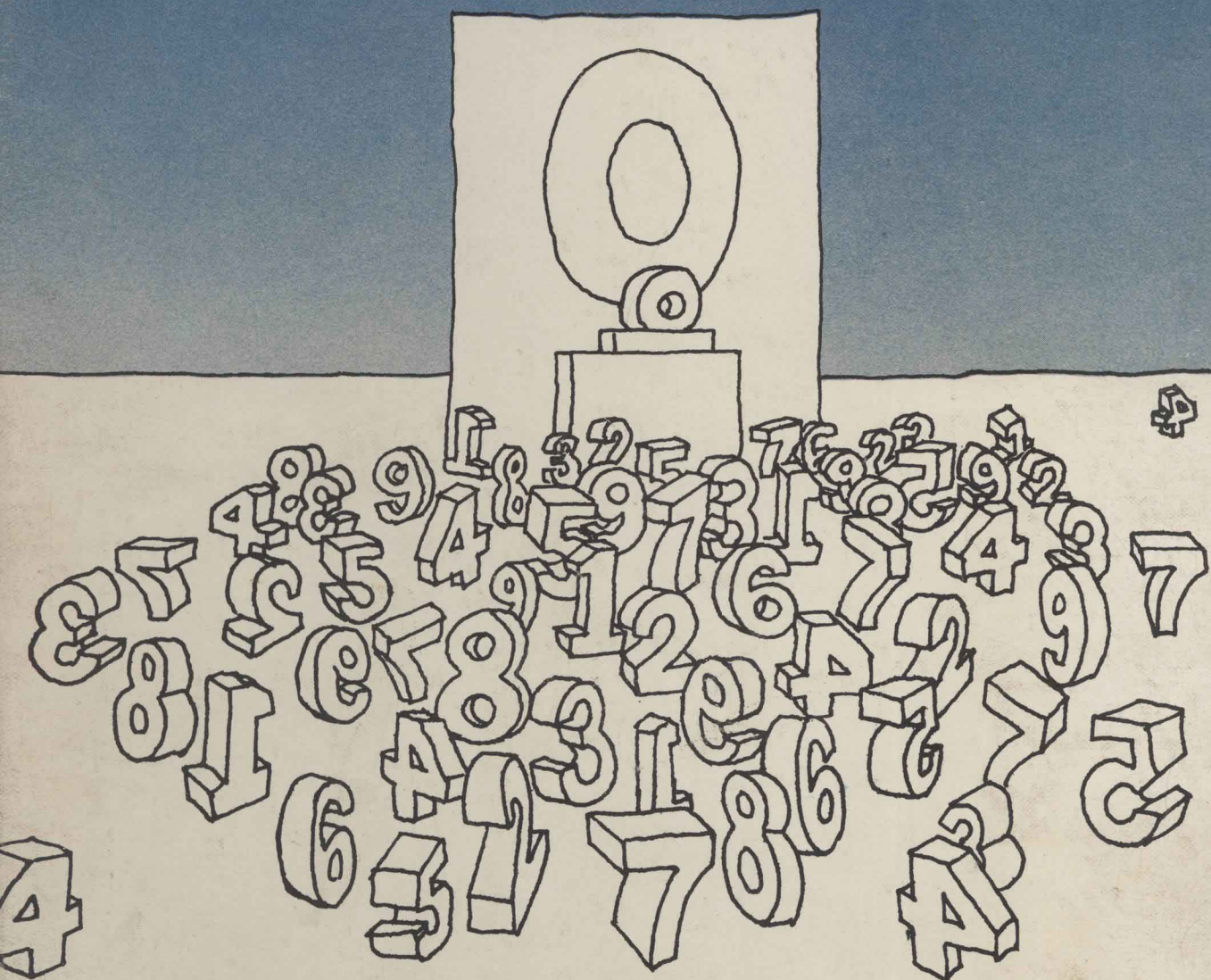


# College Algebra and Trigonometry

C.L. Johnston



# College Algebra and Trigonometry

**C.L. Johnston**

**The Foundation for Books to China**

美国友好书刊基金会

88

**Dellen Publishing Company**  
**San Francisco**

© Copyright 1978 by Dellen Publishing Company, San Francisco,  
California

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

**Library of Congress Cataloging in Publication Data**

Johnston, Carol Lee.

College algebra and trigonometry.

Includes index.

1. Algebra. 2. Trigonometry. I. Title.  
QA154.2.J62 512.9 77-21206

ISBN 0-89517-001-9

# **College Algebra and Trigonometry**

# Preface

Our major objective throughout the writing of this book has been to present the concepts of college algebra, plane trigonometry, and analytic geometry to the student in an easily understood language supplemented with illustrations, solved problems, and other aids. We assume that the students taking this course have had at least an introductory course in algebra and plane geometry.

Some of the special features of this book are:

1. The contents are arranged in small, easy-to-read segments using a one-concept-at-a-time approach. Each essential manipulation is followed by several specific examples. And examples are followed by exercises that employ the concept under discussion.
2. Liberal use of visual aids, such as shading, dotted lines, and arrows, helps clarify concepts and manipulations.
3. One of the innovations of this book is the order in which trigonometric topics are presented. After an introduction to trigonometry we take just one trigonometric function at a time and carry it through evaluations, trigonometric equations, applications to both the right and general triangles, and graphs. Once the student has used the sine function through these applications, it has become thoroughly established in his or her mind and will not easily be forgotten. Next, the cosine function is discussed, and then the tangent function. With this thorough knowledge of the sine, cosine, and tangent functions, the student has little difficulty extending ideas to include the definitions and applications of the reciprocal functions.
4. The unit circle has been used extensively as an aid in defining the trigonometric functions and in developing formulas and identities.
5. The text contains over 700 complete examples and approximately 3,000 exercises.

6. Each exercise set is divided into two equal parts marked by the letters A and B in the left margin. The answers are given in the back of the book for the exercises of the first group. The answers for the second group of exercises are *not* given in this book. They are included, with complete solutions, in a solutions manual that is available at a nominal cost. The first problem of each of the groups (A and B) is the same kind of problem. The second problem of each group is the same kind of problem, and so on, throughout the two groups of exercises. This makes it easy for a student to find the solution to a problem in the solutions manual similar to the problem in part A with which he or she might be having difficulty.
7. A chapter summary and a set of review exercises are included at the end of each chapter.
8. A diagnostic test is included at the end of each chapter. Complete solutions for the diagnostic test (with section references) are given in the back of the book.
9. An instructor's manual contains two different tests for each chapter and two final examinations. These may be easily removed and duplicated for class use. The tests include adequate space for students to work the problems. Answer keys and solutions for these tests are provided in the manual.
10. The use of calculators is encouraged and exercises to be solved with a calculator are provided. These are marked with a special symbol in the left margin to indicate that a calculator may be used in the solution. If calculators are not available, appropriate tables, which are included in the back of the book, can be used.
11. The instructor has the option of having students use calculators exclusively, or tables exclusively, or both calculators and tables in doing computational work.
12. Important concepts and algorithms are enclosed in boxes for easy identification and quick location.
13. "Words of Caution" help students avoid common mathematical errors. These are marked with a special symbol in the left margin.
14. Much of the analytic geometry is integrated with the algebra.
15. Symbols, basic formulas, and the basic trigonometric identities are listed inside the cover so as to be immediately available when needed.
16. The text can be easily adapted for use in a variety of instructional programs: conventional lecture courses, learning laboratories, or self-study programs.

I wish to thank Don Dellen of the Dellen Publishing Company for his encouragement and for soliciting critical reviews of the manuscript from Dale R. Bedgood, East Texas State University; Charles M. Biles, Humboldt State University; Boyd L. Cardon, Ricks College; John Distad, University of Alaska; Thomas Drouet, East Los Angeles College; Paul Drowne, Mount Wachusett Community College; Clifton Gary, Oscar Rose College; O. T. Gilbert, Johnson County Community College; Donald W. Gladstone, Orange County Community College; Frederic Gooding, Gallaudet College; Joel Greenstein, New York City Community College; Mark Hale, University of Florida; Nancy J. Halford, Rio Hondo College; George L. Holloway, California State University, Northridge; James Kinney, George C. Wallace State Junior College; Rodney Kohler, Central Oregon Community College; Jeanne Lazaris, East Los Angeles College; Doyle McCown, Oklahoma State University Technical Institute; William Mech, Boise State University; William A. Neal, Fresno City College; Kenneth J. Shabell, Riverside City College; Robert Shloming, Essex County College; Rose Marie Smith, Texas Woman's University; Lynn Tooley, Bellevue Community College; Jack Twitchell, Mesa Community College; Irene Verner, Baruch College, City University of New York; Richard G. Vinson, University of South Alabama; and Jack D. Wilson, Murray State University. I also want to thank Linda Thompson and Phyllis Niklas for their meticulous editing of the final manuscript. To all of these I wish to express my sincere appreciation for their valuable help.

Finally, I owe a great deal to my students who always inspire one to produce a book worthy of their time and to my wife for her encouragement and moral support over long periods of writing.

*Whittier, California*

*C. L. Johnston*

# Contents

<b>Preface</b>	<b>xiii</b>
<b>1 Basic concepts of real numbers</b>	<b>1</b>
1 Sets	2
2 Real numbers	6
3 Inequality symbols	8
4 Absolute value of a number	9
5 Some properties of operation for real numbers	10
6 Integral exponents	13
7 Roots of numbers	16
8 Combining like terms and removing grouping symbols	18
9 Order of operations	22
10 Scientific notation	24
11 Evaluating formulas	30
Summary	32
Review exercises	34
Diagnostic test	36
<b>2 Polynomials</b>	<b>39</b>
1 Definitions	40
2 Addition, subtraction, and multiplication of polynomials	43
3 Quotients of polynomials	47
4 Basic types of factoring	56
5 Factoring by completing the square	61
6 Factoring by using synthetic division	63
Summary	67
Review exercises	67
Diagnostic test	68



<b>3</b>	<b>Fractions</b>	<b>71</b>
1	Definitions	72
2	Multiplication and division of fractions	76
3	Adding and subtracting fractions	78
4	Complex fractions	84
	Summary	90
	Review exercises	91
	Diagnostic test	92
<b>4</b>	<b>Rational exponents, radicals, and complex numbers</b>	<b>95</b>
1	Rational exponents	96
2	Radicals	100
3	Complex numbers	106
4	Algebraic operations on complex numbers	109
	Summary	111
	Review exercises	113
	Diagnostic test	115
<b>5</b>	<b>Equations and inequalities</b>	<b>117</b>
1	Definitions	118
2	Solving elementary equations	119
3	Quadratic equations	126
4	The discriminant of a quadratic equation; forming equations when the roots are known	130
5	Equations in quadratic form	135
6	Applications	137
7	First-degree inequalities with one unknown	141
	Summary	154
	Review exercises	156
	Diagnostic test	158
<b>6</b>	<b>Functions, linear equations and inequalities, and graphs</b>	<b>159</b>
1	The rectangular coordinate system	160
2	Relations and functions	161
3	Equations of lines	167
4	Distance between two points	177
5	Graphs of first-degree inequalities	182
	Summary	190
	Review exercises	192
	Diagnostic test	193

<b>7</b>	<b>Angular measure and the sine function</b>	<b>195</b>
1	Angles and angular measure	196
2	Circular arc length	206
3	Special right triangles	208
4	Finding coordinates of a point	213
5	The sine function	214
6	Sine values of angles	218
7	Using the tables of sine values	221
8	Using the sine function to solve right triangles	224
9	The law of sines	226
10	The ambiguous case	229
11	Trigonometric equations	232
12	Graph of the sine function	235
	Summary	239
	Review exercises	240
	Diagnostic test	241
<b>8</b>	<b>Cosine and tangent functions</b>	<b>243</b>
1	The cosine function	244
2	Complementary angles	245
3	The cosine and sine functions on the unit circle	246
4	A special relationship	248
5	Interpolation	252
6	Accuracy in the solution of triangles	257
7	Application of the sine and cosine function to right triangles	257
8	The law of cosines	258
9	Graph of the cosine function	263
10	Graphing by the addition of ordinates	266
11	Definition of the tangent function	268
12	The sign of the tangent of an angle	269
13	Bearing and angles of elevation and depression	272
14	Graph of the tangent function	276
	Summary	277
	Review exercises	279
	Diagnostic test	281
<b>9</b>	<b>The reciprocal trigonometric functions</b>	<b>283</b>
1	Definitions of the cotangent, secant, and cosecant functions	284
2	Trigonometric functions of complementary angles	286
3	Graph of the cotangent function	290
4	Graph of the cosecant function	292
5	Graph of the secant function	293

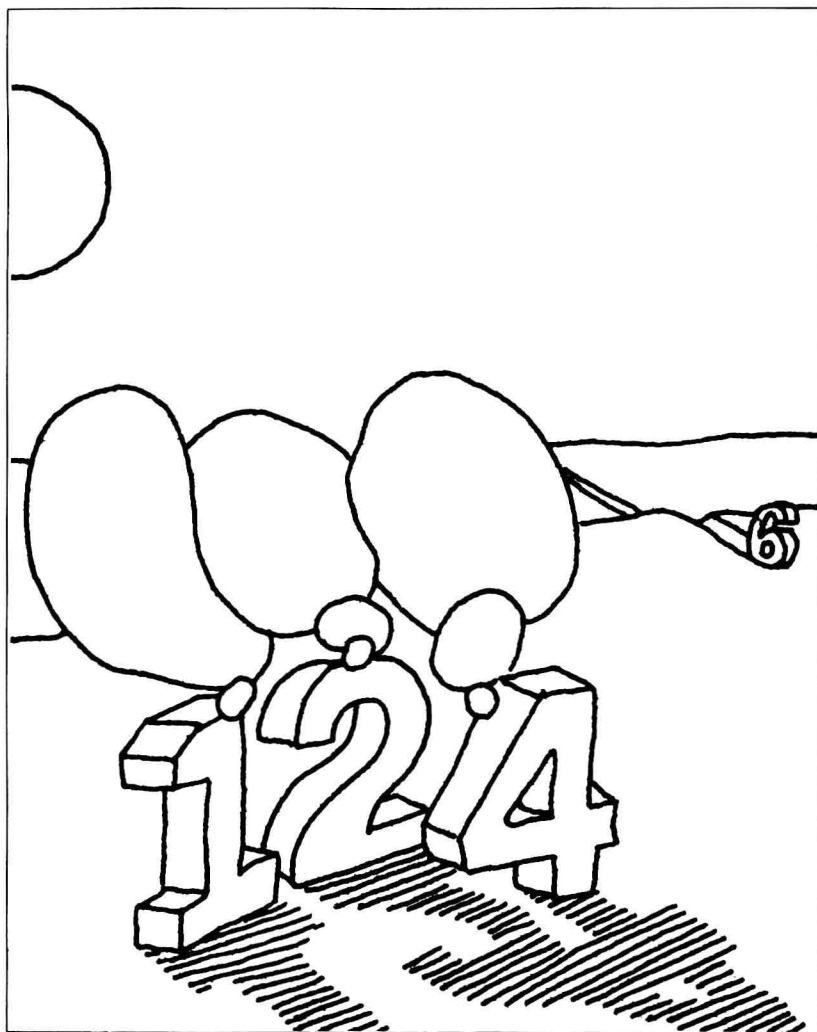
	Summary	294
	Review exercises	295
	Diagnostic test	296
<b>10</b>	<b>Trigonometric identities and conditional equations</b>	<b>297</b>
1	The eight fundamental identities	298
2	Proving trigonometric identities	303
3	Trigonometric equations	306
4	Functions of $-\theta$	309
5	$\cos(A + B)$ and $\cos(A - B)$	311
6	$\sin(A + B)$ and $\sin(A - B)$	316
7	$\tan(A + B)$ and $\tan(A - B)$	316
8	Trigonometric functions of twice an angle	319
9	Trigonometric functions of half an angle	322
10	Products of sines and cosines expressed as sums	326
11	Sums of sines and cosines expressed as products	328
12	Reduction of $a \cos \theta + b \sin \theta$ to $c \cos(\theta - \alpha)$	331
	Summary	335
	Review exercises	337
	Diagnostic test	339
<b>11</b>	<b>General methods of graphing trigonometric functions</b>	<b>341</b>
1	Analysis of the general sine and cosine curves	342
2	Analysis of the general tangent and cotangent curves	348
3	Analysis of the general secant and cosecant curves	352
4	Graphing by the addition of ordinates	353
	Summary	354
	Review exercises	356
	Diagnostic test	357
<b>12</b>	<b>Exponential and logarithmic functions</b>	<b>359</b>
1	Exponential functions	360
2	Laws of exponents	362
3	Definition of a logarithm	363
4	Fundamental laws of logarithms	365
5	Systems of logarithms	369
6	Logarithms to the base 10	369
7	Rule for the characteristic of a common logarithm	371
8	Interpolation	374
9	Computations with logarithms	376
10	Graphs of logarithmic functions	379

11	Exponential and logarithmic equations and inequalities	381
	Summary	384
	Review exercises	385
	Diagnostic test	386
<b>13</b>	<b>Area of triangles, sectors, and segments</b>	<b>389</b>
1	Logarithms of trigonometric functions	390
2	Oblique triangles	392
3	Area of a triangle	393
4	Areas of sectors and segments of circles	397
	Summary	400
	Review exercises	402
	Diagnostic test	402
<b>14</b>	<b>Inverse trigonometric functions</b>	<b>403</b>
1	Inverse trigonometric relations	404
2	Graphs of the inverse trigonometric relations	408
3	Principal values of the inverse trigonometric functions	410
	Summary	419
	Review exercises	420
	Diagnostic test	421
<b>15</b>	<b>Polar coordinates and complex numbers</b>	<b>423</b>
1	Graphical representation of complex numbers	424
2	Polar coordinates	425
3	Polar or trigonometric forms of a complex number	428
4	Multiplication and division of complex numbers in polar form	432
5	Powers of complex numbers; De Moivre's theorem	434
6	Roots of complex numbers	438
	Summary	442
	Review exercises	443
	Diagnostic test	444
<b>16</b>	<b>Analytic geometry</b>	<b>447</b>
1	Introduction	448
2	Inclination and slope of a straight line	449
3	Symmetry	456
4	Conic sections	458
5	The circle	460
6	The parabola	465
7	The ellipse	471

8	The hyperbola	478
9	Translation of axes	485
10	Standard and general forms of the equations of conic sections	489
	Summary	494
	Review exercises	496
	Diagnostic test	497
<b>17</b>	<b>Systems of linear and quadratic equations</b>	<b>499</b>
1	Definitions	500
2	Graphical method for solving a linear system	500
3	Substitution method for solving a linear system	504
4	Addition-subtraction method for solving a linear system	505
5	Systems of linear equations in three unknowns	509
6	Solving linear systems by matrices	513
7	Determinants	519
8	Cramer's rule	525
9	Homogeneous linear equations	530
10	Systems of second-degree equations in two variables	533
	Summary	542
	Review exercises	544
	Diagnostic test	546
<b>18</b>	<b>Partial fractions</b>	<b>547</b>
1	Partial fractions	548
2	Denominator containing nonrepeating linear and quadratic factors	550
3	Denominator containing repeating linear factors	552
4	Denominator containing quadratic and linear factors, some repeating	554
	Summary	555
	Review exercises	556
	Diagnostic test	556
<b>19</b>	<b>The binomial theorem and mathematical induction</b>	<b>559</b>
1	Factorial notation	560
2	The binomial theorem for positive integral exponents	562
3	The general term of the binomial expansion	568
4	Mathematical induction	570
5	Proof of the binomial theorem	574
	Summary	575
	Review exercises	576
	Diagnostic test	577

<b>20</b>	<b>Topics from the theory of equations</b>	<b>579</b>
1	Polynomial equations	580
2	The remainder theorem	581
3	The factor theorem	584
4	Number and nature of the roots of a polynomial equation	586
5	Formation of an equation with given roots	587
6	Rational roots of an equation	590
7	The graph of a polynomial	600
	Summary	602
	Review exercises	604
	Diagnostic test	605
<b>21</b>	<b>Sequences and series</b>	<b>607</b>
1	Basic definitions	608
2	The summation symbol	610
3	Arithmetic progressions	612
4	Geometric progressions	616
5	Infinite geometric series	622
	Summary	626
	Review exercises	627
	Diagnostic test	628
	<b>Appendix</b>	<b>631</b>
Table 1	Squares and square roots	632
Table 2	Common logarithms	633
Table 3	Four-place trigonometric functions	635
Table 4	Logarithms of trigonometric functions	642
Table 5	Trigonometric functions—radian and degree measures	649
Table 6	Degrees and minutes in radians	650
	<b>Answers</b>	<b>651</b>
	<b>Index</b>	<b>711</b>

# 1 Basic concepts of real numbers



It is our purpose in this course to prepare you for a successful study of calculus. We assume that you have had the equivalent of two years of high school algebra and a course in plane geometry. However, for various reasons, you may find a brief review of some of your earlier work in mathematics helpful. For that reason much of the work of the early chapters of this book is devoted to a review of your former mathematics.

A mathematical system usually begins with some undefined words or terms, some defined words or terms, a set of elements, one or more relations between the elements, one or more operations defined on that set, and some statements we accept as true without proof (**axioms**). Using these tools, we can (usually) prove many other statements (**theorems**), and thus build a mathematical structure.

In this chapter we give a brief review of some information on sets that we shall be using in this course and some of the properties of the real number system. In algebra we work with letters that represent numbers and must therefore deal with the letters according to the laws of numbers.

## 1 Sets

**A set is a collection of particular things.** For example, we speak of the set of numbers between 2 and 7, the set of points on a line segment, or within a circle, and so on. In this course we shall be concerned only with sets of numbers. The objects that make up the set are called its **elements** (or **members**). Sets are often represented by capital letters or by listing their elements within braces  $\{ \}$ . For example,  $A = \{1, 2, 3\}$ . If the braces have nothing within them, the set is called the **empty set** (or **null set**). An example of an empty set would be the set of all people in your class who are 10 feet tall. The empty set is represented by the symbol  $\{ \}$  or preferably,  $\emptyset$ .



**A WORD OF CAUTION** The set  $\{0\}$  is not an empty set because it contains the element “0.” Also, the set  $\{\emptyset\}$  is not an empty set because it contains the symbol  $\emptyset$ .

## ROSTER METHOD OF REPRESENTING A SET

A class roster is a list of the members of the class. The roster of a set of numbers is a list of the numbers in the set. If, for example, the numbers in a set are 1, 3, 5, and 7, the **roster method** of representing these numbers is  $\{1, 3, 5, 7\}$ .

The expression  $B = \{1, 3, 5\}$  is read “ $B$  is the set whose elements are 1, 3, and 5.” The symbol  $\in$  is read “is an element of.” It is used in set notation to denote that an element is a member of a particular set. For example, in the set  $D = \{5, 8, 9\}$  the symbol  $5 \in D$  tells us that “5 is an element of  $D$ .” In like manner the symbols  $8 \in D$  and  $9 \in D$  tell us that 8 and 9 are elements of  $D$ .

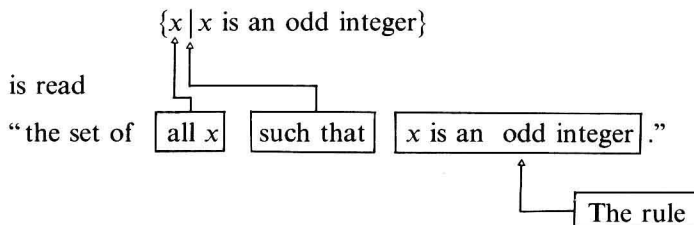
The slash line ( $/$ ) negates most symbols. The symbol  $\notin$  is read “is not



an element of.” For example, if  $A = \{8, 9\}$ , then  $4 \notin A$ . This is read “4 is not an element of  $A$ .”

### SET-BUILDER METHOD OF REPRESENTING A SET

A set can be represented by giving a property of its members. For example, the expression



**EXAMPLE 1** Write  $\{x \mid x \text{ is an even number between 1 and 9}\}$  in roster notation.

**SOLUTION**  $\{x \mid x \text{ is an even number between 1 and 9}\} = \{2, 4, 6, 8\}$ .

**EXAMPLE 2** Write  $\{3, 5, 7\}$  in set-builder notation.

**SOLUTION**  $\{3, 5, 7\} = \{x \mid x \text{ is an odd number between 2 and 8}\}$ .

### SUBSETS

Set  $A$  is a **subset** of set  $B$  if every element of set  $A$  is contained in set  $B$ . Thus, if  $A = \{1, 2, 3\}$  and  $B = \{1, 2, 3, 4\}$ , then  $A$  is a subset of  $B$ . This is written “ $A \subseteq B$ .” Every set is a subset of itself. The null set,  $\emptyset$ , is, by definition, a subset of every set.

**EXAMPLE 3** The subsets of the set  $\{3, 4, 6\}$  are  $\{3\}$ ,  $\{4\}$ ,  $\{6\}$ ,  $\{3, 4\}$ ,  $\{3, 6\}$ ,  $\{4, 6\}$ ,  $\{3, 4, 6\}$ , and  $\emptyset$ .

### EQUAL SETS

Two sets  $A$  and  $B$  are said to be equal, written  $A = B$ , if and only if  $A \subseteq B$  and  $B \subseteq A$ ; that is, if they contain the same elements. The order in which the elements are written and the number of times a particular element is repeated within a given set does not matter.

**EXAMPLE 4** Examples of equal sets.

- a.  $\{1, 2, 3\} = \{2, 1, 3\} = \{3, 1, 2\} = \{1, 2, 2, 3, 2\}$
- b.  $\{a, c, e\} = \{a, e, c, e, a\}$

### UNION AND INTERSECTION OF SETS

Sets are often represented by closed geometric figures, where it is understood that all points within each respective figure represent the elements of a set. For example, in Figure 1 we show that sets  $A$  and  $B$  are subsets of  $U$ .

In Figure 2, the interiors of circles  $A$  and  $B$  represent different sets.