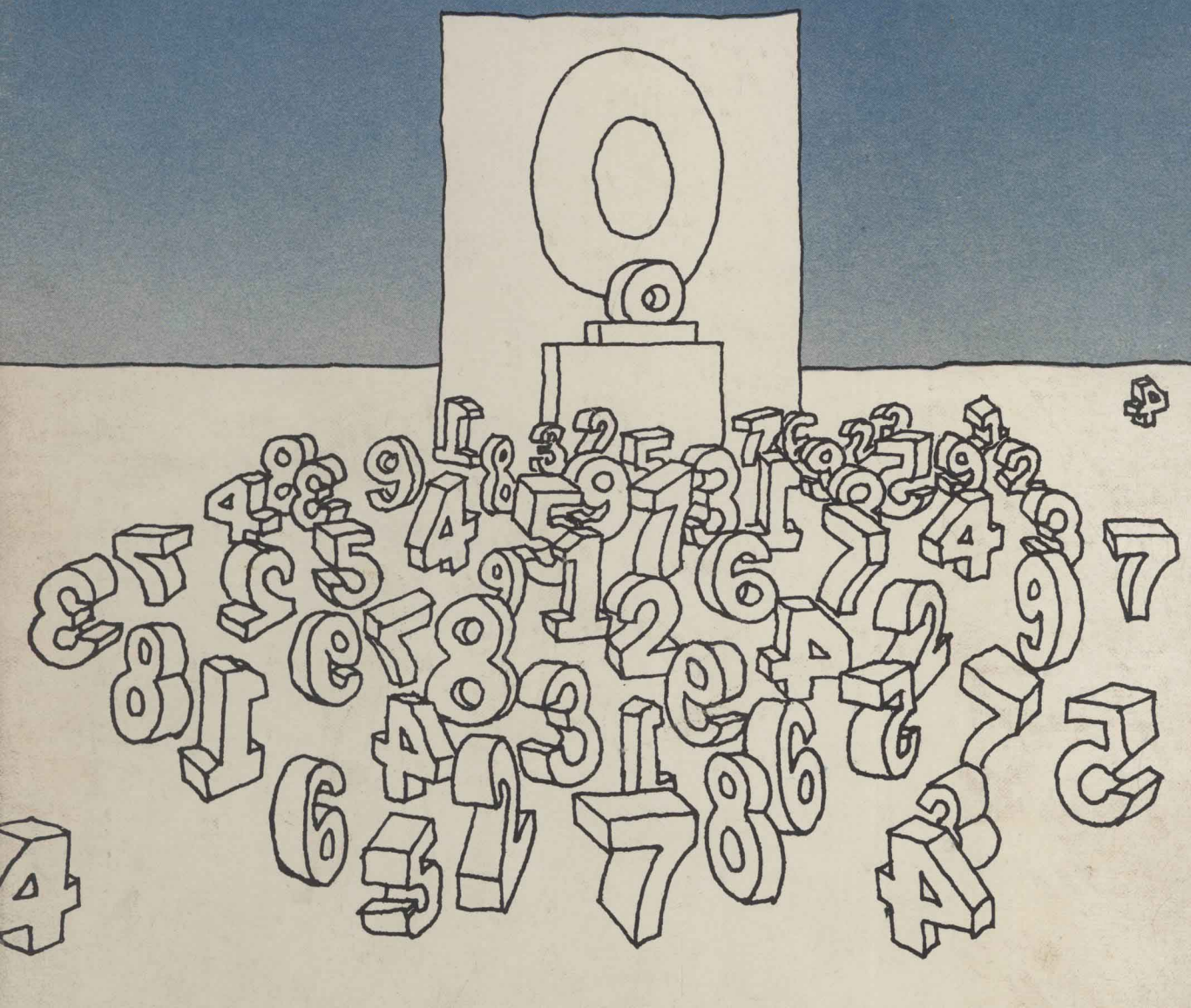
College Algebra and Trigonometry

C.L. Johnston



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Dellen Publishing Company
San Francisco

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Library of Congress Cataloging in Publication Data

Johnston, Carol Lee.
College algebra and trigonometry.

Includes index.
1. Algebra. 2. Trigonometry. I. Title.
QA154.2.J62 512.9 77-21206

ISBN 0-89517-001-9

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Preface

Our major objective throughout the writing of this book has been to present the concepts of college algebra, plane trigonometry, and analytic geometry to the student in an easily understood language supplemented with illustrations, solved problems, and other aids. We assume that the students taking this course have had at least an introductory course in algebra and plane geometry.

Some of the special features of this book are:

1. The contents are arranged in small, easy-to-read segments using a one-concept-at-a-time approach. Each essential manipulation is followed by several specific examples. And examples are followed by exercises that employ the concept under discussion.
2. Liberal use of visual aids, such as shading, dotted lines, and arrows, helps clarify concepts and manipulations.
3. One of the innovations of this book is the order in which trigonometric topics are presented. After an introduction to trigonometry we take just one trigonometric function at a time and carry it through evaluations, trigonometric equations, applications to both the right and general triangles, and graphs. Once the student has used the sine function through these applications, it has become thoroughly established in his or her mind and will not easily be forgotten. Next, the cosine function is discussed, and then the tangent function. With this thorough knowledge of the sine, cosine, and tangent functions, the student has little difficulty extending ideas to include the definitions and applications of the reciprocal functions.
4. The unit circle has been used extensively as an aid in defining the trigonometric functions and in developing formulas and identities.
5. The text contains over 700 complete examples and approximately 3,000 exercises.

6. Each exercise set is divided into two equal parts marked by the letters A and B in the left margin. The answers are given in the back of the book for the exercises of the first group. The answers for the second group of exercises are *not* given in this book. They are included, with complete solutions, in a solutions manual that is available at a nominal cost. The first problem of each of the groups (A and B) is the same kind of problem. The second problem of each group is the same kind of problem, and so on, throughout the two groups of exercises. This makes it easy for a student to find the solution to a problem in the solutions manual similar to the problem in part A with which he or she might be having difficulty.
7. A chapter summary and a set of review exercises are included at the end of each chapter.
8. A diagnostic test is included at the end of each chapter. Complete solutions for the diagnostic test (with section references) are given in the back of the book.
9. An instructor's manual contains two different tests for each chapter and two final examinations. These may be easily removed and duplicated for class use. The tests include adequate space for students to work the problems. Answer keys and solutions for these tests are provided in the manual.
10. The use of calculators is encouraged and exercises to be solved with a calculator are provided. These are marked with a special symbol in the left margin to indicate that a calculator may be used in the solution. If calculators are not available, appropriate tables, which are included in the back of the book, can be used.
11. The instructor has the option of having students use calculators exclusively, or tables exclusively, or both calculators and tables in doing computational work.
12. Important concepts and algorithms are enclosed in boxes for easy identification and quick location.
13. "Words of Caution" help students avoid common mathematical errors. These are marked with a special symbol in the left margin.
14. Much of the analytic geometry is integrated with the algebra.
15. Symbols, basic formulas, and the basic trigonometric identities are listed inside the cover so as to be immediately available when needed.
16. The text can be easily adapted for use in a variety of instructional programs: conventional lecture courses, learning laboratories, or self-study programs.

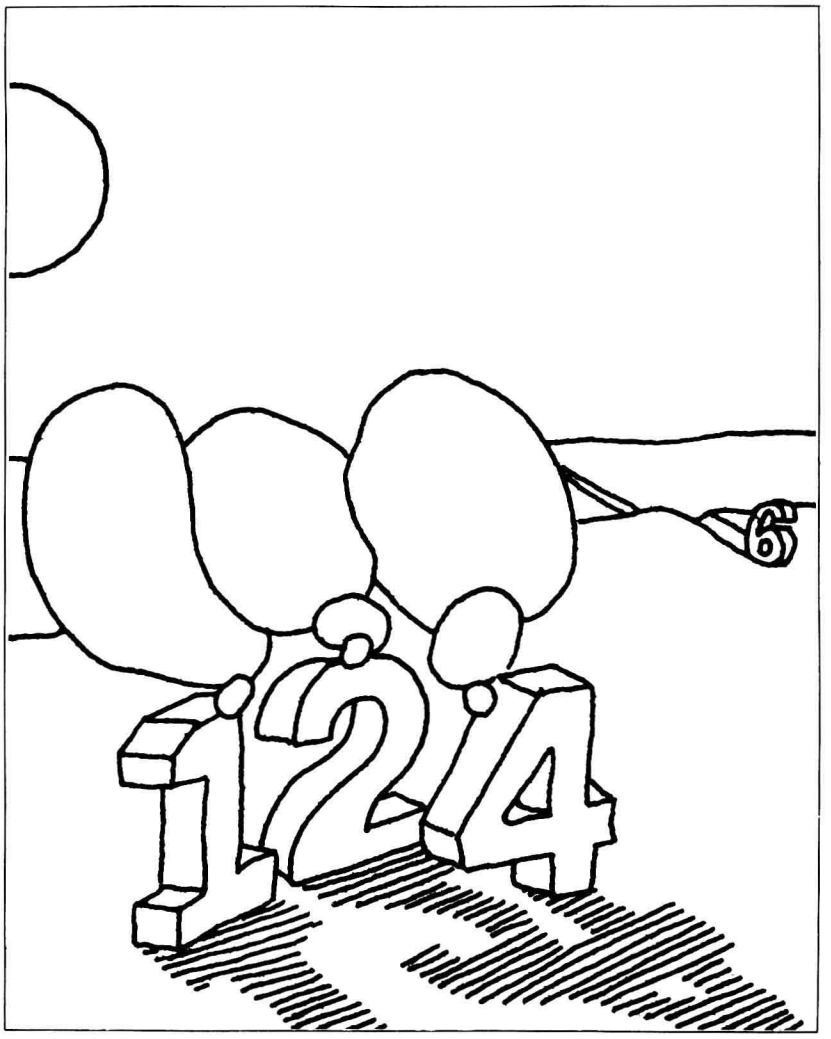
I wish to thank Don Dellen of the Dellen Publishing Company for his encouragement and for soliciting critical reviews of the manuscript from Dale R. Bedgood, East Texas State University; Charles M. Biles, Humboldt State University; Boyd L. Cardon, Ricks College; John Distad, University of Alaska; Thomas Drouet, East Los Angeles College; Paul Drowne, Mount Wachusett Community College; Clifton Gary, Oscar Rose College; O. T. Gilbert, Johnson County Community College; Donald W. Gladstone, Orange County Community College; Frederic Gooding, Gallaudet College; Joel Greenstein, New York City Community College; Mark Hale, University of Florida; Nancy J. Halford, Rio Hondo College; George L. Holloway, California State University, Northridge; James Kinney, George C. Wallace State Junior College; Rodney Kohler, Central Oregon Community College; Jeanne Lazaris, East Los Angeles College; Doyle McCown, Oklahoma State University Technical Institute; William Mech, Boise State University; William A. Neal, Fresno City College; Kenneth J. Shabell, Riverside City College; Robert Shloming, Essex County College; Rose Marie Smith, Texas Woman's University; Lynn Tooley, Bellevue Community College; Jack Twitchell, Mesa Community College; Irene Verner, Baruch College, City University of New York; Richard G. Vinson, University of South Alabama; and Jack D. Wilson, Murray State University. I also want to thank Linda Thompson and Phyllis Niklas for their meticulous editing of the final manuscript. To all of these I wish to express my sincere appreciation for their valuable help.

Finally, I owe a great deal to my students who always inspire one to produce a book worthy of their time and to my wife for her encouragement and moral support over long periods of writing.

Whittier, California *C. L. Johnston*

Contents

1 Basic concepts of real numbers



It is our purpose in this course to prepare you for a successful study of calculus. We assume that you have had the equivalent of two years of high school algebra and a course in plane geometry. However, for various reasons, you may find a brief review of some of your earlier work in mathematics helpful. For that reason much of the work of the early chapters of this book is devoted to a review of your former mathematics.

A mathematical system usually begins with some undefined words or terms, some defined words or terms, a set of elements, one or more relations between the elements, one or more operations defined on that **set**, and some statements we accept as true without proof (**axioms**). Using these tools, we can (usually) prove many other statements (**theorems**), and thus build a mathematical structure.

In this chapter we give a brief review of some information on sets that we shall be using in this course and some of the properties of the real number system. In algebra we work with letters that represent numbers and must therefore deal with the letters according to the laws of numbers.

1 Sets

A set is a collection of particular things. For example, we speak of the set of numbers between 2 and 7, the set of points on a line segment, or within a circle, and so on. In this course we shall be concerned only with sets of numbers. The objects that make up the set are called its **elements** (or **members**). Sets are often represented by capital letters or by listing their elements within braces { }. For example, $A = \{1, 2, 3\}$. If the braces have nothing within them, the set is called the **empty set** (or **null set**). An example of an empty set would be the set of all people in your class who are 10 feet tall. The empty set is represented by the symbol { } or preferably, $\varnothing$.

A WORD OF CAUTION The set $\{0\}$ is not an empty set because it contains the element "0." Also, the set $\{\varnothing\}$ is not an empty set because it contains the symbol $\varnothing$.

ROSTER METHOD OF REPRESENTING A SET

A class roster is a list of the members of the class. The roster of a set of numbers is a list of the numbers in the set. If, for example, the numbers in a set are 1, 3, 5, and 7, the **roster method** of representing these numbers is $\{1, 3, 5, 7\}$.

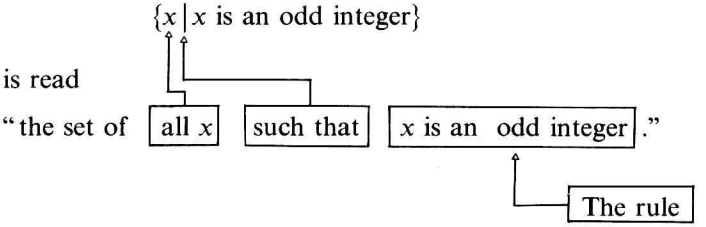
The expression $B = \{1, 3, 5\}$ is read "B is the set whose elements are 1, 3, and 5." The symbol $\in$ is read "is an element of." It is used in set notation to denote that an element is a member of a particular set. For example, in the set $D = \{5, 8, 9\}$ the symbol $5 \in D$ tells us that "5 is an element of D." In like manner the symbols $8 \in D$ and $9 \in D$ tell us that 8 and 9 are elements of D.

The slash line (/) negates most symbols. The symbol $\notin$ is read "is *not*

an element of." For example, if $A = \{8, 9\}$, then $4 \notin A$. This is read "4 is not an element of A."

SET-BUILDER METHOD OF REPRESENTING A SET

A set can be represented by giving a property of its members. For example, the expression



EXAMPLE 1 Write $\{x \mid x$ is an even number between 1 and 9$\}$ in roster notation.

SOLUTION $\{x \mid x$ is an even number between 1 and 9$\} = \{2, 4, 6, 8\}$.

EXAMPLE 2 Write $\{3, 5, 7\}$ in set-builder notation.

SOLUTION $\{3, 5, 7\} = \{x \mid x$ is an odd number between 2 and 8$\}$.

SUBSETS

Set A is a **subset** of set B if every element of set A is contained in set B. Thus, if $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$, then A is a subset of B. This is written "$A \subseteq B$." Every set is a subset of itself. The null set, $\varnothing$, is, by definition, a subset of every set.

EXAMPLE 3 The subsets of the set $\{3, 4, 6\}$ are $\{3\}$, $\{4\}$, $\{6\}$, $\{3, 4\}$, $\{3, 6\}$, $\{4, 6\}$, $\{3, 4, 6\}$, and $\varnothing$.

EQUAL SETS

Two sets A and B are said to be equal, written $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$; that is, if they contain the same elements. The order in which the elements are written and the number of times a particular element is repeated within a given set does not matter.

EXAMPLE 4 Examples of equal sets.

a. $\{1, 2, 3\} = \{2, 1, 3\} = \{3, 1, 2\} = \{1, 2, 2, 3, 2\}$

b. $\{a, c, e\} = \{a, e, c, e, a\}$

UNION AND INTERSECTION OF SETS

Sets are often represented by closed geometric figures, where it is understood that all points within each respective figure represent the elements of a set. For example, in Figure 1 we show that sets A and B are subsets of U.

In Figure 2, the interiors of circles A and B represent different sets.