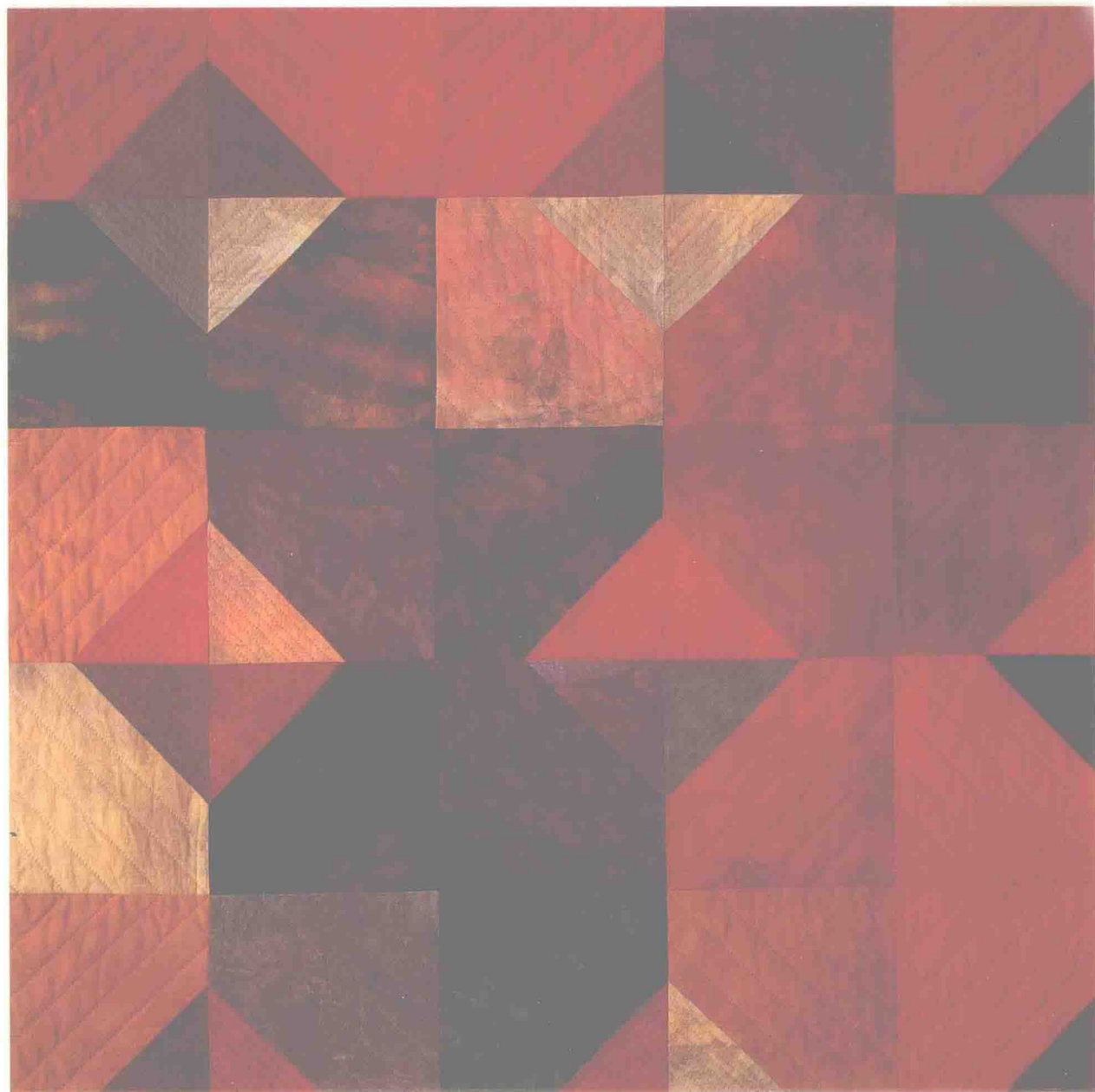


Elementary

A Graphing Approach

Algebra



HUBBARD ■ ROBINSON



Elementary Algebra

A Graphing Approach

Elaine Hubbard
Ronald D. Robinson

Kennesaw State College

D. C. HEATH AND COMPANY

Lexington, Massachusetts Toronto

Address editorial correspondence to:

D. C. Heath and Company
125 Spring Street
Lexington, MA 02173

Acquisitions: Charles Hartford

Development: Kathleen Sessa-Federico

Editorial Production: Melissa Ray, Craig Mertens

Design: Cornelia Boynton

Art Editing: Gary Crespo

Production Coordination: Richard Tonachel

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Preface

The Approach

Every learning theory emphasizes the value of widening the sensory spectrum. To accomplish this in teaching mathematics, we believe one must give students the opportunity to progress beyond mere symbol manipulation by providing the visual connection that allows students to “see” mathematics in context.

In *Elementary Algebra: A Graphing Approach*, we use our extensive teaching experience to create a balance between traditional algebra and the use of the graphing calculator. We have developed an approach to teaching with a calculator that works successfully for us, for our colleagues, and for students. In fact, we have not hesitated to include ideas that are based on our students’ observations.

We have learned how to use the graphing calculator to provide an efficient and manageable way for students to begin at a visual, concrete level before moving to the more formal, abstract level of mathematics. Our students are surprised and pleased with their ability to control their own environment and to make things happen. We have found that collaboration evolves naturally.

Students can create a visual illustration of a concept, see relationships, and estimate outcomes. By asking “What if?” questions, they can experiment, explore, and discover. In short, the graphing calculator allows our students to be active participants in the learning process.

Our classroom teaching experience has been that a graphing calculator, far from replacing traditional mathematics, actually motivates it. Students benefit from the graphics overview that they can create for themselves, but they invariably come to understand the need for the refinements and precision that formal algebraic methods provide. The power in this pedagogical approach lies in the student’s eventual motivation to formulate definitions and rules and to develop methods for accomplishing mathematical tasks.

These positive outcomes are nurtured by the protocols that lie at the heart of this textbook. Whenever possible, we *explore*, *estimate*, and *discover* graphically; we *verify*, *generalize*, and *determine* algebraically. This process preserves mathematics as the authority.

In this book we strive to maintain a proper perspective. The focus is on mathematics, with the graphing calculator serving as a tool for better understanding. We remind students that a calculator is designed to execute the rules of mathematics, not the other way around. And we encourage students to view the calculator as a means to an end rather than an end itself.

This text is written for students at the beginning or elementary level of algebra. Even in Chapter 1, we use a calculator to explore the rules for performing operations with signed numbers. The graphing calculator begins to play a prominent role with the introduction of the coordinate plane in Chapter 2. In all of the succeeding chapters, a calculator-based approach is used in the exposition of all topics appropriate to the elementary level of study.

In the following pages, the special features of the text are highlighted and discussed in detail.

Key Features

Chapter Opener

Each chapter begins with a short introduction to a real-data application that is covered later in the chapter as well as an accompanying graph of the data to motivate students to want to learn more about the related mathematical ideas. This introduction also includes a helpful overview of the topics that will be covered in the chapter and a list of the section titles.

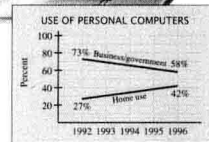
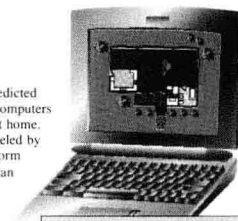
5 Systems of Linear Equations

- 5.1 The Graphing Method
- 5.2 The Addition Method
- 5.3 The Substitution Method
- 5.4 Applications
- 5.5 Systems of Linear Inequalities

The accompanying graph shows a predicted comparison between the percentages of personal computers used in business and government and those used at home. The data in each category can be modeled by a linear equation, and the two equations together form a **system of equations**. By solving the system we can

the percentages for the same. (For more on , see Exercises 71–74.)

chapter is devoted to the topic equations in two variables, and algebraic methods for use these skills in solving tion problems. In the final the solutions of systems



(Source: Channel Marketing Group.)

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SECTION 2.2 The Coordinate Plane 87

2.2 The Coordinate Plane

Familiar Graphs • Graphs in Mathematics • Coordinate Systems on a Calculator

Familiar Graphs

Bar graphs and line graphs appear frequently in the news media. The bar graph in Fig. 2.2 and the line graph in Fig. 2.3 are visual ways of showing the relationship between a person's weight and the number of calories that are burned when a person walks at a normal pace for 30 minutes. In both graphs, weights are given along the horizontal axis, and calories are reported along the vertical axis.

Figure 2.2

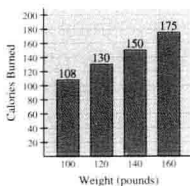
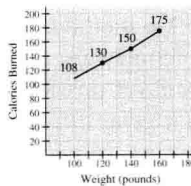


Figure 2.3



We can begin at either axis and read to the other axis. The following are examples of information that can be obtained from either graph.

- Beginning at 120 pounds on the horizontal axis, we move up to the top of the bar (or to the line in the line graph) and then left to the vertical axis. The corresponding number is 130 calories.
- Beginning at 175 calories on the vertical axis, we move right to the top of the bar (or to the line in the line graph) and then down to the horizontal axis. The corresponding number is 160 pounds.

We can use the data in Figs. 2.2 and 2.3 to create the following table, which shows each pairing of weight and calories consumed. In the third column, we show the pairings in the form (weight, calories).

Weight	Calories Consumed	(Weight, Calories)
100	108	(100, 108)
120	130	(120, 130)
140	150	(140, 150)
160	175	(160, 175)

Section Opener

Each section begins with a list of subsection titles that provides a brief outline of the material that follows.

Exploration / Discovery Examples

Whenever possible, we introduce topics via Exploration/Discovery examples. These titled examples help students to discover the rules and properties of algebra themselves. Usually these examples encourage students to experiment with their graphing calculators and answer “What if?” questions about the relationships between equations, graphs, and specific data points. Asking guiding questions in the Exploration and then showing how one looks for patterns and makes generalizations in the Discovery helps students obtain a deeper understanding of the mathematical relationships as well as develop problem-solving skills, key goals of the NCTM Standards.

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CHAPTER 2 Algebra Basics, Equations, and Inequalities

$$\begin{array}{lll}
 \text{(b)} & 2 < 7 & -1 > -2 \\
 & -2(2) > -2(7) & -2(-1) < -2(-2) \\
 & -4 > -14 & 2 < 4 \\
 & -6 < 3 & 8 > -2 \\
 & -2(-6) > -2(3) & -2(8) < -2(-2) \\
 & 12 > -6 & -16 < 4
 \end{array}$$

The direction of the inequality symbol in the original inequality.

The results of Exploration 1 suggest the following property of inequalities.

Multiplication Property of Inequalities

For real numbers a , b , and c ,

- if $c > 0$, then the inequalities $a < b$ and $ac < bc$ are equivalent.
- if $c < 0$, then the inequalities $a < b$ and $ac > bc$ are equivalent.

A similar result holds for $>$, \leq , and \geq .

Because division is defined in terms of multiplication, the same rules hold for division.

In words, we can multiply or divide both sides of an inequality by a positive number, and the direction of the inequality symbol is not changed. If we multiply or divide both sides by a negative number, then the direction of the inequality symbol is reversed.

EXAMPLE 2 Using the Multiplication Property of Inequalities

Solve each inequality and graph the solution set.

$$\text{(a)} -6x \geq 24 \qquad \text{(b)} 3x < -15 \qquad \text{(c)} -\frac{1}{3}x \leq -2$$

Solution

$$\begin{array}{l}
 \text{(a)} -6x \geq 24 \\
 \frac{-6x}{-6} \geq \frac{24}{-6} \\
 x \leq -4
 \end{array}$$

Reverse the direction of the inequality symbol.

$$\begin{array}{l}
 \text{(b)} 3x < -15 \\
 \frac{3x}{3} < \frac{-15}{3} \\
 x < -5
 \end{array}$$

The direction of the inequality symbol is unchanged.



SECTION 2.8 Inequalities: Algebraic Methods

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NOTE: As with equations, we can exchange the left and right sides of inequalities. However, we must also take care to retain the inequality. In part (b) of Example 1, exchanging the sides of $-1 \leq y$ also changes the inequality symbol to \geq .

Multiplication Property of Inequalities

We can multiply or divide both sides of an equation by any nonzero number. Can we do the same with inequalities? As Exploration 1 shows, the answer is a qualified yes.

EXPLORATION 1

Multiplying Both Sides of an Inequality

Consider the following four true inequalities.

$$2 < 7 \qquad -1 > -7 \qquad -6 < 3 \qquad 8 > -2$$

- (a) In the following we multiply both sides of each inequality by 4. Insert an inequality symbol in each blank to make the inequality true.

$$\begin{array}{ll}
 4(2) \underline{\hspace{1cm}} 4(7) & 4(-1) \underline{\hspace{1cm}} 4(-7) \\
 4(-6) \underline{\hspace{1cm}} 4(3) & 4(8) \underline{\hspace{1cm}} 4(-2)
 \end{array}$$

Compare the direction of the inequality symbol in the original inequality and in the result.

- (b) In the following, we multiply both sides of each inequality by -2 . Insert an inequality symbol in each blank to make the inequality true.

$$\begin{array}{ll}
 -2(2) \underline{\hspace{1cm}} -2(7) & -2(-1) \underline{\hspace{1cm}} -2(-7) \\
 -2(-6) \underline{\hspace{1cm}} -2(3) & -2(8) \underline{\hspace{1cm}} -2(-2)
 \end{array}$$

Compare the direction of the inequality symbol in the original inequality and in the result.

Discovery

$$\begin{array}{ll}
 \text{(a)} & 2 < 7 \qquad -1 > -7 \\
 & 4(2) < 4(7) \qquad 4(-1) > 4(-7) \\
 & 8 < 28 \qquad -4 > -28 \\
 & -6 < 3 \qquad 8 > -2 \\
 & 4(-6) < 4(3) \qquad 4(8) > 4(-2) \\
 & -24 < 12 \qquad 32 > -8
 \end{array}$$

The direction of the inequality symbol in the result is the same as in the original inequality.

Standard Examples

All sections contain numerous standard, titled examples, many with multiple parts graded by difficulty, that serve to reinforce the concepts and techniques introduced in the Exploration/Discovery examples. Helpful comments appear to the right of the detailed solution steps.

Key Word Icons



Whenever we introduce a mathematical technique that can be performed on a graphing calculator, we also include a graphing calculator-related word, usually in the margin. Each word references a discussion in the accompanying *Graphing Calculator Keystroke Guide*, where specific keystroke information for several popular calculator models can be found. Students need only look up the key word in the guide to obtain the location of the appropriate keystroke discussion. We repeat key word icons whenever we feel that students would benefit from a review of that particular calculator technique.

2.3 Evaluating Expressions Graphically

Evaluating Expressions Repeatedly • Evaluating Expressions with Graphs

Evaluating Expressions Repeatedly

In Section 2.1 we considered one method for using a calculator to evaluate an algebraic expression. On the home screen, we store the value of the variable and then enter the expression. The calculator reports the value of the expression.

When an expression must be evaluated repeatedly, a more efficient method is to enter the expression on the Y screen rather than on the home screen. Then, each time we store a value for the variable, we can obtain the corresponding value of the expression.



EXAMPLE 1

Evaluating an Expression on the Y Screen

Evaluate $x + 4$ for $x = -6, -4, 0, 2$, and 5 .

Solution Begin by entering the expression $x + 4$ as Y_1 on the Y screen (see Fig. 2.9). In effect, we are letting $y_1 = x + 4$.

Figure 2.9

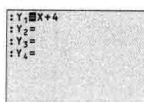
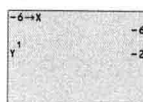


Figure 2.10



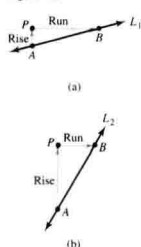
Now return to the home screen, store the first value of x , and retrieve Y_1 , which is the value of $x + 4$ (see Fig. 2.10). Repeat this process for the remaining of x . The following table summarizes the results.

	-6	-4	0	2	5
$x + 4$	-2	0	4	6	9

Evaluating Expressions with Graphs

We use a graph to visualize the relationship between the values of a variable and corresponding values of an expression containing that variable. For the expression $x + 4$ in Example 1, we can write ordered pairs in which the first coordinate corresponds to the value of the variable x and the second coordinate

Figure 4.6



Exploration 1 suggests the following general statements regarding the graph of $y = mx + b$.

1. The sign of m determines whether the line rises or falls from left to right.
2. The absolute value of m determines the steepness of the line.

The word *steepness* is too vague for our purposes. To be more precise, we need a numerical measure of a line's steepness. We call this numerical measure the **slope** of the line.

Consider the lines L_1 and L_2 in Fig. 4.6. Intuitively, we say that L_2 is steeper than L_1 . Now suppose that we wished to move from point A to point B . One path is from A to P (a change called the **rise**) and then from P to B (a change called the **run**). For L_1 , the rise is considerably less than the run, whereas for L_2 , the rise is considerably greater than the run. This observation suggests that the slope of a line can be determined by comparing the rise of the line to the run. We can use a ratio to make such comparisons.

Definition of the Slope of a Line

Given a line and two points A and B of the line, the **rise** of the line is the vertical change from A to B , and the **run** of the line is the horizontal change from A to B . The **slope** of the line is the ratio of the rise to the run.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

NOTE: The triangle determined by the points A , P , and B in Fig. 4.6 is called the **slope triangle**.

The definition of slope does not indicate what two points to use for determining the rise and the run. Exploration 2 clarifies this matter.

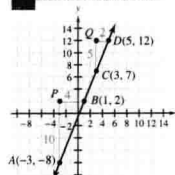


EXPLORATION 2

Using Points of a Line to Determine the Slope

The figure shows a line with certain points highlighted.

- (a) Determine the slope of the line by using points A and B .
- (b) Determine the slope of the line by using points C and D .
- (c) What do your results suggest about the points to use for determining slope?



- (a) Using the slope triangle APB , we see that the rise (AP) is 10 units and the run (PB) is 4 units.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{10}{4} = \frac{5}{2}$$

Graphs

Numerous graphs throughout the exposition and exercises help students develop important visualization skills. Graphing calculator displays are proportioned like actual calculator screens to resemble what students obtain on their own calculators. Traditional coordinate plane graphs are also included where appropriate.

All curves have been computer-generated for accuracy. In addition, the axes and the curves appear in the same color, eliminating inaccuracies caused by color registration problems during printing.

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CHAPTER 3 Applications

NOTE: Although a formula is simply an set of instructions for calculating some formula $I = Prt$ instructs us to calculate

Many formulas come from geometry. Sober π , which is the ratio of the circumference number π is an irrational (nonterminating) approximately equal to 3.1416. Most calculators

EXAMPLE 2 Using Geometry Formulas

The circumference C of a circle with diameter d is given by the formula $C = \pi d$. The area A of a circle is given by $A = \pi r^2$, where r is the radius. What are the circumference and the area of a circle with a radius of 3 feet?

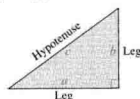
Solution The diameter of a circle is twice its radius. Because the radius is 3 feet, the diameter is 6 feet. The display shows how we store the values for d and r and evaluate the formulas for the circumference and the area. The circumference is approximately 18.85 feet, and the area is approximately 28.27 feet.

6 → D	6.00
πD	18.85
3 → R	3.00
πR^2	28.27

Another important formula from geometry is contained in the **Pythagorean Theorem**.

The Pythagorean Theorem

A triangle is a right triangle if and only if the sum of the squares of the legs is equal to the square of the hypotenuse. Symbolically, $a^2 + b^2 = c^2$.



NOTE: The legs of a right triangle are **perpendicular**, which means that they form a right angle. The hypotenuse is the longest side of a right triangle and lies directly across from the right angle.

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CHAPTER 4 Properties of Lines

EXAMPLE 2 Graphing Linear Inequalities

Graph the solution set of $x + 2y < 10$.

Solution

$$x + 2y < 10$$

$$2y < -x + 10$$

$$y < -\frac{1}{2}x + 5$$

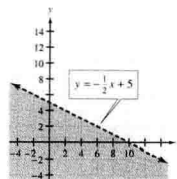
Solve the inequality for y .

Subtract x from both sides.

Divide both sides by 2.

Draw the boundary line $y = -\frac{1}{2}x + 5$. The line is dashed because the inequality does not include the equal sign. Because the inequality is in the form $y < mx + b$, we shade the region below the boundary line.

Figure 4.23



The point $(3, 7)$ lies above the boundary line. We can test this point to determine whether it represents a solution.

$$x + 2y < 10$$

$$3 + 2 \cdot 7 < 10$$

$$3 + 14 < 10$$

$$17 < 10$$

The original inequality

Replace x with 3 and y with 7.

False

Because $(3, 7)$ does not satisfy the inequality, the half-plane above the line should be shaded. This result confirms the graph in Figure 4.23.

Graphs of linear inequalities can also be produced on a calculator. Figure 4.24 shows a typical display of the graph in Example 2.

Figure 4.24



NOTE: Calculator graphs of linear inequalities do not distinguish between solid and dashed boundary lines.

Screen Displays of Calculations

In selected locations, especially in the early chapters when students might need guidance on how to use their calculators, we have included screen displays of calculations from a graphing calculator. These show students how a calculation will appear on their screens if it has been entered correctly.

Exercises

This text contains over 7200 exercises. At the beginning of the problem sets, exercises are usually graded and paired and are intended to follow up on the examples in the text. Later in the set, we provide mixed exercises for the synthesis of concepts and methods. The answers to the odd-numbered section exercises are included at the back of the text.

Writing Exercises

Most exercise sets begin with a pair of simple writing exercises to help students gain confidence in their ability to write about mathematics. Additional writing exercises are scattered throughout the sets and provide a way for students to develop a more complete understanding of the important ideas.

Special Cases

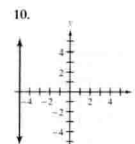
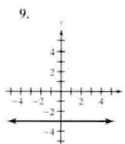
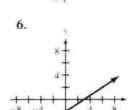
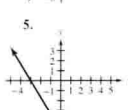
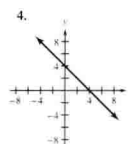
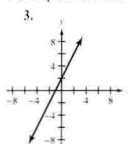
- If c is a constant, then
 1. the graph of $y = c$ is a horizontal line whose slope is 0.
 2. the graph of $x = c$ is a vertical line whose slope is undefined.

4.3 Exercises

1. Consider a very steep, but not vertical cliff. From the bottom to the top of the cliff, how would you compare the rise with the run?

2. Describe a line that has a
- (a) positive slope.
 - (b) negative slope.

In Exercises 3–10, refer to the graph to determine the slope of the line.



11. Suppose that you know the coordinates of four points of a line. Which points can be used to calculate the slope of the line with the Slope Formula?

12. Suppose that you use $x_2 - x_1$ to represent the run from $A(x_1, y_1)$ to $B(x_2, y_2)$. What expression must you use to represent the rise? Why?

In Exercises 13–36, determine the slope of the line that contains the given points.

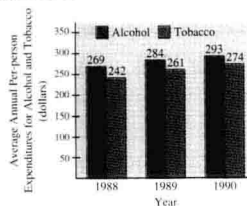
13. (1, 3), (4, 9)
14. (-2, 1), (2, 13)
15. (3, 3), (5, 5)
16. (0, 0), (6, -6)
17. (-2, -4), (0, -6)
18. (-5, 4), (1, 0)
19. (-1, 5), (3, 5)
20. (2, -4), (-1, -4)
21. (-6, 0), (-6, 2)
22. (1, -7), (1, 3)
23. (-5, -3), (7, -1)
24. (-3, 5), (6, -7)
25. (-8, -2), (-4, -6)
26. (-7, -4), (-3, -1)
27. (6, -2), (1, -3)
28. (-2, 3), (-6, 4)
29. (2, -5), (17, -11)
30. (0, 7), (-9, 0)
31. (-3, 7), (-7, -3)
32. (-5, 8), (10, 2)

Exploring with Real Data

In Exercises 77–80, refer to the following background information and data.

The bar graph shows the average annual per-person expenditures for alcoholic beverages and tobacco products during the period 1988–1990. (Source: U.S. Bureau of Labor Statistics.)

Figure for 77–80



If we let x represent the number of years since 1988, then the average annual per-person expenditures can be modeled by the following equations.

$$A = 12x + 270 \quad \text{Alcohol}$$

$$T = 16x + 243 \quad \text{Tobacco}$$

77. Together the two equations form a system. Without graphing or solving, how can you tell that the system has a unique solution?

78. Estimate the solution of the system by graphing.
79. What is your interpretation of the solution of the system?
80. Does the greater slope for the tobacco products equation imply that the number of smokers is increasing faster than the number of drinkers? Explain.

Challenge

In Exercises 81–84, use the graphing method to estimate the solution of the given system of three equations in two variables.

81. $y = x - 12$
82. $x + y = 3$
- $2y + x = 0$
- $y = \frac{2}{5}x + 10$
- $2y = 3x - 32$
- $-3x + 2y = 31$
83. $y = 9 - x$
84. $x - y = -7$
- $3y + 3x = -21$
- $x + y = 7$
- $2x = 18 - 2y$
- $2y = x - 10$

85. Determine the vertices of a triangle formed by the graphs of $y = \frac{1}{2}x + 13$, $2y + 3x = 0$, and $x = 12$. Is it a right triangle?

86. Determine the vertices of a quadrilateral formed by the graphs of the equations $y = x + 6$, $y = x - 8$, $y = 7$, and $y = -4$. Is it a parallelogram?

Exploring with Real Data

Most sections contain real-data problems with source acknowledgments that show the relevancy of mathematics to a wide variety of subject areas. Typically, the student is given a mathematical model of the data and is asked to calculate, predict, and interpret. A unique feature is the frequent use of a concluding discussion question of an interdisciplinary nature. An index of the real-data exercises is on the inside front cover.

Other Applications

There are numerous other interesting real-world applications, with some sections devoted entirely to them. A complete list of applications, organized by subject matter, appears in the Index of Applications.

Concept Extension Exercises

* Groups of problems that are not specifically illustrated by examples are highlighted in the Instructor's Annotated Edition. (For example, see problems 61–64 on page 235.) These Concept Extension exercises offer slight deviations from the kinds of examples presented in the exposition, and thus require students to extend the ideas and techniques they have learned to solve them.

$$49. x + 5 = -2 \quad 50. y + 3 = 0$$

$$51. 6 + y = 4 \quad 52. 7 = x - 1$$

This group helps students recognize special cases.

* In Exercises 53–60, determine whether the graph of the equation is horizontal, vertical, or neither.

$$53. 3x - 12 = 0 \quad 54. 3y = 0$$

$$55. y = 5x \quad 56. x = 2y$$

$$57. -3y = 6 \quad 58. x + 4 = 1$$

$$59. y = 7 - 0x \quad 60. 2x - y = 0$$

* In Exercises 61–64, write two ordered pairs whose coordinates satisfy the given conditions. Then translate the conditions into an equation. If possible, produce the graph of the equation and trace it to verify that your two ordered pairs are represented by points of the graph.

61. The x -coordinate is always 7.

62. The y -coordinate is always 2.

63. The difference between twice the y -coordinate and 6 is always 8.

64. Three more than the x -coordinate is always 12.

In Exercises 65–68, write the equation of the given graph.

65.  66. 

SECTION 4.2 Intercepts and Special Cases

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* In Exercises 69–72, the graph of a linear equation in two variables is described. Write an equation.

69. A line contains $(4, -2)$ and is parallel to the y -axis.

70. A line contains $(-1, -5)$ and is parallel to the x -axis.

71. A line contains $(-3, 7)$ and is perpendicular to the y -axis.

72. A line contains $(2, 6)$ and is perpendicular to the x -axis.

In Exercises 73–76, use the indicated assignments of variables and write an equation in the form $y = mx + b$. Then use the integer setting to produce the graph of the equation.

73. A window washer has 22 windows to wash. Let x represent the number of windows that have been washed and y represent the number of windows that have not yet been washed.

(a) Trace the graph to estimate the intercepts.

(b) How can the intercepts be interpreted for this problem?

74. A boy is 3 years younger than his sister. Let x represent the boy's age and y represent his sister's age.

(a) Trace the graph to estimate the intercepts.

(b) For this problem only one of the intercepts is meaningful. Why? Interpret the meaningful intercept.

75. A tool rental store rents a high-powered drill for \$5 plus \$1.25 per hour. Let x represent the number of hours for which the drill was rented, and y represent the rental charge.

(a) Trace the graph to estimate the cost of renting the drill for 6 hours.

(b) Trace the graph to estimate the y -intercept and then interpret it.

76. A person is reading a 20-page report at a rate of 1 page per minute. Let x represent the number of minutes the person has been reading and y represent the number of pages that remain to be read.

(a) Trace the graph to estimate the intercepts.

(b) How can the intercepts be interpreted for this problem?

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CHAPTER 5 Systems of Linear Equations

In Exercises 41–44, write a system of equations and solve the system algebraically.

41. The difference of two numbers is 16. The sum of the larger number and 3 times the smaller number is -24 . What are the two numbers?

42. One number is 4 less than another. The difference of twice the smaller number and 5 times the larger number is -11 . What are the numbers?

43. The width of a rectangle is 1 foot less than half its length. If the perimeter is 64 feet, what are the dimensions?

44. The length of a rectangle is 4 inches more than its width. If the perimeter is 48 inches, what are the dimensions?

Exploring with Real Data

In Exercises 45–48, refer to the following background information and data.

Pretzels and potato chips are staples in Americans' salted snack food diets. In 1993, revenues from the sales of pretzels were up 25% from the previous year to \$1.1 billion. Revenues from the sales of potato chips were up 4% to \$4.6 billion. (Source: Snack Food Association.)

45. Let $x = 0$ for 1992 and $x = 1$ for 1993. Use the given information to write a linear equation for the revenue y for each product during this period.

46. Assume that your equations are accurate models for revenues for the indefinite future. Use the substitution method to solve the system consisting of your two equations.

47. How many years will elapse from 1992 until the revenues from sales of pretzels and potato chips are the same?

48. Suppose that there is a government proposal to tax unhealthy foods or otherwise discourage their consumption. Give arguments for and against such a plan.

Challenge

49. Determine k_1 such that the system in (a) has no solution, and determine k_2 such that the system in (b) has infinitely many solutions.

$$(a) \begin{cases} k_1y + 12 = x \\ 3y - x = 21 \end{cases}$$

$$(b) \begin{cases} 3y - 2x = k_2 \\ y = \frac{2}{3}x + 1 \end{cases}$$

In Exercises 50 and 51, use the substitution method to solve the system of three linear equations in three variables.

$$50. \begin{cases} x + 2y - z = -6 \\ y + z = 4 \\ 3z = 9 \end{cases}$$

$$51. \begin{cases} x - 2 = 5 \\ x + 2y = 13 \\ 2x - y + z = 9 \end{cases}$$

52. Determine the vertices of a quadrilateral bounded by the graphs of $y = x - 5$, $y + x = 3$, $x + y + 2 = 0$, and $y - x = 7$. What specific kind of quadrilateral is this figure?

53. Determine the vertices of a triangle bounded by the graphs of $x + 3 = 0$, $y = 6$, and $6y - 7x = 15$. An **isosceles triangle** is a triangle in which two of the sides are of equal length. Is this triangle an isosceles triangle?

Geometric Connections Exercises



Problems that specifically reference topics and formulas in geometry appear throughout the exercise sets. A useful list of common geometry formulas is printed on the inside front cover.

Challenge Exercises

These problems appear at the end of most exercise sets and offer more challenging work than the standard and Concept Extension problems.

Quick Reference Summary

Quick References appear at the end of all sections except those that are devoted exclusively to applications. These detailed summaries of the important definitions, rules, properties, and procedures are grouped by subsection for a handy reference and review tool.

Chapter Review Exercises

Each chapter ends with a set of review exercises. These exercises include helpful section references that direct students to the appropriate sections for review. The answers to the odd-numbered review exercises are included at the back of the text.

Chapter Tests

A chapter test follows each chapter review. The answers to all the test questions, with the appropriate section references, are included at the back of the text.

Cumulative Tests

A cumulative test appears at the end of Chapters, 3, 5, 8, and 10. The answers to all the test questions, with section references, are included at the back of the text.

2.8 Quick Reference

Addition Property of Inequalities

- For real numbers a , b , and c , the inequalities $a < b$ and $a + c < b + c$ are equivalent. A similar result holds for $>$, \leq , and \geq .

Multiplication Property of Inequalities

- For real numbers a , b , and c ,
 - if $c > 0$, then the inequalities $a < b$ and $ac < bc$ are equivalent.
 - if $c < 0$, then the inequalities $a < b$ and $ac > bc$ are equivalent.

2 Chapter Review Exercises

Section 2.1

- Evaluate the expression for the given value of the variable.

(a) $-4x + 7$; $x = -6$

(b) $-x^2 + 1$; $x = 1$

- Evaluate the expression for the given values of the variables.

(a) $-2(x - y) + 3(x + y)$;

$x = 4$, $y = -3$

(b) $3a$

- Evaluate the expression.

- Identify the expression.

(a) x^2

- Combine like terms.

(a) $3xy$

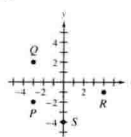
(b) $8e^2$

- Simplify the expression.

- Simplify the expression.

- Simplify the expression.

Figure for 11–14



2 Chapter Test

- Evaluate $\frac{2x}{y} - (x + y)$ for $x = 2$ and $y = -4$.
- Combine like terms: $7a - b + 3b - 6a$.
- Simplify $-(x - 1) + 4(2x + 3)$.
- What is the coefficient of $\frac{x - 9}{2}$? Why?
- Name the quadrant in which all points have the coordinate sign pattern $(-, +)$.
- Describe the set of all points in the coordinate plane whose x -coordinates are 0.

4-5 Cumulative Test

- Complete the table so that each pair is a solution of $x - y = 7$.

x	6
y	-2
	3

- Produce the graph of $y = \frac{1}{2}x - 5$ on your calculator. Then trace the graph to determine a and b so that $A(a, -3)$ and $B(-4, b)$ are points of the graph.
- Determine algebraically the intercepts of the graph of $4x - 3y = -12$.
- Write an equation of the line that contains $P(2, -5)$ and is
 - parallel to the y -axis.
 - perpendicular to the y -axis.
- Determine the slope of a line that contains $P(-3, 5)$ and $Q(7, -4)$.
- State whether the described line has a slope that is positive, negative, 0, or undefined.
 - The line contains $(2, 5)$ and $(-2, 5)$.
 - The line is perpendicular to the x -axis.
 - The line contains $(-3, 9)$ and $(3, 8)$.
- Use the slope-intercept form of $5x - 3y = 9$ to determine the slope and y -intercept of the graph.
- Sketch the line that has a slope of $-\frac{3}{4}$ and contains $P(-7, 5)$.
- Which of the following equations has a graph that is not parallel to the other two?
 - $3x - 4y = 20$
 - $5y - 3x = 25$
 - $y = \frac{3}{4}x + 4$
- The equation of L_1 is $2x + 5y = 20$, and the equation of L_2 is $y = kx - 3$. If L_1 is perpendicular to L_2 , what is the value of k ?
- Write the standard form of the equation of the line containing $A(-6, 2)$ and $B(5, 11)$.
- Write the slope-intercept form of the equation of L_1 that contains $(2, 3)$ and is parallel to L_2 , whose equation is $4x + y = 13$.

Supplements

Instructor's Annotated Edition This edition is a reproduction of the student text with answers to the even-numbered exercises; teaching tips highlighted in blue; and as mentioned previously, asterisks highlighting the Concept Extension exercises.

Graphing Calculator Keystroke Guide This guide provides helpful keystroke instruction for various graphing calculator models and is referenced by the key word icons in the text.

Instructor's Resource Guide with Tests This item contains worked-out solutions to the even-numbered exercises in the text as well as formatted chapter and cumulative tests and their answers.

Student Solutions Guide Worked-out solutions to the odd-numbered text exercises are included in this guide.

Computerized Testing A computerized test bank of multiple-choice and free response questions for the IBM PC (DOS version) and the Apple Macintosh is available to instructors free of charge.

Tutorial Software This supplement offers students additional problem-solving practice and concept reinforcement. This software is available for Windows and Macintosh operating systems.

Videotapes A series of videotapes provide concept review and worked-out examples to reinforce the text presentations.

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Elaine Hubbard
Ronald D. Robinson



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