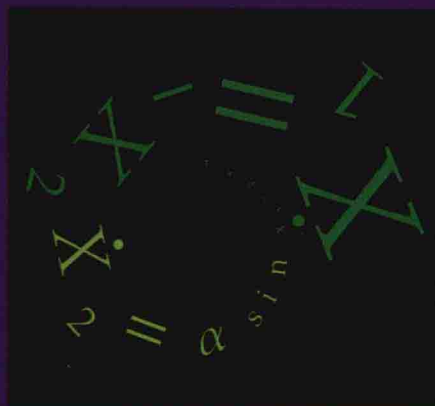


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# A Practical Approach to Dynamical Systems for Engineers

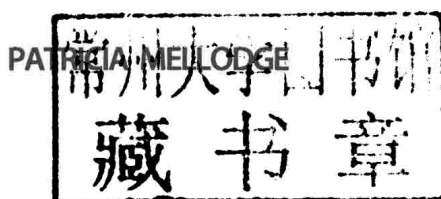
Patricia Mellodge



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# A Practical Approach to Dynamical Systems for Engineers



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## ABOUT THE AUTHOR

Patricia Mellodge is an associate professor of electrical and computer engineering at the University of Hartford. She received a BS in electrical engineering from the University of Rhode Island. Her graduate work was completed at Virginia Tech where she received an MS in mathematics and an MS and PhD in electrical engineering.

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# PREFACE

Dynamical systems are an important topic in engineering. Applications are prevalent within mechanical, electrical, and biomedical engineering and can be found within robotic, automotive, aerospace, and human systems, among others. The purpose of this book is to present dynamical systems topics in a way that is relevant for practicing engineers. It is intended for engineers who need to understand both the background theory *and* how to apply it. As such, there is an attempt to create a bridge between the theory and the application. Every abstract concept is discussed in depth, described in a readable and down-to-earth manner, and illustrated using practical examples. The intended audience is engineers who are working in industry, graduate students who are taking courses or doing research related to dynamical systems, and undergraduate students who are taking courses in control systems. This is not a textbook, and there are no end-of-chapter problems. Rather, it should be considered an application guide for those in the trenches of working with and learning about dynamical systems.

It is assumed that readers have a solid mathematical foundation in calculus, differential equations, and matrix theory. In presenting the material, the emphasis is on applying the theory, so there are relatively few theorems and no proofs. However, there is a *lot* of mathematics. Much mathematical detail is given that is missing from other texts on these topics. The reason for this level of detail is to help readers understand the complete application in real-world systems. The focus is on depth and not breadth. In covering the selected topics at this level of detail, unfortunately, the number of topics had to be limited. As such, this is not a complete and comprehensive presentation of all topics in dynamical systems. However, this book attempts to cover many relevant topics that an engineer in the field would encounter and provide a foundational understanding for further study.

It is also assumed that the reader has some understanding of MATLAB and Simulink, although expertise is not required. Many of the concepts are demonstrated using real-world examples in MATLAB or Simulink. For the earlier chapters, the MATLAB code is explained line by line to show how various concepts are implemented. These explanations are gradually decreased throughout the book.

The book transitions from topics commonly found at the undergraduate level in engineering, to those covered in graduate courses, to those that engineers

may never see in a course. As such, the coverage changes in its approach and assumptions about what the reader knows. The layout of the topics is as follows. Chapter 1 introduces dynamical systems, provides motivation for why it's important to study them, and discusses different types of systems. This material should be familiar from undergraduate engineering courses in linear systems and control theory. There is high-level discussion of this material, but the mathematics starts early with definitions of the different classes of systems.

Chapter 2 discusses modeling and covers differential and difference equations, transfer functions, state-space models, eigenvalues, eigenvectors, and singular value decomposition. Although many of these topics are familiar from courses in differential equations, control systems, and linear algebra, the emphasis is on putting them in the context of dynamical systems. There is also an emphasis on working through examples in MATLAB and giving details of the implementation, which may not be covered in those courses.

Chapter 3 focuses on solutions of dynamical equations, equilibrium points, and stability. These concepts are often encountered in introductory graduate-level courses in dynamical systems and control theory. Again, the emphasis is not on deriving the results but applying them. As such, several of the relevant theorems are presented and applied.

Chapter 4 discusses nonlinear systems and some rich behavior that is only found in them such as limit cycles, bifurcations, chaos, and linearization. These are topics typically found in graduate-level engineering courses. There is a minimal amount of theoretical coverage, and the behaviors are described through examples.

Finally, Chapter 5 introduces Hamiltonian systems, which typically fall in the realm of physicists. However, undamped vibrational systems and their equivalent are an important class of Hamiltonian systems, and there is much rich theory in this area. As with Chapter 4, there is minimal theoretical coverage, and the focus is placed more on the introduction of the concepts through examples.

Many people contributed to the creation of the book. Acknowledgment goes out to colleagues and students at the University of Hartford, particularly Lee Townsend and Iman Salehi for their supportive ideas and engaging discussion; colleagues in the van Rooy Center for Complexity and Conflict Analysis for the productive biweekly meetings; Harriet Clayton and Glyn Jones at Elsevier for the support and feedback; and Joe Romagnano for his editorial skills.

**Patricia Mellodge**  
West Hartford, CT

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