as the representative graphing calculator because it is the most popular model at the colleges we surveyed. Identical Calculator Perspectives, including screen shots and step-by-step keystroke instructions, are provided for other graphing calculator models at www.mhhe.com/hallmercer. (Some colleges require calculators rather than list them as optional because this is advantageous for students whose grants and scholarships are based on need.) (See AMATYC Standard P-1.)

Estimation Skills and Error Analysis Exercises

Estimation skills, concern for reasonable answers, and the ability to detect calculator errors should be developed by students at the same time they develop their calculator skills. Many examples and exercises in the book are specifically designed to help the students develop these skills. They are clearly labeled as such in the exercise sets. (See AMATYC Standard C-1.)

Mathematical Notes

Mathematical notes throughout the book give the students a sense of historical perspective and connect mathematics to other disciplines. These short vignettes give the origin of some of the symbols and terms that we now use and provide brief glimpses into the lives of some of the men and women of mathematics. (See AMATYC Standard P-3.)

Self-Checks

Self-checks and Self-check answers in each section help students become active learners and monitor their own progress. (See AMATYC Standard I-7.)

Example Format

The example format provides a clear model students can use to work the exercises. Sidebar explanations limit wordiness and allow students with different ability levels to use the examples in different ways. Examples are demonstrated from multiple perspectives so students can compare algebraic, numerical, graphical, and verbal approaches to a given problem. (See AMATYC Standard P-4.)

Geometrical-Based Problems

Examples and exercises based on geometric shapes are placed throughout the textbook. Many exercise sets have problems involving perimeter, area, and volume. (See AMATYC Standard C-3.)

Design of Exercises

Many exercises are composed of multiple parts to help the students face common misconceptions about the language and symbolism of mathematics. A few examples are:

Exercise 1.6, #13	**a.** Simplify $(6 + 4)^2$
	b. Simplify $6^2 + 4^2$
Chapter 5, Review #71	**a.** Expand $2x(x + 4y) - 3y(x + 4y)$
	b. Factor $2x(x + 4y) - 3y(x + 4y)$
Exercise 7.5, #55	**a.** Solve $(5m - 3)(m - 2) = 0$
	b. Simplify $(5m - 3)(m - 2)$

Group Discussion Questions and Group Projects

Each exercise set has group discussion questions, and there is a group project at the end of each of the first ten chapters. Many of these exercises build bridges to past or future material, and many seek to engage students orally and in writing to enable interactive and collaborative learning. (See AMATYC Standard P-2.)

Key Concepts, Chapter Review, and Mastery Test

Each chapter ends with these features. The Key Concepts outline the main points covered in that chapter. The Chapter Review contains a selection of exercises designed to help students review material from the chapter and to gauge their readiness for an exam. The Chapter Review is longer than an hour exam, and the order of the questions may not parallel the order of each topic within the chapter. The Mastery Test is more directed in its purpose: each of its questions matches an objective stated at the beginning of one of the sections in the chapter. Students can use the Mastery Test diagnostically to determine which sections and objectives have been mastered and on which they need more work.

Diagnostic Review of Beginning Algebra

A comprehensive review of the first five chapters appears between Chapters 5 and 6. Every question is categorized to help the instructor assign questions for a specific purpose. For each problem, the student is provided with the correct answer and directed to specific examples in the first five chapters of the text for personal review. We have found this review especially useful for students entering directly into the Intermediate Algebra portion of the book.

SUPPLEMENTS FOR THE INSTRUCTOR

Instructor's Edition

This ancillary contains answers to problems and exercises in the text, including answers to all Language and Symbolism of Mathematics vocabulary questions, all end-of-section exercises, all end-of-chapter review exercises, and all end-of-chapter mastery tests.

Computerized Test Bank

The computerized test bank allows you to create well-formatted quizzes or tests using a large bank of algorithmically generated and static questions through an intuitive Windows or Macintosh interface. When creating a quiz or test, you can manually choose individual questions or have the software randomly select questions based on section, question type, difficulty level, and other criteria. Instructors also have the ability to add or edit test bank questions to create their own customized test bank. In addition to printed tests, the test generator can deliver tests over a local area network or the World Wide Web, with automatic grading.

Instructor's Solutions Manual

Prepared by Mark Smith of College of Lake County, this supplement contains detailed solutions to all the exercises in the text not included in the Student Solution's Manual (see below). The methods used to solve the problems in the manual are the same as those used to solve the examples in the textbook.

Student Study Guide

This supplement provides a great many student benefits (see below), but also has notable benefits for instructors. Novice instructors can use it as the framework for day-by-day class plans. Veteran teachers have testified to its helpfulness in making better use of class time and keeping students focused on active learning.

Online Learning Center

Web-based interactive learning is available for your students on the Online Learning Center, located at www.mhhe.com/hallmercer. Student resources are located in the Student Center, and include interactive applications, algorithmically generated practice exams and quizzes, audiovisual tutorials, and web links. Instructor resources are located in the Instructor Center and include links to PageOut, ALEKS®, and other recommended sites.

PageOut

PageOut is McGraw-Hill's unique point-and-click course website tool, enabling you to create a full-featured, professional-quality course website without knowing HTML coding. With PageOut you can post your course syllabus, assign McGraw-Hill Online Learning Center content, add links to important off-site resources, and maintain student results in the online grade book. You can send class announcements, copy your course site to share with colleagues, and upload original files. PageOut is free for every McGraw-Hill user, and if you're short on time, we even have a team ready to help you create your site!

SUPPLEMENTS FOR THE STUDENT

Student's Solutions Manual

Prepared by Mark Smith of College of Lake County, the Student's Solutions Manual contains complete worked-out solutions to all the odd-numbered exercises from the text, including all end-of-section exercises, all end-of-chapter review exercises, and all end-of-chapter mastery tests.

Student Study Guide

This supplement provides key language and symbolism, definitions, and procedures so students can avoid using valuable class time recopying these items. Its examples are consistent with those in the text, and its structure models the framework of a class lecture. When completed, it provides an organized set of notes for later study.

Hall/Mercer Video Series

The video series is composed of 11 videocassettes (one for each chapter of the text). An on-screen instructor introduces topics and works through examples using the methods presented in the text, including step-by-step instruction on graphing calculator operations. The video series is also available on video CDs.

Hall/Mercer Tutorial CD-ROM

This interactive CD-ROM is a self-paced tutorial linked directly to the text that reinforces topics through unlimited opportunities to review concepts and practice problem solving. The CD-ROM provides algorithmically generated "bookmarkable" practice exercises (including hints), section- and chapter-level testing with gradebook capabilities, and a built-in graphing calculator. This product requires virtually no computer training on the part of students and supports Windows and Macintosh systems.

Online Learning Center

Student resources are located in the Student Center on the Online Learning Center (OLC) and include interactive applications, algorithmically generated "bookmarkable" practice exercises (including hints), section- and chapter-level testing, a built-in graphing calculator, a glossary, audiovisual tutorials, and links to PageOut, NetTutor, and other fun and useful algebra websites. The OLC is located at www.mhhe.com/hallmercer, and a free password card is included with each new copy of this text.

ALEKS

ALEKS® (**A**ssessment and **LE**arning in **K**nowledge **S**paces) is an artificial intelligence-based system for individualized math learning, available over the World Wide Web. ALEKS® delivers precise, qualitative diagnostic assessments of students' math knowledge, guides them in the selection of appropriate new study material, and records their progress toward mastery of curricular goals in a robust classroom management system. It interacts with the student much as a skilled human tutor would, moving between explanation and practice as needed, correcting and analyzing errors, defining terms, and changing topics on request. By sophisticated modeling of a student's "knowledge state" for a given subject matter, ALEKS® can focus clearly on what the student is most ready to learn next, building a learning momentum that fuels success.

To learn more about ALEKS®, including purchasing information, visit the ALEKS® website at www.highed.aleks.com.

NetTutor

NetTutor is a revolutionary system that enables students to interact with a live tutor over the World Wide Web. Students can receive instruction from live tutors using NetTutor's Web-based, graphical chat capabilities. They can also submit questions and receive answers, browse previously answered questions, and view previous live chat sessions.

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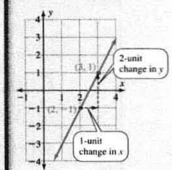
Walk-Through

discussed in Section 1.3.

$x_2 - x_1$ = change in x

Figure 3.1.2

Slope of a Line Through (x_1, y_1) and (x_2, y_2)

ALGEBRAICALLY	VERBALLY	NUMERICAL EXAMPLE	GRAPHICAL EXAMPLE
$m = \frac{y_2 - y_1}{x_2 - x_1}$ for $x_1 \neq x_2$.	The slope of a line is the ratio of the change in y to the change in x.	The slope of the line through the points (2, −1) and (3, 1) is $m = \frac{1-(-1)}{3-2}$ $m = \frac{2}{1}$	

A Mathematical Note

The origin of the use of *m* to designate slope is unknown. In his book *Mathematical Circles Revisited*, Howard Eves says

polynomials, you can use tables as illustrated in Example 8.

usually start by factoring out the greatest common factor. The **greatest common factor (GCF)** of the terms of a polynomial is the common factor that contains the largest possible coefficient and the largest possible exponent on each variable factor. For example, the GCF of $12x^4y^5$ and $15x^3y^6$ is $3x^3y^5$.

EXAMPLE 9 Using the Distributive Property to Factor Out a Greatest Common Factor

Use the distributive property to expand the expressions in parts **(a)**, **(c)**, and **(e)** and to factor the expressions in parts **(b)**, **(d)**, and **(f)**.

SOLUTIONS

(a) Expand $3x^2(5x - 4y)$.	$3x^2(5x - 4y) = (3x^2)(5x) + (3x^2)(-4y)$ $= 15x^3 - 12x^2y$	Distribute the factor of $3x^2$ to each term of the trinomial. Then simplify using the product rule for exponents.
(b) Factor $15x^3 - 12x^2y$.	$15x^3 - 12x^2y = (3x^2)(5x) + (3x^2)(-4y)$ $= 3x^2(5x - 4y)$	$3x^2$ is the greatest common factor of each term. Use the distributive property to factor out this common factor.
(c) Expand $8xy(2x + 3y)$.	$8xy(2x + 3y)$ $= (8xy)(2x) + (8xy)(3y)$ $= 16x^2y + 24xy^2$	Distribute the factor of $8xy$ to each term of the binomial. Then simplify using the product rule for exponents.
(d) Factor $16x^2y + 24xy^2$.	$16x^2y + 24xy^2$ $= (8xy)(2x) + (8xy)(3y)$ $= 8xy(2x + 3y)$	$8xy$ is the greatest common factor of each term. Use the distributive property to factor out this common factor.
(e) Expand $2x(5x - 4) + 3(5x - 4)$.	$2x(5x - 4) + 3(5x - 4)$ $= (2x)(5x) + (2x)(-4) + (3)(5x) + (3)(-4)$ $= 10x^2 - 8x + 15x - 12$ $= 10x^2 + 7x - 12$	Distribute the factor of $2x$ to each term of $5x - 4$ and distribute the factor 3 to each term of $5x - 4$. Then simplify using the product rule for exponents. Lastly, add like terms.
(f) Factor $2x(5x - 4) + 3(5x - 4)$.	$2x(5x - 4) + 3(5x - 4)$ $= (2x + 3)(5x - 4)$	Use the distributive property to factor out the common factor of $5x - 4$.

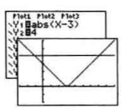
Solve $|4x$

We first used this strategy in Section 2.3 to solve linear equations in one variable.

We can use two-dimensional graphs to solve absolute value equations and inequalities in one variable. In the following calculator perspective we start by using y_1 to represent the absolute value expression.

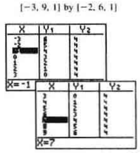
CALCULATOR PERSPECTIVE 4.4.1 Using a Graph and a Table to Solve an Absolute Value Equation or Inequality

To solve $|x - 3| = 4$, $|x - 3| < 4$, and $|x - 3| > 4$ from Example 2 using a TI-83 Plus calculator, enter the following keystrokes:



[−3, 9, 1] by [−2, 6, 1]

Press the down arrow key repeatedly to obtain the next screen.



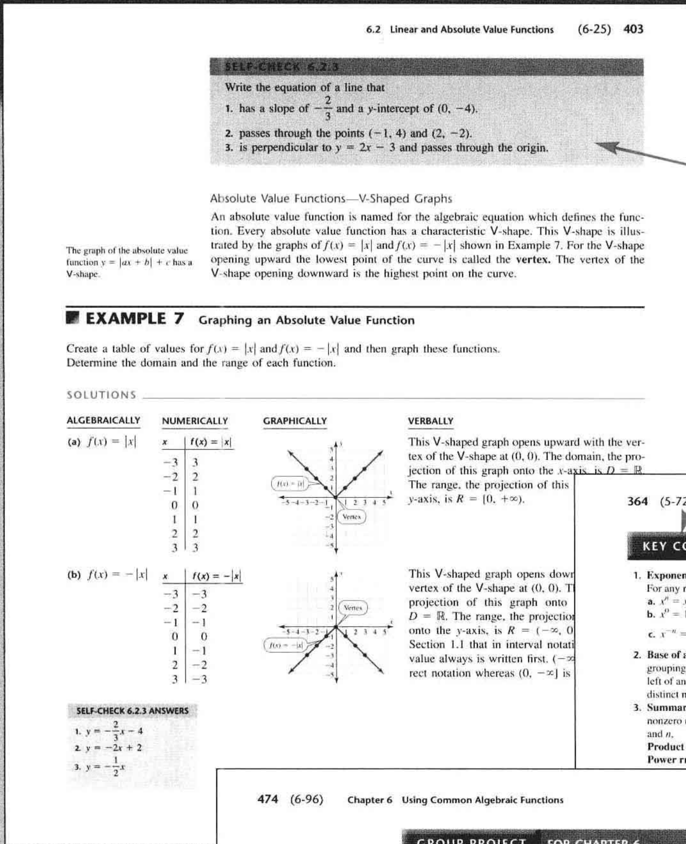
Note: Both in the table and on the graph $|x - 3| = 4$ for $x = -1$ or $x = 7$. Also $|x - 3| < 4$ for the x-values between −1 and 7, the interval (−1, 7); and $|x - 3| > 4$ for $x < -1$ or for $x > 7$, the set $(-\infty, -1) \cup (7, +\infty)$.

Tools for Learning

Multiple Perspective boxes—Demonstrating concepts in multiple perspectives— algebraically (symbolically), numerically, graphically, and verbally— helps students develop a deeper understanding of mathematics.

Examples—Each chapter includes many worked examples. These examples advance skills, develop concepts, connect concepts, and show application of concepts. Many examples are presented using multiple perspectives. These problems are worked in the same format students should use and have explanations horizontally aligned with the steps.

Calculator Perspective boxes—The TI-83 Plus™ calculator is used to work sample problems as concepts are developed. The location of the Calculator Perspectives within the textbook is not only convenient for the student, it also helps to keep the focus on the mathematical concepts while showing the appropriate use of technology for exploration and for computation. For students with calculators other than the TI-83 Plus™, every one of these Calculator Perspectives has been rewritten for TI-86™ and Casio CFX-9850GB Plus™ graphing calculators and provided in a clear, printable format on the text's Online Learning Center, complete with screenshots and step-by-step instructions.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SELF-CHECK 6.2.3

Write the equation of a line that

1. has a slope of $-\frac{2}{3}$ and a y-intercept of (0, −4).
2. passes through the points (−1, 4) and (2, −2).
3. is perpendicular to $y = 2x - 3$ and passes through the origin.

Absolute Value Functions—V-Shaped Graphs

The graph of the absolute value function $y = |ax + b| + c$ has a V-shape.

An absolute value function is named for the algebraic equation which defines the function. Every absolute value function has a characteristic V-shape. This V-shape is illustrated by the graphs of $f(x) = |x|$ and $f(x) = -|x|$ shown in Example 7. For the V-shape opening upward the lowest point of the curve is called the **vertex**. The vertex of the V-shape opening downward is the highest point on the curve.

EXAMPLE 7 Graphing an Absolute Value Function

Create a table of values for $f(x) = |x|$ and $f(x) = -|x|$ and then graph these functions. Determine the domain and the range of each function.

SOLUTIONS

ALGEBRAICALLY / NUMERICALLY / GRAPHICALLY / VERBALLY

(a) $f(x) = |x|$

x	$f(x) = \|x\|$
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3

This V-shaped graph opens upward with the vertex of the V-shape at (0, 0). The domain, the projection of this graph onto the x-axis, is $D = \mathbb{R}$. The range, the projection of this y-axis, is $R = [0, +\infty)$.

(b) $f(x) = -|x|$

x	$f(x) = -\|x\|$
−3	−3
−2	−2
−1	−1
0	0
1	−1
2	−2
3	−3

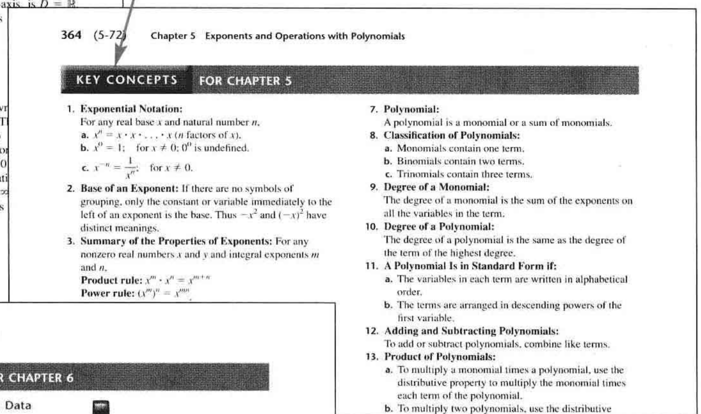
This V-shaped graph opens down vertex of the V-shape at (0, 0). T projection of this graph onto $D = \mathbb{R}$. The range, the projectio onto the y-axis, is $R = (-\infty, 0$ Section 1.1 that in interval notati value always is written first. $(-\infty$ rect notation whereas $(0, -\infty]$ is

SELF-CHECK 6.2.3 ANSWERS

1. $y = -\frac{2}{3}x - 4$
2. $y = -2x + 2$
3. $y = -\frac{1}{2}x$

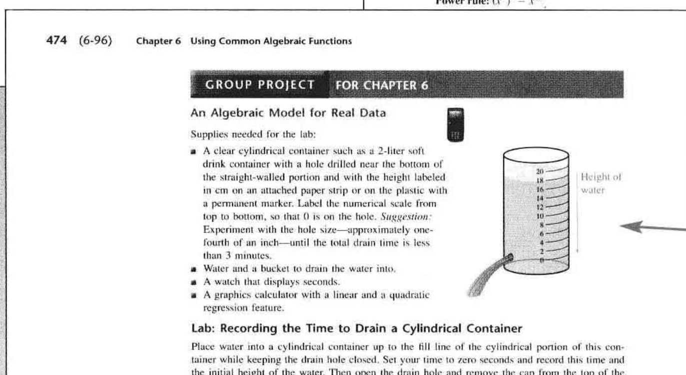
Self-Checks—Self-Checks and nearby answers in each section help students to become active learners and to monitor their own progress.

Key Concepts—Following each chapter is a Key Concepts section highlighting the chapter's key terms, principles, and procedures to assist in the students' review.


364 (5-72) Chapter 5 Exponents and Operations with Polynomials

KEY CONCEPTS FOR CHAPTER 5

1. **Exponential Notation:** For any real base x and natural number n,
   a. $x^n = x \cdot x \cdot \ldots \cdot x$ (n factors of x).
   b. $x^0 = 1$; for $x \neq 0$; 0^0 is undefined.
   c. $x^{-n} = \frac{1}{x^n}$; for $x \neq 0$.
2. **Base of an Exponent:** If there are no symbols of grouping, only the constant or variable immediately to the left of an exponent is the base. Thus $-x^2$ and $(-x)^2$ have distinct meanings.
3. **Summary of the Properties of Exponents:** For any nonzero real numbers x and y and integral exponents m and n,
   Product rule: $x^m \cdot x^n = x^{m+n}$
   Power rule: $(x^m)^n = x^{mn}$
7. **Polynomial:** A polynomial is a monomial or a sum of monomials.
8. **Classification of Polynomials:**
   a. Monomials contain one term.
   b. Binomials contain two terms.
   c. Trinomials contain three terms.
9. **Degree of a Monomial:** The degree of a monomial is the sum of the exponents on all the variables in the term.
10. **Degree of a Polynomial:** The degree of a polynomial is the same as the degree of the term of the highest degree.
11. **A Polynomial Is in Standard Form if:**
    a. The variables in each term are written in alphabetical order.
    b. The terms are arranged in descending powers of the first variable.
12. **Adding and Subtracting Polynomials:** To add or subtract polynomials, combine like terms.
13. **Product of Polynomials:**
    a. To multiply a monomial times a polynomial, use the distributive property to multiply the monomial times each term of the polynomial.
    b. To multiply two polynomials, use the distributive


474 (6-96) Chapter 6 Using Common Algebraic Functions

GROUP PROJECT FOR CHAPTER 6

An Algebraic Model for Real Data

Supplies needed for the lab:

- A clear cylindrical container such as a 2-liter soft drink container with a hole drilled near the bottom of the straight-walled portion and with the height labeled in cm on an attached paper strip or on the plastic with a permanent marker. Label the numerical scale from top to bottom, so that 0 is on the hole. *Suggestion:* Experiment with the hole size—approximately one-fourth of an inch—until the total drain time is less than 3 minutes.
- Water and a bucket to drain the water into.
- A watch that displays seconds.
- A graphics calculator with a linear and a quadratic regression feature.

Lab: Recording the Time to Drain a Cylindrical Container

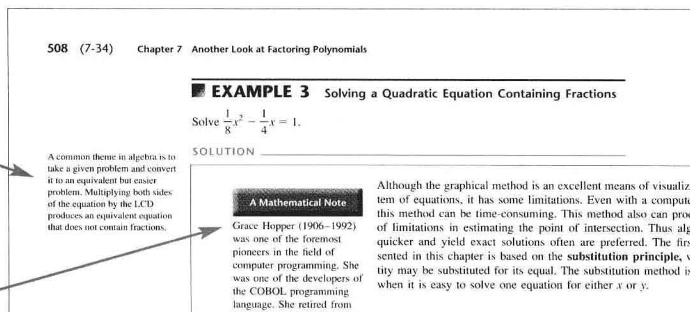
Place water into a cylindrical container up to the fill line of the cylindrical portion of this container while keeping the drain hole closed. Set your time to zero seconds and record this time and the initial height of the water. Then open the drain hole and remove the cap from the top of the

Group Project—Chapters end with Group Projects that encourage collaborative problem-solving and reinforce the links between past and future material.

Commentary and Guidance

Marginal Notes—The margins of the text are populated with brief hints and explanations of nearby material that will help teachers teach and students learn.

Mathematical Notes—These short vignettes describe the origin of some of the symbols and terms used in the text and provide brief glimpses into the lives of some of the men and women of mathematics.

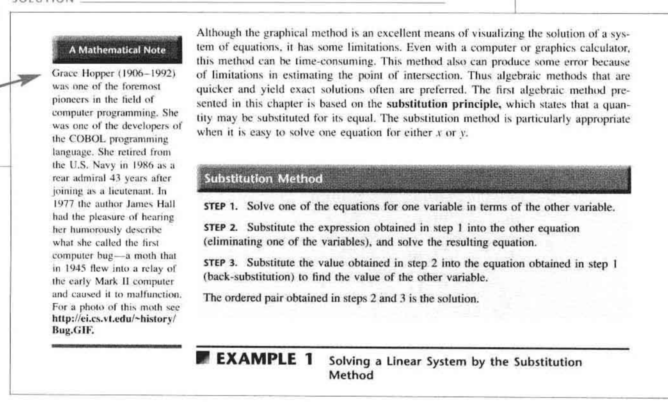

508 (7-34) Chapter 7 Another Look at Factoring Polynomials

EXAMPLE 3 Solving a Quadratic Equation Containing Fractions

Solve $\frac{1}{8}x^2 - \frac{1}{4}x = 1$.

SOLUTION

A common theme in algebra is to take a given problem and convert it to an equivalent but easier problem. Multiplying both sides of the equation by the LCD produces an equivalent equation that does not contain fractions.


A Mathematical Note

Grace Hopper (1906–1992) was one of the foremost pioneers in the field of computer programming. She was one of the developers of the COBOL programming language. She retired from the U.S. Navy in 1986 as a rear admiral 43 years after joining as a lieutenant. In 1977 the author James Hall had the pleasure of hearing her humorously describe what she called the first computer bug—a moth that in 1945 flew into a relay of the early Mark II computer and caused it to malfunction. For a photo of this moth see **http://ei.cs.vt.edu/~history/Bug.GIF.**

Although the graphical method is an excellent means of visualizing the solution of a system of equations, it has some limitations. Even with a computer or graphics calculator, this method can be time-consuming. This method also can produce some error because of limitations in estimating the point of intersection. Thus algebraic methods that are quicker and yield exact solutions often are preferred. The first algebraic method presented in this chapter is based on the **substitution principle,** which states that a quantity may be substituted for its equal. The substitution method is particularly appropriate when it is easy to solve one equation for either x or y.

Substitution Method

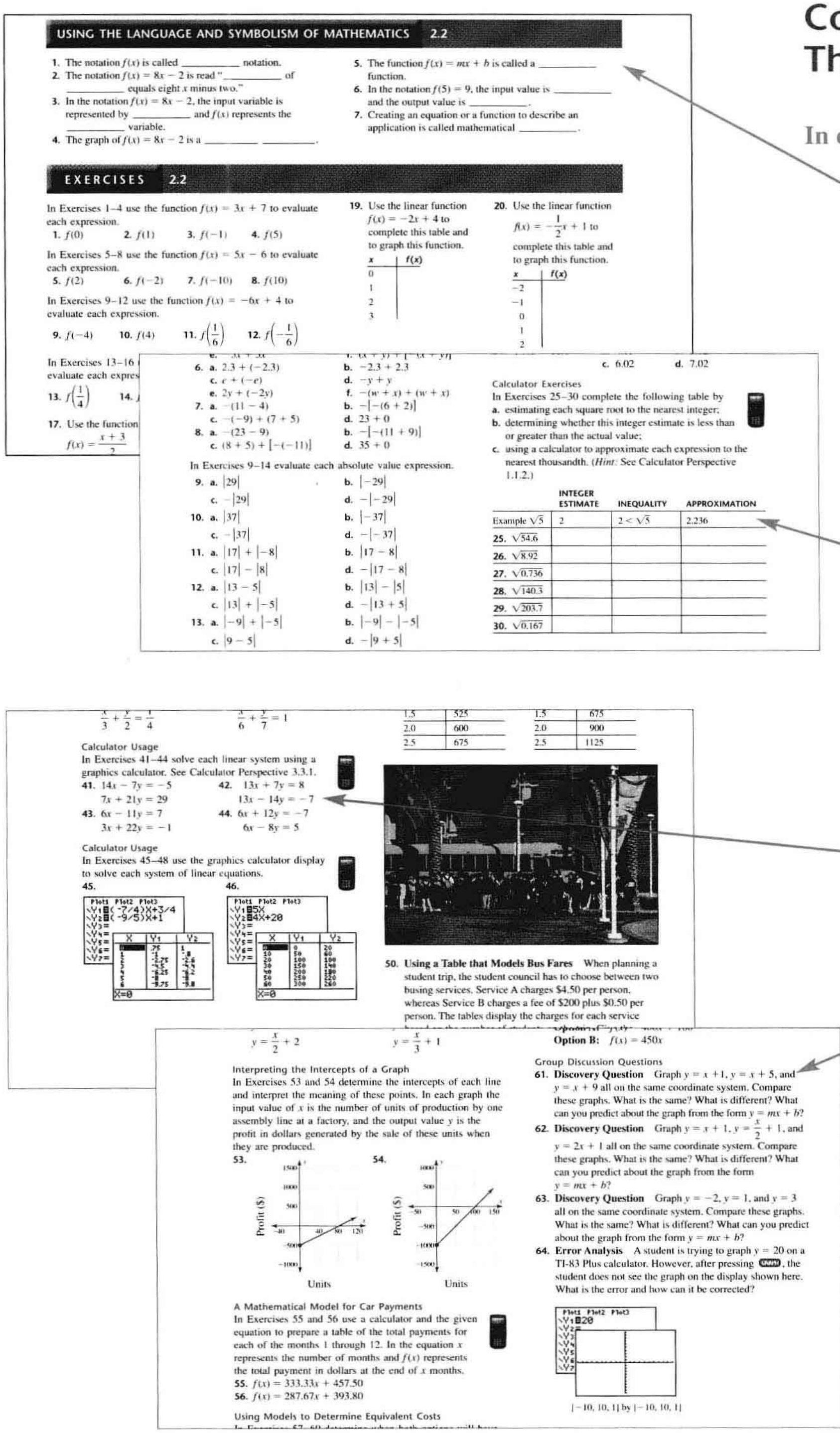
STEP 1. Solve one of the equations for one variable in terms of the other variable.

STEP 2. Substitute the expression obtained in step 1 into the other equation (eliminating one of the variables), and solve the resulting equation.

STEP 3. Substitute the value obtained in step 2 into the equation obtained in step 1 (back-substitution) to find the value of the other variable.

The ordered pair obtained in steps 2 and 3 is the solution.

EXAMPLE 1 Solving a Linear System by the Substitution Method



Conceptual Understanding Through Practice

In every section of the text:

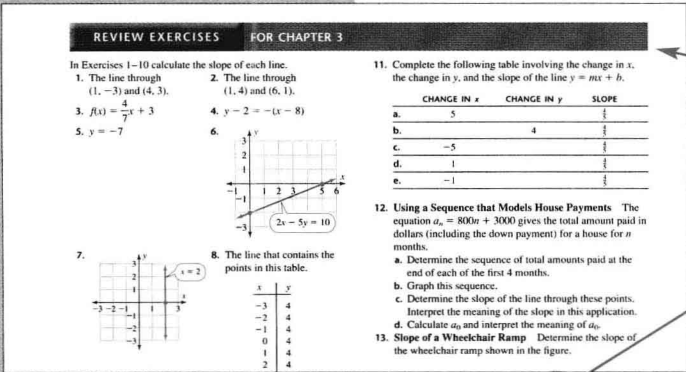
Using the Language and Symbolism of Mathematics—Every section includes a series of fill-in-the-blank questions to help students gain fluency in the language and symbolism of mathematics.

Exercises—Each end-of-section exercise set is carefully constructed to develop and to reinforce the skills and concepts of algebra, and to provide an appropriate review of the section. Exercise sets include:

Estimation Skills and Error Analysis Exercises—Estimation skills, concern for reasonable answers, and the ability to detect calculator errors are critical elements of students' mathematical knowledge. There are examples and exercises in the book specifically designed to help the students develop these skills.

Calculator Exercises—When the use of a calculator is appropriate to the solution of an exercise, it is indicated in the text by a calculator icon, right next to the exercise.

Group Discussion Questions—These exercises involve students in interactive and collaborative learning and encourage them to communicate mathematics both orally and in writing.


REVIEW EXERCISES FOR CHAPTER 3

In Exercises 1–10 calculate the slope of each line.

1. The line through $(1, -3)$ and $(4, 3)$.
2. The line through $(1, 4)$ and $(6, 1)$.
3. $f(x) = \frac{4}{7}x + 3$
4. $y - 2 = -(x - 8)$
5. $y = -7$
6.

7.

8. The line that contains the points in this table.

x	y
-3	4
-2	4
-1	4
0	4
1	4
2	4

11. Complete the following table involving the change in x, the change in y, and the slope of the line $y = mx + b$.

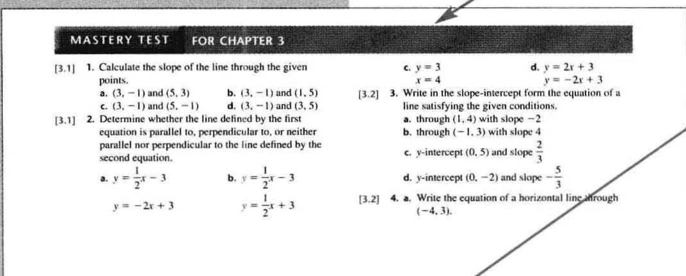
	CHANGE IN x	CHANGE IN y	SLOPE
a.	5		$\frac{4}{5}$
b.		4	$\frac{4}{5}$
c.	-5		$\frac{4}{5}$
d.	1		$\frac{4}{5}$
e.	-1		$\frac{4}{5}$

12. **Using a Sequence that Models House Payments** The equation $a_n = 800n + 3000$ gives the total amount paid in dollars (including the down payment) for a house for n months.
    a. Determine the sequence of total amounts paid at the end of each of the first 4 months.
    b. Graph this sequence.
    c. Determine the slope of the line through these points. Interpret the meaning of the slope in this application.
    d. Calculate a_0 and interpret the meaning of a_0.
13. **Slope of a Wheelchair Ramp** Determine the slope of the wheelchair ramp shown in the figure.

In every chapter of the text:

Chapter Review Exercises—These comprehensive exercise sets provide ample and well-distributed practice on the topics of the chapter.

Chapter Mastery Test—These tests are written specifically to cover each objective presented in the chapter.


MASTERY TEST FOR CHAPTER 3

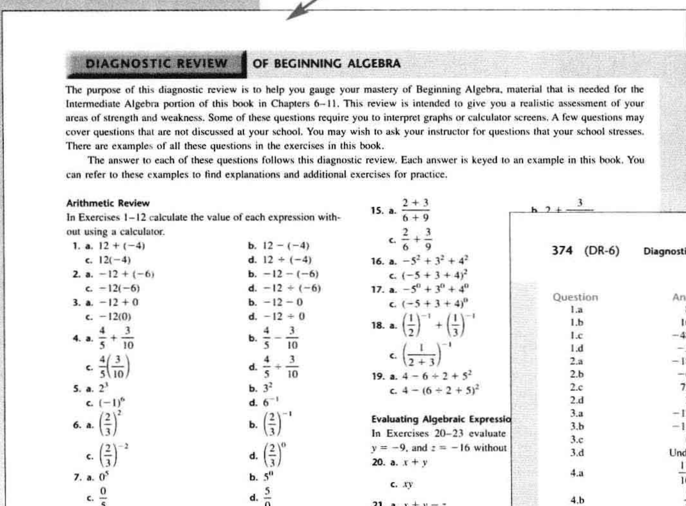
[3.1] 1. Calculate the slope of the line through the given points.
   a. $(3, -1)$ and $(5, 3)$ b. $(3, -1)$ and $(1, 5)$
   c. $(3, -1)$ and $(5, -1)$ d. $(3, -1)$ and $(3, 5)$

[3.1] 2. Determine whether the line defined by the first equation is parallel to, perpendicular to, or neither parallel nor perpendicular to the line defined by the second equation.
   a. $y = \frac{1}{2}x - 3$, $y = -2x + 3$ b. $y = \frac{1}{2}x - 3$, $y = \frac{1}{2}x + 3$
   c. $y = 3$, $x = 4$ d. $y = 2x + 3$, $y = -2x + 3$

[3.2] 3. Write in the slope-intercept form the equation of a line satisfying the given conditions.
   a. through $(1, 4)$ with slope -2
   b. through $(-1, 3)$ with slope 4
   c. y-intercept $(0, 5)$ and slope $\frac{2}{3}$
   d. y-intercept $(0, -2)$ and slope $-\frac{5}{3}$

[3.2] 4. a. Write the equation of a horizontal line through $(-4, 3)$.

Diagnostic Review—To ensure students have mastered the concepts of the previous chapters, a comprehensive diagnostic review, complete with answers, appears between Chapters 5 and 6. It will prove especially helpful to students who are entering the book at this intermediate junction and who need to assess their beginning algebra skills. Direct references to the relevant examples in Chapters 1 to 5 are provided alongside the answers for students who need to refresh or reinforce their mastery of beginning algebra concepts.


DIAGNOSTIC REVIEW OF BEGINNING ALGEBRA

The purpose of this diagnostic review is to help you gauge your mastery of Beginning Algebra, material that is needed for the Intermediate Algebra portion of this book in Chapters 6–11. This review is intended to give you a realistic assessment of your areas of strength and weakness. Some of these questions require you to interpret graphs or calculator screens. A few questions may cover questions that are not discussed at your school. You may wish to ask your instructor for questions that your school stresses. There are examples of all these questions in the exercises in this book.

The answer to each of these questions follows this diagnostic review. Each answer is keyed to an example in this book. You can refer to these examples to find explanations and additional exercises for practice.

Arithmetic Review

In Exercises 1–12 calculate the value of each expression without using a calculator.

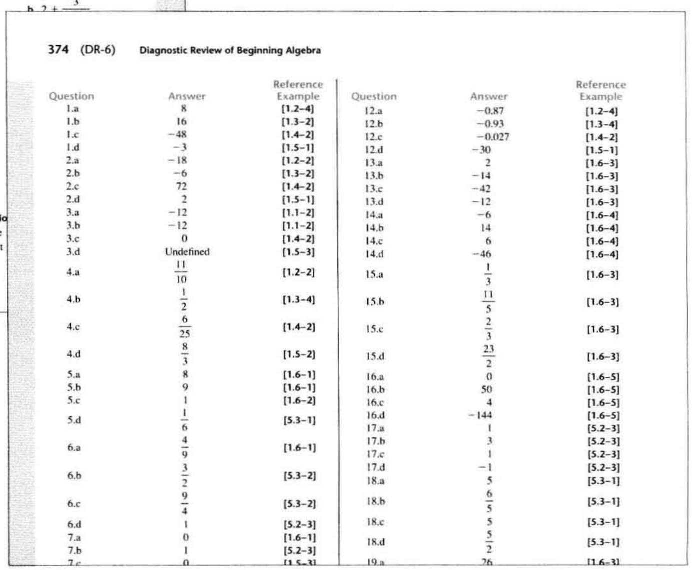
1. a. $12 + (-4)$ b. $12 - (-4)$ c. $12(-4)$ d. $12 \div (-4)$
2. a. $-12 + (-6)$ b. $-12 - (-6)$ c. $-12(-6)$ d. $-12 \div (-6)$
3. a. $-12 + 0$ b. $-12 - 0$ c. $-12(0)$ d. $-12 \div 0$
4. a. $\frac{4}{5} + \frac{3}{10}$ b. $\frac{4}{5} - \frac{3}{10}$ c. $\frac{4}{5}\left(\frac{3}{10}\right)$ d. $\frac{4}{5} \div \frac{3}{10}$
5. a. 2^3 b. 3^2 c. $(-1)^6$ d. 6^{-1}
6. a. $\left(\frac{2}{3}\right)^2$ b. $\left(\frac{2}{3}\right)^{-1}$ c. $\left(\frac{2}{3}\right)^{-2}$ d. $\left(\frac{2}{3}\right)^0$
7. a. 0^5 b. 5^0 c. $\frac{0}{5}$ d. $\frac{5}{0}$

15. a. $\frac{2+3}{6+9}$ b. $2 + \frac{3}{}$ c. $\frac{2}{6} + \frac{3}{9}$
16. a. $-5^2 + 3^2 + 4^2$ c. $(-5 + 3 + 4)^2$
17. a. $-5^0 + 3^0 + 4^0$ c. $(-5 + 3 + 4)^0$
18. a. $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1}$ c. $\left(\frac{1}{2+3}\right)^{-1}$
19. a. $4 - 6 \div 2 + 5^2$ c. $4 - (6 \div 2 + 5)^2$

Evaluating Algebraic Expressio

In Exercises 20–23 evaluate
$y = -9$, and $z = -16$ without

20. a. $x + y$ c. xy
21. a. $x + y - z$


374 (DR-6) Diagnostic Review of Beginning Algebra

Question	Answer	Reference Example
1.a	8	[1.2–4]
1.b	16	[1.3–2]
1.c	−48	[1.4–2]
1.d	−3	[1.5–1]
2.a	−18	[1.2–2]
2.b	−6	[1.3–2]
2.c	72	[1.4–2]
2.d	2	[1.5–1]
3.a	−12	[1.1–2]
3.b	−12	[1.1–2]
3.c	0	[1.4–2]
3.d	Undefined	[1.5–3]
4.a	$\frac{11}{10}$	[1.2–2]
4.b	$\frac{1}{2}$	[1.3–4]
4.c	$\frac{6}{25}$	[1.4–2]
4.d	$\frac{8}{3}$	[1.5–2]
5.a	8	[1.6–1]
5.b	9	[1.6–1]
5.c	1	[1.6–2]
5.d	$\frac{1}{6}$	[5.3–1]
6.a	$\frac{4}{9}$	[1.6–1]
6.b	$\frac{3}{2}$	[5.3–2]
6.c	$\frac{9}{4}$	[5.3–2]
6.d	1	[5.2–3]
7.a	0	[1.6–1]
7.b	1	[5.2–3]
7.c	0	[1.5–3]

Question	Answer	Reference Example
12.a	−0.87	[1.2–4]
12.b	−0.93	[1.3–4]
12.c	−0.027	[1.4–2]
12.d	−30	[1.5–1]
13.a	2	[1.6–3]
13.b	−14	[1.6–3]
13.c	−42	[1.6–3]
13.d	−12	[1.6–3]
14.a	−6	[1.6–4]
14.b	14	[1.6–4]
14.c	6	[1.6–4]
14.d	−46	[1.6–4]
15.a	$\frac{1}{3}$	[1.6–3]
15.b	$\frac{11}{5}$	[1.6–3]
15.c	$\frac{2}{3}$	[1.6–3]
15.d	$\frac{23}{2}$	[1.6–3]
16.a	0	[1.6–5]
16.b	50	[1.6–5]
16.c	4	[1.6–5]
16.d	−144	[1.6–5]
17.a	1	[5.2–3]
17.b	3	[5.2–3]
17.c	1	[5.2–3]
17.d	−1	[5.2–3]
18.a	5	[5.3–1]
18.b	$\frac{6}{5}$	[5.3–1]
18.c	5	[5.3–1]
18.d	$\frac{5}{2}$	[5.3–1]
19.a	26	[1.6–3]