Beginning Algebra

Dennis T. Christy

Robert Rosenfeld

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Nassau Community College

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Beginning Algebra

To Margaret and Leda, Thank You

Preface

Audience

This book is intended for students who need a concrete approach to mathematics. It is written for college students who have never studied algebra or who need a review course in beginning algebra. The thorough pedagogical features of the text and the associated ancillary package ensure that the student has a wealth of helpful material.

Approach

This book is written with the belief that current textbooks must provide a dynamic approach to problem solving that allows students to monitor their progress and that effectively integrates graphics and calculator use. Our approach in these four areas is explained next.

Problem-Solving Approach

Our experience is that students who take beginning algebra learn best by "doing." Examples and exercises are crucial since it is usually in these areas that the students' main interactions with the material take place. The problem-solving approach contains brief, precisely formulated paragraphs, followed by many detailed examples. A relevant word problem introduces *every* section of the text, and word problems or other motivational problems are included in *every* section exercise set.

Graphics

A major component of a problem-solving approach with intuitive concept development is a strong emphasis on graphics. Students are given, and are encouraged to draw for themselves, visual representations of the concepts they are analyzing and the problems they are solving. Effective use of color enhances the many images in concept developments and in exercise sets.

Interactive Approach

Because students learn best by doing, progress check exercises are associated with each example problem in the text. By doing these exercises, students obtain immediate feedback on their understanding of the concept being discussed.

Calculator Use

The text encourages the use of calculators and discusses how they can be used effectively. It is assumed in the discussions that students have scientific calculators that use the algebraic operating system (AOS). Calculator illustrations show primarily the keystrokes required on a Texas Instruments TI-30-SLR+.

Features

- Problem-solving approach with intuitive concept developments
- Extensive and varied word problems and application problems
- "Think About It" exercises to develop critical-thinking skills
- Section introductions that include an interest-getting applied problem that is solved as an example in the text
- Discussions about basic concepts in geometry
- Effective use of color with an emphasis on graphics
- Over 6,300 exercises and 430 examples
- "Progress Check" exercises that allow for instant self-evaluation
- Problem sets of graduated difficulty that are closely matched to the example problems
- "Remember This" exercises that end each section and are crafted to provide smooth transition to the next section as well as spiral review of previous material
- Abundant chapter review exercises

- A chapter test and a cumulative review test for each chapter
- Boxes with labels for important definitions and rules
- "Note" and "Caution" remarks that provide helpful insights and point out potential student errors
- Unique chapter summaries that highlight specific objectives and key terms and concepts at the end of each chapter
- Instructions on calculator use
- Rigorous accuracy checking to avoid errors in the text
- Complete instructional package

Pedagogy

Section Introductions

In the spirit of problem solving, each section opens with a problem that should quickly involve students and teachers in a discussion of an important section concept. These problems are later worked out as an example in each section.

Keyed Section Objectives

Specific objectives of each section are listed at the beginning of the section, and the portion of the exposition that deals with each objective is signaled by numbered symbols such as [1].

Systematic Review

Students benefit greatly from a systematic review of previously learned concepts. At the end of each chapter there are a detailed chapter summary that includes a checklist of objectives illustrated by example problems, abundant chapter review exercises, a chapter test, and a cumulative review test. In addition, each section exercise set is concluded with a short set of "Remember This" exercises that review previous concepts, with particular emphasis on skills that will be needed in the next section.

"Think About It" Exercises

Each exercise set is followed immediately by a set of "Think About It" exercises. Although some of these problems are challenging, this section is not intended as a set of "mind bogglers." Instead, the goal is to help develop critical-thinking skills by asking students to create their own examples, express concepts in their own words, extend ideas covered in the section, and analyze topics slightly out of the mainstream. These exercises are an excellent source of nontemplate problems and problems that can be assigned for group work.

For the Instructor

The *Annotated Instructor's Edition* provides a convenient source for answers to exercise problems. Each answer is placed near the problem and appears in red. This feature eliminates the need to search through a separate answer key and helps instructors forecast effective problem assignments. Suggestions and comments based on our experiences in developmental mathematics are also provided to complement your teaching techniques, and relevant historical asides that can enliven the course material are often given. We mention three sources helpful for this historical material. *A History of Mathematics* by Victor J. Katz from HarperCollins, which emphasizes the influence of the most important textbooks of the past; *A History of Mathematical Notations* by Florian Cajori from Open Court, an older work that traces the origins and development of familiar mathematical symbols; and *The History of Mathematics: An Introduction,* 2e, by David M. Burton from Wm. C. Brown Publishers, a good discoursive general introduction.

The *Instructor's Resource Manual* includes a guide to supplements that accompany *Beginning Algebra,* reproducible tests, and transparency masters of key concepts and procedures.

The *Instructor's Solutions Manual* contains solutions to every problem in the text, including solutions to the "Think About It" problems. These solutions are intended for the use of the instructor only.

WCB Computerized Testing Program provides you with an easy-to-use computerized testing and grade management program. No programming experience is required to generate tests randomly, by objective, by section, or by selecting specific test items. In addition, test items can be edited and new test items can be added. Also included with the *WCB Computerized Testing Program* is an on-line testing option which allows students to take tests on the computer. Tests can then be graded and the scores forwarded to the grade-keeping portion of the program.

The *Test Item File* is a printed version of the computerized testing program that allows you to examine all of the prepared test items and choose test items based on chapter, section, or objective. The objectives are taken directly from *Beginning Algebra.*

For the Student

The *Student's Solutions Manual and Study Guide* provides a summary of the objectives, vocabulary, rules and formulas, and key concepts for each section. Detailed solutions are given for every-other odd-numbered section exercise. Additional practice is available for each section, and each chapter ends with a sample practice test. The *Student's Solutions Manual and Study Guide* is available for student purchase through the bookstore.

Videotapes and Software that are text specific have been developed to reinforce the skills and concepts presented in *Beginning Algebra*. Contact your Wm. C. Brown Publishers representative for detailed descriptions.

Acknowledgments

A project of this magnitude is a team effort that develops over many years with the input of many talented people. We are indebted to all who contributed. In particular, we wish to thank the reviewers of this text, who are listed separately; Carol Hay, Linda J. Murphy, and Nancy K. Nickerson, who checked for accuracy in our final manuscript; Carole D. Carney, who checked for accuracy in the typeset text; Mary Trerice, who helped formulate exercise answers; Amy Driscoll, who did a prompt and accurate job of typing the manuscript; Carol Beal, who skillfully copyedited the manuscript; Dwala Canon, Eugenia M. Collins, Theresa Grutz, Barbara Hodgson, Earl McPeek, Linda Meehan, and Diane Saeugling, Wm. C. Brown Publishers; Deborah Schneck, Schneck-DePippo Graphics. To our parents, a special thank you. In each case they have always supported our efforts and taught us to persevere and overcome obstacles. Finally, but most important, we thank our wives, Margaret and Leda, and dedicate this book to them. They have given that special help and understanding only they could provide.

Dennis Christy
Robert Rosenfeld

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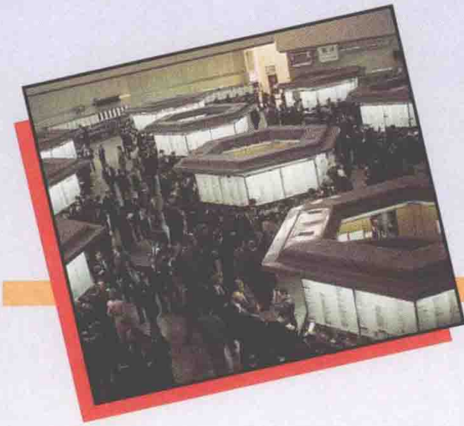


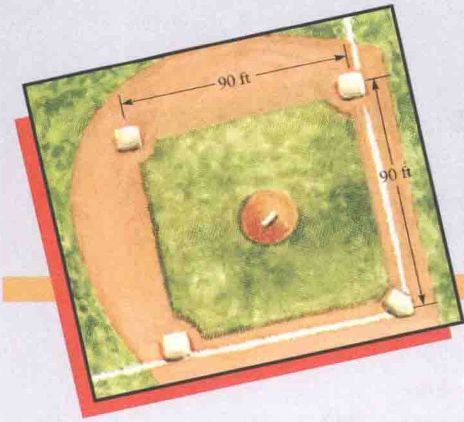
Calculator Use TO THE STUDENT

A scientific hand-held calculator is now standard equipment for beginning algebra and beyond. These ten-dollar wonders provide you with the benefits of electronic computation that is fast, accurate, and easy to learn. Most important, efficient calculator use helps you focus on important mathematical ideas. To understand and apply mathematical concepts is our fundamental aim, and calculators are marvelous aids in attaining this goal.

A scientific calculator (the type you need) contains at least the following special features: algebraic keys x^2, $\sqrt{x}$, $1/x$, y^x or x^y, $\sqrt[x]{y}$; parentheses keys (,); a scientific notation key EE or EXP; and one memory that can store and recall.

In this book we also assume a scientific calculator using the algebraic operating system (AOS). Texas Instruments, Sharp, and Casio produce scientific calculators using this system. With AOS you can key in the problem exactly as it appears, and the calculator is programmed to use the order of operations discussed in Section 1.2. For example, since multiplication is done before addition, 2 [+] 3 [×] 4 [=] 14. If your calculator displays 20 when you key in this sequence, it is operating on left-to-right logic. You must then be careful to key in the problem so the correct order of operations is followed. Calculator illustrations in this text show primarily the keystrokes required on a Texas Instruments TI-30-SLR+. In any case, you should read the owner's manual that comes with your calculator to familiarize yourself with its specific keys and limitations.

One other introductory note—a calculator *computes*, that's all. You do the important part—you *think*. You analyze the problem, decide on the significant relationships, and determine if the solution makes sense in the real world. It's nice not to get bogged down in certain calculations and tables, but critical thinking has always been the main goal.

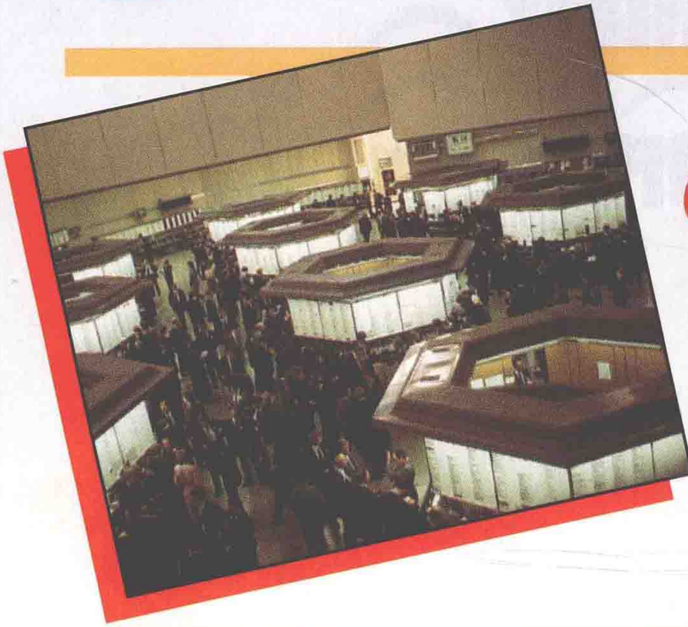


Contents

Beginning Algebra

CHAPTER

1 From Numbers to Algebra



One share of stock in Apple Computer opens a week with a share price of $39\frac{7}{8}$ and closes the week with a share price of $41\frac{1}{4}$ dollars. For this week, find the dollar gain for an investor who owns 400 shares of this stock. (See Example 10 of Section 1.1.)

THE TRANSITION from arithmetic to algebra requires a firm grasp of number concepts. In this chapter we review some important procedures and vocabulary from arithmetic, and we discuss in detail the basic properties and operations associated with real numbers. In the process we will begin to study numerical relations in a more general way by using symbols, such as x, that may be replaced by numbers. By using such symbols, we create a generalized version of arithmetic, which we call algebra, that is a powerful tool in analyzing concepts in a wide variety of fields.

1.1 Whole Numbers and Fractions

OBJECTIVES

1. Graph a set of whole numbers.
2. Identify and use the symbols $=, <, >, \neq, \leq, \geq$.
3. Write a number as a product of its prime factors.
4. Express a fraction in lowest terms.
5. Multiply, divide, add, and subtract fractions.

1 To discuss number concepts, it is useful to have names for some special collections of numbers. Our most basic need is for the numbers that are used in counting: 1, 2, 3, 4, and so on. We write three dots ". . . ," called **ellipses,** to mean "and so on," and we call this set* of numbers the natural numbers.

$$\text{Natural numbers} = \{1,2,3,4, \ldots\}$$

By attaching 0 to the list of natural numbers, we obtain the set of whole numbers.

$$\text{Whole numbers} = \{0,1,2,3,4, \ldots\}$$

A good way to describe sets of numbers is to picture them on a number line. For example, Figure 1.1 shows the picture, or graph, of the set of numbers $\{1,3,5,7\}$. To construct this graph, draw a horizontal line; then place a mark to represent 0 at a convenient point on the line. Choose a unit length, and mark off and label consecutive whole numbers moving to the right. Now to graph $\{1,3,5,7\}$, just place dots at the points corresponding to 1, 3, 5, and 7. We call a dot corresponding to a number the **graph** of the number.

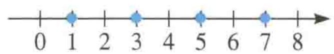


Figure 1.1

EXAMPLE 1 Graph the set of even whole numbers from 0 to 8.

Solution Even numbers are exactly divisible by 2, so the set of even whole numbers from 0 to 8 is $\{0,2,4,6,8\}$. To graph this set of numbers, place dots on the number line at 0, 2, 4, 6, and 8, as shown in Figure 1.2.

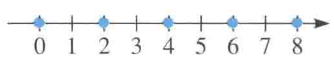


Figure 1.2

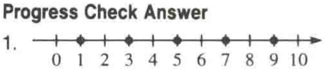
Note An important objective of this chapter is to develop numbers so that we can fill in the number line. Then each point on the line will correspond to a number, and each number will correspond to a point on the number line.

PROGRESS CHECK 1 Graph the set of odd whole numbers from 1 to 9. Odd whole numbers are whole numbers that are not exactly divisible by 2.

2 An important property of any set of numbers that can be associated with points on the number line is that the numbers can be put in numerical order. If we compare two numbers, a and b, then either a is less than b, a is greater than b, or a equals b. These order relations are symbolized as follows.

*A **set** is simply a collection of objects, and we may describe a set by listing the objects or members of the collection within braces.

Progress Check Answer

1. 

Statement	Read	Comment
$a = b$	a equals b	a and b represent the same number
$a < b$	a is less than b	the symbol $<$ points to a, the smaller number
$a > b$	a is greater than b	the symbol $>$ points to b, the smaller number

To illustrate, because 2 is smaller than 5, we may write

$$2 < 5, \quad \text{read} \quad 2 \text{ is less than } 5,$$
$$\text{or} \quad 5 > 2, \quad \text{read} \quad 5 \text{ is greater than } 2.$$

$2 < 5$ or $5 > 2$

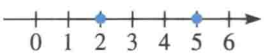


Figure 1.3

Relations of "less than" and "greater than" can be seen easily on the number line, as shown in Figure 1.3. The graph of the larger number is to the right of the graph of the smaller number. Note that $2 < 5$ may be written as $5 > 2$, and vice versa.

EXAMPLE 2 Insert the proper symbol ($<, >, =$) to indicate the correct order.

a. 9 ____ 4 **b.** 4 ____ 9
c. $2 + 3$ ____ $7 - 2$ **d.** $7 - 5$ ____ $5 - 5$

Solution

a. Because 9 is greater than 4, the correct order is given by $9 > 4$.
b. Because 4 is less than 9, write $4 < 9$.
c. $2 + 3$ and $7 - 2$ both represent 5, so $2 + 3 = 7 - 2$.
d. $7 - 5 > 5 - 5$ because $7 - 5$ is 2, $5 - 5$ is 0, and $2 > 0$.

PROGRESS CHECK 2 Insert the proper symbol ($<, >, =$) to indicate the correct order.

a. 0 ____ 5 **b.** 5 ____ 0
c. $9 - 3$ ____ $2 + 3$ **d.** $3 + 7$ ____ $7 + 3$

The symbols $\neq$, $\leq$, and $\geq$ are also commonly used to indicate an inequality relation.

$a \neq b$ means a is not equal to b.

$a \leq b$ means a is less than or equal to b.

$a \geq b$ means a is greater than or equal to b.

In the following example note that statements like $a \leq b$ are true if either the "less than" part is true or the "equal" part is true. It is never possible for both the "less than" part and the "equal" part to be true simultaneously.

EXAMPLE 3 Classify each statement as true or false.

a. $2 \leq 5$ **b.** $2 \geq 2$
c. $5 \leq 2$ **d.** $2 + 3 \neq 6$

Solution

a. $2 \leq 5$ is read "2 is less than or equal to 5." This statement is true because "2 is less than 5" is true.
b. $2 \geq 2$ is read "2 is greater than or equal to 2." Because "2 is equal to 2" is true, $2 \geq 2$ is true.
c. $5 \leq 2$ is false because $5 < 2$ is false and $5 = 2$ is false.

Progress Check Answers

2. (a) $0 < 5$ (b) $5 > 0$ (c) $9 - 3 > 2 + 3$ (d) $3 + 7 = 7 + 3$

d. $2 + 3 \neq 6$ is read "$2 + 3$ does not equal 6." Because $2 + 3$ represents 5, the statement is true.

PROGRESS CHECK 3 Classify each statement as true or false.

a. $5 \geq 2$ **b.** $0 \geq 1$
c. $5 \leq 5$ **d.** $4 + 6 \neq 6 + 4$

3 Another important set of numbers is the prime numbers. A **prime number** is a natural number greater than 1 that is exactly divisible only by itself and 1. The first 10 prime numbers are

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots,$$

and the list continues indefinitely. It is often useful to write a number as a product of prime factors. To illustrate what this means, consider the terminology associated with the multiplication problem below that uses a raised dot to indicate multiplication.

$$\overbrace{2 \cdot 3 \cdot 5}^{\text{product}}$$
$$\uparrow \quad \uparrow \quad \uparrow$$
$$\text{factors}$$

We call $2 \cdot 3 \cdot 5$ a **product,** and 2, 3, and 5 **factors** in the product. Because $2 \cdot 3 \cdot 5$ equals 30, we express 30 as a product of prime factors by writing $2 \cdot 3 \cdot 5$, and we say this product expresses 30 in **prime-factored form.** A procedure for expressing a number as a product of its prime factors is shown in the next example.

EXAMPLE 4 Express 350 as a product of its prime factors.

Solution Perform successive divisions using prime numbers until the result of the division is a prime number. Step 1 is at the bottom of the calculations.

7	Step 4. The result (7) is a prime number.
$5\overline{)35}$	Step 3. Divide the result (35) by 5.
$5\overline{)175}$	Step 2. Divide the result (175) by 5.
$2\overline{)350}$	Step 1. Divide 350 by 2.

Thus, $350 = 2 \cdot 5 \cdot 5 \cdot 7$. Check the answer by doing the multiplication to obtain 350.

PROGRESS CHECK 4 Express 114 as a product of its prime factors.

4 Prime numbers have important applications in work with fractions. Recall from arithmetic that fractions such as

$$\frac{1}{2}, \quad \frac{6}{18}, \quad \frac{15}{11}, \quad \text{and} \quad \frac{0}{1}$$

are numerals written in the form a/b, where a is called the **numerator** and b is called the **denominator.** The fraction a/b is equivalent to the division $a \div b$, so the denominator of a fraction cannot equal 0 because division by 0 is not defined. (We show why in Section 1.8.) Prime numbers may be used to formulate a general procedure for simplifying fractions. For example, to simplify $\frac{6}{9}$, we note that $6 = 2 \cdot 3$ while $9 = 3 \cdot 3$. Thus,

$$\frac{6}{9} = \frac{2 \cdot \overset{1}{\cancel{3}}}{3 \cdot \underset{1}{\cancel{3}}}.$$

Because 3 is a common factor of both 6 and 9, we may divide out this common factor

Progress Check Answers

3. (a) T (b) F (c) T (d) F

4. $2 \cdot 3 \cdot 19$

by dividing both the numerator and the denominator by 3 to obtain

$$\frac{6}{9} = \frac{2 \cdot 3}{3 \cdot 3} = \frac{2}{3}.$$

This example illustrates a general principle in fractions that is used often.

Fundamental Principle of Fractions

If b and k are not zero, then

$$\frac{a \cdot k}{b \cdot k} = \frac{a}{b}.$$

When we simplify fractions by the fundamental principle, it is important to recognize that we may divide out only *nonzero factors* of the numerator and the denominator. In the next example the directions ask for a fraction in lowest terms. A fraction is in **lowest terms** when there is no natural number besides 1 that is a factor of both the numerator and the denominator. If the numerator and denominator of a fraction are equal (and are not both zero), then the fraction is equal to 1.

EXAMPLE 5 Express each fraction in lowest terms.

a. $\frac{12}{28}$ **b.** $\frac{15}{56}$

Solution

a. Express 12 as $2 \cdot 2 \cdot 3$, and 28 as $2 \cdot 2 \cdot 7$. Then apply the fundamental principle.

$$\frac{12}{28} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 7}$$ Write 12 and 28 in prime-factored form.

$$= \frac{3}{7}$$ Divide out the common factor $2 \cdot 2$ according to the fundamental principle.

In lowest terms, $\frac{12}{28}$ is expressed as $\frac{3}{7}$.

b. Express 15 and 56 in prime-factored form.

$$\frac{15}{56} = \frac{3 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 7}$$

Because 15 and 56 share no common prime factors, $\frac{15}{56}$ is in lowest terms.

Note To check that two fractions are equal, use the principle

$$\frac{a}{b} = \frac{c}{d} \quad \text{provided} \quad ad = bc \quad (b,d \neq 0).$$

For example, we know $\frac{12}{28} = \frac{3}{7}$ is a true statement because $12 \cdot 7 = 28 \cdot 3$ is a true statement.

PROGRESS CHECK 5 Express each fraction in lowest terms.

a. $\frac{42}{63}$ **b.** $\frac{45}{76}$

Example 5 shows that we may use the following procedure to express a fraction in lowest terms.

Progress Check Answers

5. (a) $\frac{2}{3}$ (b) already in lowest terms