

Roger Penrose Collected Works

VOLUME 1

1953-1967



OXFORD SCIENCE PUBLICATIONS

**ROGER PENROSE
COLLECTED WORKS**

VOLUME 1

1953-1967



OXFORD
UNIVERSITY PRESS

OXFORD

UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide in

Oxford New York

Auckland Cape Town Dar es Salaam Hong Kong Karachi
Kuala Lumpur Madrid Melbourne Mexico City Nairobi
New Delhi Shanghai Taipei Toronto

With offices in

Argentina Austria Brazil Chile Czech Republic France Greece
Guatemala Hungary Italy Japan Poland Portugal Singapore
South Korea Switzerland Thailand Turkey Ukraine Vietnam

Oxford is a registered trade mark of Oxford University Press
in the UK and in certain other countries

Published in the United States
by Oxford University Press Inc., New York

© Roger Penrose 2011

The moral rights of the authors have been asserted
Crown copyright material is reproduced under Class Licence
Number C01P0000148 with the permission of OPSI
and the Queen's Printer for Scotland

Database right Oxford University Press (maker)

First published 2011

All rights reserved. No part of this publication may be reproduced,
stored in a retrieval system, or transmitted, in any form or by any means,
without the prior permission in writing of Oxford University Press,
or as expressly permitted by law, or under terms agreed with the appropriate
reprographics rights organization. Enquiries concerning reproduction
outside the scope of the above should be sent to the Rights Department,
Oxford University Press, at the address above

You must not circulate this book in any other binding or cover
and you must impose the same condition on any acquirer

British Library Cataloguing in Publication Data
Data available

Library of Congress Control Number: 2010926298

Typeset by Newgen Imaging Systems Pvt Ltd., Chennai, India
Printed in Great Britain
on acid-free paper by
CPI Antony Rowe, Chippenham, Wiltshire

ISBN 978-0-19-921936-0 (Vol. 1)
978-0-19-921944-5 (Set)

1 3 5 7 9 10 8 6 4 2

PREFACE

The topics addressed, in these *Collected Works*, vary over quite an extensive range, the boundaries between their areas being sometimes quite blurred. It would have been hard to classify all these articles and arrange them according to individual topics. Moreover, my own mode of working has been, on the whole, to keep most of my different scientific or mathematical interests running concurrently. Accordingly, it has seemed appropriate not to attempt to group the papers together into separate tidy categories. To do so would have provided a somewhat misleading picture of my overall mode of thinking. Instead, the ordering of articles that has been adopted is essentially chronological, at least with regard to the date of publication. The resulting somewhat higgledy-piggledy appearance that we find in the sometimes abrupt juxtaposition of different topics is, perhaps, fairly illustrative of my actual mode of working.

There are two main kinds of drive that underlie these works. On the one hand, some of the articles have arisen mainly from purely mathematical considerations, which can be ultimately aesthetic or just matters of simple curiosity. On the other hand, motivations for many others of the articles came primarily from a strong desire to discover more about the underlying principles governing the workings of the physical world. Yet, one of the deep mysteries about what has been discovered about these principles is the great breadth of intrinsically appealing mathematics that is found to underlie the actual operation of the physics of the universe, so these two drives are found to relate closely to one another. The reader may perhaps note the distinct geometrical flavour of much of my own mathematics, evidencing the strong appeal that geometry exerts on me, an appeal that is not infrequently intertwined with desire for algebraic elegance. Indeed, I have found it remarkable how much of the workings of our universe appear to depend upon sophisticated and powerful geometry and algebra.

My own very geometrical perspectives, as opposed to purely computational ones, are perhaps made more obvious in the later volumes of this series than in the early ones, but this appearance is deceptive. Very early in my career as a Cambridge graduate student, I had developed a graphical notation which enabled me to carry out many kinds of calculation in an entirely diagrammatic way. For a long time I was rather secretive of this procedure, and in my earlier writings would take care to remove all trace of these visual origins. I do not doubt that my predilections for such geometric or algebraically elegant forms of understanding have influenced the specific choices of direction in which my endeavours towards furthering our understandings of the physics of the world have taken.

The focus of these directions has become more clarified as time has moved on, and has narrowed down to such a degree that there has emerged a somewhat disturbing repetitiveness in many of the later articles. A good number of these provide rather non-technical accounts. An example of such repetitiveness occurs with my descriptions of gravitational collapse to a black hole. There is, however, a gradually developing outlook on this issue, leading to an emphasis on the fundamental contrast in character between the space-time singularities that inevitably appear within black holes and the space-time singularity that constitutes the big bang, this contrast being central to the existence of a second law of thermodynamics in our universe. The same kind of repetitiveness applies to my frequently repeated claim of a need for a fundamental change in the basic laws of quantum theory, where only towards the end do my suggestions become particularly specific. Likewise, many mildly differing accounts are provided outlining the formalism and the basic ideas that underlie twistor theory. The later ones lead up to various viewpoints on the 'googly' problem, which has constituted the main obstruction to the finding of a full understanding of general relativity in twistor terms. Despite such repetition, it is my hope that there is a sufficient variety of emphasis and of historical development in such articles for it to have been worth while to include them all here.

Several colleagues have collaborated with me in writing the works presented here, and I can mention but only a few. Foremost, I have collaborated long and particularly fruitfully with Ezra T. Newman, especially with regard to the asymptotics of general relativity, and also with Wolfgang Rindler in the development of the formalism of 2-spinor calculus. Other collaborators who have played important roles in helping to develop the mathematics of discrete physics and twistor theory have been Erwin Kronheimer, Michael Eastwood, Ronny Wells, and Denny Hill. Among my collaborators, providing very significant inputs, who were former students in my research group, are Andrew Hodges, Paul Tod, Lane Hughston, George Sparling, Richard Ward, Nick Woodhouse, and Lionel Mason, but there were many others. Although not playing roles as actual collaborators, fundamental influences in my research have come from Dennis Sciama and Michael Atiyah.

Roger Penrose
April 2010

BIOGRAPHY

Early years and influences

I was born in Colchester, Essex, UK on 8 August 1931. My father, Lionel Sharples Penrose, FRS, was a highly talented and intelligent man, with many interests. He, himself, had come from a Quaker family, his own father (James Doyle Penrose) having been a professional artist and his mother (Josephine Peckover) was from a well-to-do Quaker banking family. Lionel's primary professional interest was in the causes of hereditary mental disease. He was Galton Professor of Human Genetics at University College London from 1945 and, following retirement, Director of the Kennedy-Galton Institute, St Albans. He had a very considerable artistic talent and a keen interest in music, especially Mozart, but also Bach and other classical composers, dabbling a little in composition himself, particularly when there was a puzzle aspect in what was involved (where he once wrote a piece that could be played equally well upside down as right-way up). He liked making puzzles of various kinds in wood, and spent much of his later life designing wooden models illustrating self-reproduction. He was recognized as a fine composer of chess problems. He had a definite fondness for mathematics and was an innovative statistician. He loved explaining matters of science and mathematics. On one occasion, when I told him that my mathematics teacher at school had informed us that he would be explaining calculus the next day, Lionel immediately took me aside so as to be sure to be the first to teach me about the beauties of calculus. This made a great impression on me. Yet he was an extremely difficult person to communicate with on personal matters.

My mother, born Margaret Leathes, was a charming and caring woman with a very subtle mind. She had been Head Girl at Bedales school, and became medically qualified, but found she was not able to practice after her marriage. After Lionel died, in 1972, she married Max Newman FRS, a fascinating man, who had been Professor of Mathematics in Manchester and, as we later learned, a key figure at Bletchley Park during the war. Margaret's father, John Beresford Leathes, FRS, had been a distinguished professor of physiology, and her mother, a concert pianist, born Sara Mara Natanson from a Jewish family in Latvia, had disowned her family (then in St Petersburg), becoming completely secretive about her origins, and even about her maiden name. Margaret enjoyed geometry at school, but did not

communicate much on this topic with me. My older brother Oliver was very precocious, and I learnt a lot of physics and mathematics from him as I was growing up. He later became a distinguished mathematical physicist (statistical mechanics) and FRS. My younger brother Jonathan excelled at games, particularly chess, and later became British Chess Champion a record 10 times, and then a top correspondence chess player. My much younger sister Shirley Victoria, now Shirley Hodgson, became a highly regarded Professor of Cancer Genetics.

This atmosphere of science, mathematics, art, and music, and of puzzles and games, had been an important part of my upbringing. During the war years, the family lived in London, Ontario, Canada, where Lionel became Director of Psychiatric Research at the Ontario Hospital. We returned to England in 1945, where I attended University College School, London. As an undergraduate, I studied Mathematics at University College London, from 1951 to 1954; then I went to St John's College Cambridge to do research in algebraic geometry, initially under William Hodge and subsequently John Todd. In my same year, under Hodge, was Micheal Atiyah, who was an inspirational, though somewhat intimidating colleague. Frank Adams and Christopher Zeeman (see Chapter 13) were contemporaries who also influenced me, and I became good friends with Hans Liebeck and Douglas Munn (see Chapter 3), who were close research-student colleagues at St John's.

Yet, the colleague who had the greatest influence on the development of my research during this period, and for many years later, was the cosmologist Dennis Sciama. Even before studying at Cambridge, I had a chance meeting with him during a visit to my brother Oliver in Cambridge in which I raised a question that had worried me in relation to the inspiring series of BBC radio talks given by Fred Hoyle in 1951 (The nature of the universe. A series of broadcast lectures). On the strength of this encounter (at which I used a space-time diagram with tilted light cones to make my point), Dennis had evidently formed the opinion that it would be worth while to interest me more in physics, even trying, later, to persuade me to switch my topic of research to cosmology. I was then too committed to my many pure-mathematical activities, but I learnt a great deal of physics from Dennis, and of the excitements involved in striving to uncover secrets of the universe. Another influential colleague was Felix Pirani, from whom I learnt much about developments in general relativity.

Pivotal were the superb lecture courses of Bondi, on general relativity, and Dirac, on quantum mechanics. On the pure-mathematical side, I recall particularly an elegant information-packed algebra course by Philip Hall. Lectures by Steen on mathematical logic turned out to influence me strongly towards my later viewpoint concerning the relation between computation and conscious understanding (e.g. Chapters 169, 184, 186).

Degrees

- | | |
|------|---|
| 1952 | BSc Mathematics (Spec) (1st Cl), University College, London |
| 1957 | PhD (Cantab), St John's College, Cambridge |

Employment

1956	NRDC London (research on numerical analysis with computers); later consultant
1956–1957	Assistant Lecturer, Bedford College, London
1957–1960	Research Fellow, St John's College, Cambridge
1959–1961	NATO Research Fellow, Princeton, Syracuse, Cornell Universities
1961–1963	Research Associate, King's College, London
1963–1964	Visiting Associate Professor, University of Texas, Austin, Texas
1964–1966	Reader (Applied Mathematics), Birkbeck College, London
1966–1967	Visiting appointments: Yeshiva, Princeton, Cornell, Chicago Universities; lecturer Battelle Inst., Seattle
1967–1973	Professor (Applied Mathematics), Birkbeck College, London
1969–1970	Visiting professor, Yeshiva New York, Princeton, and Cornell Universities
1973–1998	Rouse Ball Professor (Mathematics), Oxford; 1998– Emeritus
1978	Miller Fellow, Univ. of California, Berkeley
1982–1987	Lovett Professor (Mathematics), Rice Univ., Houston, Texas, USA (Part-time)
1987–1993	Distinguished Professor (Physics, Mathematics), Syracuse Univ. USA (Part-time)
1993–	Francis and Helen Pentz Distinguished Professor (Physics, Mathematics; part-time)
1998–2001	Gresham Professor (Geometry), Gresham College, London

Honours Prizes include 1971 Heinemann Prize; 1972 Fellow of Royal Society; 1978 Plenary Lecture, International Congress of Mathematicians, Helsinki; 1985 Royal Medal (Royal Society); 1988 Wolf Prize (physics, with Hawking); 1990 Albert Einstein Medal; 1994 knighted (services to science); 1994 Polish Academy of Sciences; 1998 US National Academy of Sciences; 2000 Order of Merit; 2004 de Morgan Medal (London Mathematical Society); 2001 Royal Irish Academy (Science); 2008 Copley Medal (Royal Society).

Marriages

1958	Joan Isabel Wedge; divorced 1980
1988	Vanessa Dee Thomas

Children

1963	Christopher Shaun
1964	Toby Nicholas
1966	Eric Alexander
2000	Maxwell Sebastian

Honorary degrees

1992	University of New Brunswick, Canada
1993	University of Surrey
1994	University of Bath
1995	University of London
1996	University of Glasgow
1996	University of Essex
1997	University of St Andrews
1998	University of Santiniketan, India
1998	Open University
2002	University of Southampton
2004	University of Waterloo, Canada
2005	University of Leuven, Belgium
2005	Warsaw University
2005	University of Athens
2006	University of York

Books

1972	Techniques of Differential Topology in Relativity (NSF and SIAM)
1984 and 1986	Spinors and Space-Time (Volumes 1 and 2; Cambridge University Press; with Rindler)
1989	The Emperor's New Mind (Oxford University Press 1989)
1994	Shadows of the Mind (Oxford University Press)
1996	The Nature of Space and Time (Princeton University Press; with Hawking)
1997	The Large, the Small and the Human Mind (Cambridge University Press)
2004	The Road to Reality (Jonathan Cape)

INTRODUCTION

On looking through the various articles that have found themselves to be in the period spanned by Volume 1 of this 6-volume set of *Collected Works*, I am struck by the number of them (published between the years 1953 and 1967) that form the basis of most of the research that has occupied me for the rest of my life. The general ideas of the algebra of tensor systems and their diagrammatic representations, as addressed by the two theses (Chapters 6 and 7) and the notation introduced in Chapter 31, have formed an important background to most of my later research. The beauty and power of 2-spinor calculus, which had intrigued me as seeming to lie outside the framework of ordinary tensor algebra, was made clear to me from lectures in Cambridge by Paul Dirac. The magnificence of general relativity was impressed upon me by the lectures of Hermann Bondi, and much of my early published research involved bringing these two spheres of understanding together.

Dennis Sciama's influence had guided me in various ways into many of the enticing mysteries of physics, but I had indeed found myself to be particularly attracted by the geometry of spinors and by the powerful principles underlying Einstein's general relativity. Accordingly, my own distinctive approaches to physical problems have concentrated on 2-spinors, spin-coefficients, and the geometry of light cones (particularly Chapters 11, 12, and 15), and this very much guided me with regard to my own peculiar type of approach to the study of space-time geometry. This, in turn, has taken me in certain other directions, one of which led to singularity theorems (Chapter 20) and to the idea of treating asymptotics in terms of conformal boundaries (Chapter 17, 18, and 22). Intrigued by issues raised by the inter-relations between quantum mechanics and space-time structure I played, early on, with ideas relating to the possibility of a discrete basis to physics (Chapters 28, 31, and 32). But as my understanding of quantum mechanics developed I became increasingly impressed by the power, mystery, and geometric subtlety bestowed by the complex number system, as opposed to that provided by discrete systems, and this led me into twistor theory (Chapter 29)—some of the more significant implications of which have only recently begun to come to light.

ACKNOWLEDGEMENTS

Oxford University Press has attempted to trace the copyright holder of all the papers included in these volumes and apologizes to copyright holders if permission to publish in this form has not been obtained.

The author and publishers thank the following individuals for their permissions:

Acta Physics Polonica B **280**

Adam Hilger Ltd **147**

American Chemical Society **206**

American Institute of Physics

American Journal of Physics **19, 24**

Journal of Maths and Physics **15, 26, 27, 29, 39, 41, 58, 84**

Errata **15**

American Mathematical Society **30, 180**

Bulletin of the American Mathematical Society **144**

Contemporary Mathematics **179**

American Physical Society

Physical Review Letters **16, 20, 23, 302**

Reviews of Modern Physics **21**

Annals of Human Genetics **100**

Asian Journal of Mathematics **281**

Astronomische Gesellschaft **295**

Benjamin **36**

Bibliopolis **175**

Blackwell **80, 166, 186, 215**

Annals of the New York Academy of Sciences **60, 190, 294**

British Broadcasting Company **59**

British Journal of Psychology **9**

- Cambridge Entomological Club **243**
- Cambridge University Press **6, 7, 46, 104, 138, 157, 172, 173, 176, 195, 218, 220, 227, 230, 240, 290, 303, 304**
- Behavioural and Brain Sciences* **193, 194, 224**
- Proceedings of the Cambridge Philosophical Society* **3, 4, 5, 32, 42, 50, 126**
- Chapman and Hall/CRC Press **107, 124, 125, 127, 128, 130, 136, 139, 142, 143, 146, 149, 150, 154, 155, 156, 160, 161, 164, 170, 171, 178, 189, 191, 198, 201, 203, 209, 229, 234, 293**
- Connected Globe **257**
- Consiglio Nazionale Della Ricerca **145**
- Cornell University Press **33**
- Éditions du Centre National de la Recherche Scientifique* **14**
- Elsevier **12, 45, 52, 68, 85, 86, 92, 93, 123, 151, 167, 185, 187, 226, 279**
- Escher, M.C. images: The M.C. Escher Company B.V. **218, 249, 306**
- Eureka* **1, 2**
- Folio Society **284**
- Freeman **56**
- Granta **297**
- Gresham College **277**
- Halsted Press **148**
- Hanser **66, 67**
- Imperial College Press **264**
- Imprint Academic **235, 245**
- Indian Academy of Sciences **244**
- Indiana University Press **25**
- Institute of Electrical and Electronics Engineers **196**
- Institute of Physics **217**
- Classical and Quantum Gravity* **253, 254, 274, 282**
- European Journal of Physics* **258**
- Journal of Physics A: Mathematical and General* **95**
- International Astronomical Union
- International Astronomical Union Symposium Proceedings Series No. 13, Cosmology* **72**

International Atomic Energy Agency

Contemporary Physics **38**

International Centre for Theoretical Physics **53, 68**

International Mathematical Union **286**

International University Bremen **305**

Inter-University Centre for Astronomy and Physics **265**

The Italian Astronomical Society

Memorie della Società Astronomica Italiana **200**

Longman **78, 79, 108, 116, 117, 134, 135, 141, 177, 188**

The Massachusetts Institute of Technology Press **228, 246**

National Institute of Sciences of Ukraine **48**

Nature **8, 44, 49**

Nature (Physical Science) **47**

New Scientist **10, 101**

New York Academy of Sciences **294**

Oxford University Press **55, 69, 121, 122, 165, 178, 184, 197, 202, 208, 216, 237, 270, 272, 276, 291, 299**

Pearson **109**

Penguin **273**

Penrose's Unpublished Papers **31**

Pergamon Press **17**

Pitman **73, 75, 76, 77, 82, 87, 89, 90, 94, 102, 103, 110**

Pontifical University **296**

Princeton University Press **28, 268**

Annals of Math **13, 131**

Proceedings of the International Congress of Mathematicians **115**

Rivista del Nuovo Cimento **40**

Routledge **169**

Royal Astronomical Society **183**

The Royal Society

Proceedings of the Royal Society Series A **22, 35, 97, 105, 106, 132**

Philosophical Transactions of the Royal Society of London. Series A **275, 283**

The Sciences **99**

Scientific American **51**

Simon and Schuster **242**

Springer **61, 63, 65, 83, 91, 98, 111, 133, 168, 204, 207, 231, 255, 263, 278, 292, 298, 300**

Artificial Intelligence Review **205**

Communications in Mathematical Physics **43, 54, 129**

Cosmology and Gravitation **120**

Foundations of Physics **140**

General Relativity and Gravitation **70, 71, 113, 114, 239, 251**

Higher-Order and Symbolic Computation **287**

International Journal of Theoretical Physics **37, 56**

Journal of Statistical Physics **233**

Lecture Notes in Physics **112, 152**

The Mathematical Intelligencer **285**

Taylor and Francis **18**

International Studies in the Philosophy of Science **256**

Surveys in High Energy Physics **119**

Trinity College, Dublin **219**

World Scientific **162, 163, 225, 236, 241, 252, 289, 301**

Universal Academy Press **266**

University of Chicago Press **96, 269**

University of Chittagong **158**

University College London **267**

The University of Oxford (Mathematics Department)

Twistor Newsletter **73, 74, 75, 76, 77, 78, 79, 81, 82, 88, 89, 90, 94, 102, 103, 107, 108, 116, 117, 118, 124, 125, 127, 128, 130, 134, 135, 136, 137, 139, 141, 142, 143, 146, 149, 150, 153, 154, 155, 156, 157, 160, 161, 164, 170, 171, 174, 181, 182, 188, 189, 191, 192, 198, 199, 201, 203, 209, 210, 211, 212, 213, 214, 221, 222, 223, 229, 232, 238, 248, 290, 250, 259, 260, 261, 262, 271, 288**

University of Utah Press **247**

CONTENTS

VOLUME 1

Introduction

xxiii

1950s

1	Morley's trisector theorem	1
2	'Construction problem' and 'Word problem'	3
3	A note on inverse semigroups	5
4	A generalized inverse for matrices	11
5	On best approximate solutions of linear matrix equations	21
6	Tensor methods in algebraic geometry	25
7	Geometrical algebras: A new approach to invariant theory	251
8	A self-reproducing analogue	351
9	Impossible objects: A special type of visual illusion	355
10	Puzzles for Christmas	359
11	The apparent shape of a relativistically moving sphere	365

1960s

12	A spinor approach to general relativity	369
13	Imbedding of manifolds in Euclidean space	403
14	General relativity in spinor form	415
15	An approach to gravitational radiation by a method of spin coefficients	421
16	Asymptotic properties of fields and space-times	437
17	The light cone at infinity	441
18	Conformal treatment of infinity	447
19	Energy conservation as the basis of relativistic mechanics	469
20	Gravitational collapse and space-time singularities	475
21	A remarkable property of plane waves in general relativity	479
22	Zero rest-mass fields including gravitation: Asymptotic behaviour	487
23	10 exact gravitationally conserved quantities	533
24	Energy conservation as the basis of relativistic mechanics II	537
25	General-relativistic energy flux and elementary optics	541
26	Note on the Bondi-Metzner-Sachs group	559

27	Some new gravitationally conserved quantities	569
28	An analysis of the structure of space-time	579
29	Twistor algebra	731
30	Conserved quantities and conformal structure in general relativity	755
31	Theory of quantized directions	769
32	On the structure of causal spaces	801

DETAILED CONTENTS

VOLUMES 1–6

VOLUME 1

1950s

1	Morley's trisector theorem	1
2	'Construction problem' and 'Word problem'	3
3	A note on inverse semigroups	5
4	A generalized inverse for matrices	11
5	On best approximate solutions of linear matrix equations	21
6	Tensor methods in algebraic geometry	25
7	Geometrical algebras: A new approach to invariant theory	251
8	A self-reproducing analogue	351
9	Impossible objects: A special type of visual illusion	355
10	Puzzles for Christmas	359
11	The apparent shape of a relativistically moving sphere	365

1960s

12	A spinor approach to general relativity	369
13	Imbedding of manifolds in Euclidean space	403
14	General relativity in spinor form	415
15	An approach to gravitational radiation by a method of spin coefficients	421
16	Asymptotic properties of fields and space-times	437
17	The light cone at infinity	441
18	Conformal treatment of infinity	447
19	Energy conservation as the basis of relativistic mechanics	469
20	Gravitational collapse and space-time singularities	475

21	A remarkable property of plane waves in general relativity	479
22	Zero rest-mass fields including gravitation: Asymptotic behaviour	487
23	10 exact gravitationally conserved quantities	533
24	Energy conservation as the basis of relativistic mechanics II	537
25	General-relativistic energy flux and elementary optics	541
26	Note on the Bondi-Metzner-Sachs group	559
27	Some new gravitationally conserved quantities	569
28	An analysis of the structure of space-time	579
29	Twistor algebra	731
30	Conserved quantities and conformal structure in general relativity	755
31	Theory of quantized directions	769
32	On the structure of causal spaces	801

VOLUME 2

1960s

33	Cosmological boundary conditions for zero rest-mass fields	1
34	On gravitational collapse and cosmology	15
35	New conservation laws for zero rest-mass fields in asymptotically flat space-time	29
36	Structure of space-time	61
37	Twistor quantization and curved space-time	177
38	On gravitational collapse	217