STABILITY, STABILIZATION AND CONTROL OF LARGE SCALE SYSTEMS

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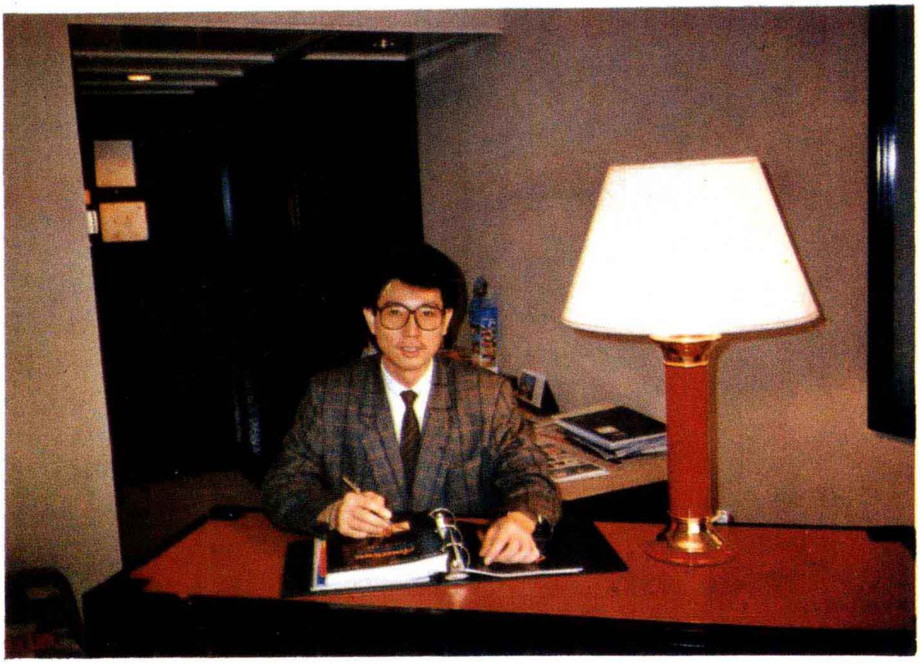
PROFESSOR LIU YONGQING was born in China in 1930. Since his graduation from Mathematics Department of Fudan University in July 1955, he has been a research scientist in Mathematics Institute, Chinese Academy of Sciences (1955—62). And he is now the Chair of System Engineering Research Division, Department of Automation, South China University of Science and Technology. Untill now, he has published 132 papers and 6 monographs on the theory and application of large scale systems at home and abroad, and won 17 prizes awarded by National Education Committee and others. Especially, in 1984 he had been recognized as one of National Scientists with distinguished achievements titled by The Ministry of Labour and Personnel of The State Council.

He is a member of Association for the Advancement of Modelling and Simulation Techniques in Enterprices (AMSE) in France, and of Editorial Board and Teams of 5 AMSE Periodicals, the fellow of International Biographical Association (IBA) in Britain and of International Biographical Institute Research Association (ABIRA) of ABI in America.

His Biography has been listed in many famous records at home and abroad, such as Chinese Science and Technology Personality Dictionary; Current Chinese Science and Technology Achievements Dictionary (1949—89); New China Science and Technology Who's Who, etc., and The International Directory of Distinguished Leadership (2nd edition, Winter, 1987 / 88) by ABI; Who's Who in Australasia and The Far East (1st edition, Winter, 1988); Dictionary of International Biography (XXI edition, 1989 / 90); International Who's Who of Intellectuals (8th edition, 1988 / 89) by IBC in Cambridge, UK.



MR. PAUL K. C. CHIAN A SCHOLAR AND AN ENTREPRENEUR WHO SETS HIS AMBITION AIMING TO BE A GREAT "MACRO SYSTEM ENGINEER EXPERT".

MR. CHIAN WAS BORN IN 1952, GREW UP IN A LITTLE ISLAND COUNTRY "SINGAPORE". AFTER OBTAINED HIS BSC DEGREE IN MECHANICAL ENGINEERING HE FURTHER PERSUED HIS ADVANCED GRADUATE STUDIES IN THREE FAMOUS GRADUATE SCHOOLS IN MANILA. THE ASIAN INSTITUTE OF MANAGEMENT – BMP – MPP, THE UNIVERSITY OF PHILIPPINES – MBA; AND THE DELA SALLE UNIVERSITY MAJOR (MASTER DEGREE) IN MECHANICAL ENGINEERING.

UNDER THE GUIDANCE OF PROF. LIU., MR. CHIAN HAS SUCCESSFULLY COMPLETED THIS " FIRST" JOINT WRITTEN BOOK. THIS BOOK ALSO SERVES AS HIS GRADUATE THESIS OF HIS PHD IN SYSTEM ENGINEERING GRADUATE STUDY IN CHINA GUANGZHOU'S MOST FAMOUS ENGINEERING UNIVERSITY – THE SOUTH CHINA UNIVERSITY.

WITH FURTHER ENCOURAGEMENT FROM HIS FAVORITE PROF. LIU, MR. CHIAN AIMS TO WRITE A "SECOND" BOOK NAMED "ABSOLUTE MACRO SYSTEM FOR NEW CHINA". A BOOK WHICH IN HIS OWN WORDS " MAY IN NEAR FURTURE HELP TO PROVIDE SOME GOOD THOUGHTS TO THE COMMUNITIES AND AIMS TO HELP RAISING THE LIVING STANDARD OF PEOPLE THROUGH THE INTRODUCTION OF GOOD MACRO SYSTEM CONCEPTS AND THOUGHTS".

PREFACE

In 1959, Chinese mathematicians Qin Yuanxun, Liu Yongqing and Wang Muqiu introduced the decomposition concept and method of the stability of large scale systems raised from engineering practice and enlarged the research domain of the stability of large scale systems.

In this monograph, the main works and contribution of Liu Yongqing and his cooperators on the stability, stabilization and control of large scale systems are summarized . Their main works include Liu's 103 papers on the stability, stabilization and control of large scale systems published at home and abroad during 1958 to 1988, the current works of Liu Yongqing and his Ph D student Paul K. C. Chian, and some chapters and paragraphs from Liu's monograph " Theory and Application of Large Scale Dynamic Systems (Vol. 1, Decomposition, stability and structure; Vol.2, Modelling, stabilization and control, which were published in Chinese in 1988 and 1989 respectively).

Chapter one gives an account of the decomposition concept and methods of the stability of large scale dynamic systems first proposed by Professor Qin Yuanxun and the application of this concept and method of Lyapunov function decomposition to engineering practice by Liu Yongqing.

In Chapter two, the extension of parametric domain of stability of large scale systems and the optimazation of parametric domain of stability are introduced . Besides, three different methods for studying the stability of large scale systems are analyzed.

Chapter three presents a new method for analyzing the stability of large scale systems and the criterion of whether all roots of an algebraic equation have negative parts.

Chapter four deals with the basic theorems and formulas of Lyapunov function for discrete systems .

In Chapter five, comparison principle of discrete systems and the stability of discrete large scale systems are dealt with.

In Chapters six and seven, the stability of continuous time lag large scale systems, comparison principle of multidelay systems, and unconditional stability of large scale time-lag systems are introduced.

Chapter eight is concerned with stabilization and suboptimal control of linear constant large scale control systems and linear time-varying large scale control systems.

In Chapters nine and ten, equivalence of linear constant cotrol system with time lags, and the unconditional stabilization and suboptimal control of control systems with time lags are introduced.

Chapter eleven discusses the case of linear stationary discrete control systems with real time-delays and the sufficient and necessary condition of unconditional stabilization of discrete systems with closed loop time-delay when the sampling period is integer or not integer times of transfer time-delays.

In Chapter twelve, unconditional robust stabilization and suboptimal control of large scale interval neutral type control systems with time-delays are dealt with.

We wish to express our deep gratitute to Professors Zhang Zhongjun and Qin Yuanxun for their instruction and support. We would like to thank Associate Professor Cai Mingnan for reviewing and revising the manuscript and to doctoral student Xu Bugun for typing the manuscript.

CONTENTS

CHAPTER 1

DECOMPOSITION CONCEPT AND METHOD OF STABILITY OF LARGE SCALE DYNAMIC SYSTEMS

1.1 Decomposition Concept of Stability

With the development of science and technology, the concept of large scale systems has been proposed. At the beginning, for a complicated so-called "large scale system (LSS)," the main problem was to overcome the difficulties incurred by increasing size and complexity of relevant mathematics models. The problem seems to be too difficult to find a solution by means of modern computers which have the limitation of computational capacity and speed. Therefore, mathematicians attempted to develop an effective method, i.e. the decomposition method of large scale systems.

Primarily, the concept of LSS was as follows: a large scale system was such a system that could be decomposited into several subsystems connected with each other and the characters of whole system might be obtained by combining characters of its subsystems. Unfortunately, till now there has been no common and strict definition of LSS, but it is true that LSS should possess the following charaters: large scale, complicated structure, synthetic function and a number of other factors.

In 1950 it was the first time that the decomposition method, which was successfully utilized in the analysis of electric networks by Kron (Ref.1) was proposed. And in 1973 the thought of decomposition methods in Himelblaus's report (Ref.2) was applied in solving algebraic equations with principal diagonal elements by Gerling early in 1843.

Although the decomposition method may make the LSS problems become more simplified and convenient it is still to have to depend upon the selection of a particular decomposition. And it is very difficult to solve subsystems with ease and without a lot of damage to the overall LSS since sometimes it is unable to express the characters of the LSS accurately by means of decomposing the LSS and taking the combination of its subsystems instead of whole characters of the LSS itself. Thereby, it is clear that during a long period of time there is no big progress on this method in some other fields, especially, in large scale dynamic systems (LSDS) after

Kron's successful work in the analysis of electric networks.

The decomposition theory of large scale dynamic systems is still nowin an early stage of less achievement. Referring to its decomposition, there are two kinds of more important and feasible principles, that is, physical and mathematical decomposition.

When a large scale system can be expressed by a structure of subsystems linked together, which has certain physical meanings, such as in an electric towing control system of ship-lifter two fifth-order nonlinear systems could be decomposited into two subsystems. For this kind of decomposition there are some limitations existed in the aspect of mathematical models, but it is possible not only to simplify numerically, but also at the same time to give out relevant information with regard to the properties of the structure of LSS. Besides, the reason of decomposition might possibly be the need of numerical simplification of mathematics, and it is possible to select variables and mathematical models of subsystems freely and universally. But either before or after the decomposition all physical meanings of variables and the system itself have been eliminated, and it is only possible to be explained through characters of final solutions with regard to the overall large scale system. Both these two aspects of decomposition principles arouse the interest of all and have been paid full attentions.

As early as in 1959, in the design of an aerocraft autopilot Y.X. Qin (Ref.3, 4) decomposited the motion stability of the aerocraft with six degrees of freedom into longitudinal and lateral stability of motion of subsystems with three freedom each, and only considered the stability of longitudinal motion subsystem. It was the first time that the concept of stability decomposition of large scale dynamic systems was proposed from the principle of mathematical and physical decomposition. For the purpose of explaining the forming procedure of the concept with the stability decomposition of large scale dynamic systems proposed by Y.X. Qin, the following examples are given:

$$\dot{x}_1=a_{11}x_1+a_{12}x_2, \quad \dot{x}_2=a_{21}x_1+a_{22}x_2 \tag{1.1}$$

and its two isolated subsystems are

$$\dot{x}_1=a_{11}x_1, \quad \dot{x}_2=a_{22}x_2 \tag{1.2}$$

where $a_{ij}(i,j=1,2)$ are constants. The characteristic equation of (1.1) is

$$D(\lambda)=\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = \lambda^2+a\lambda+b=0 \tag{1.3}$$

where

$$a=-(a_{11}+a_{22}), \qquad b=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Example 1. Achieve the asymptotical stability of compound system (1.1) through the asymptotical stability of subsystem (1.2).
SOLUTION: Assume $a_{11}<0$, $a_{22}<0$. Then the trivial solutions of subsystem (1.2) are asymptotically stable. Let $E_1=\max(/a_{12}/,/a_{21}/)$. When $E_1<\nabla_1=\sqrt{a_{11}a_{22}}$, $b>0$. Since $a=-(a_{11}+a_{22})>0$, two characteristic roots of (1.3) have negative real parts. Therefore, when $E_1<\nabla_1$, the trivial solutions of compound system (1.1) are asymptotically stable.

Example 2. Let $a_{11}<0$, $a_{22}<0$. Prove that the trivial solutions of subsystem (1.2) are asymptotically stable.
SOLUTION: Let $E_2=\min(/a_{12}/,/a_{21}/)$ and $a_{12}a_{21}>0$. When $E_2>\nabla_1$, $b<0$, since $a=-(a_{11}+a_{22})>0$, the characteristic equation (1.3) has only one positive real root. Thereby, when $E_2>\nabla_1$, the trivial solutions of compound system (1.1) are unstable.

Example 3. Prove the compound system (1.1) is asymptotically stable through the unstability of subsystem (1.3).
SOLUTION: If $a_{11}<0$, $a_{22}>0$ (or $a_{11}>0$, $a_{22}<0$), then the trivial solutions of subsystem (1.2) are unstable. Let $a_{11}+a_{22}<0$, $a_{12}a_{21}<0$. And when $E_2>\nabla_1=\sqrt{/a_{11}a_{22}/}$, $b>0$. Since $a=-(a_{11}+a_{22})>0$, both two characteristic roots of the characteristic equation (1.3) have negative real parts. Thereby, when $E_2>\nabla_1$, the trivial solutions of compound system (1.1) are asymptotically stable.

Example 4. Prove the unstability of compound system (1.1) through unstability of subsystem (1.2).
SOLUTION: Let $a_{11}>0$, $a_{22}<0$ (or $a_{11}<0$, $a_{22}>0$). Then the trivial solutions of subsystem (1.2) are unstable. If $a_{11}+a_{22}>0$, then $a=-(a_{11}+a_{22})<0$. No matter what values of a_{12} and a_{21} are taken, at least the characteristic equation (1.3) has one characteristic root with positive real part. Then the trivial solutions of compound system (1.1) are unstable.

From example 1-4, four cases can be explained:

(I) By asymptotic stability of trivial solutions of subsystem to achieve the asymptotic stability of trivial solutions of LSS (Example 1);

(II) By asymptotic stability of trivial solutions of subsystem to show trivial solutions of large scale system are unstable (Example 2);

(III) By unstability of trivial solutions of subsystem to prove trivial solutions of large scale system are asymptotic ally stable (Example 3);

(IV) By unstability of trivial solutions of subsystem to get the unstability of trivial solutions of large scale system (Example 4).

Based on above four cases, Y.X. Qin first proposed the following concepts of stability decomposition for a large scale dynamic system:

(1) The stability of a large scale system may be replaced by the stability of trivial solutions of subsystem (such as (I) and (IV));

(2) When some subsystems are stable and the others are unstable, how to deduce the stability of overall large scale system by means of choosing parameters properly (such as (III)).

Seven years later, Bailey (1966, Ref.5) proposed similar concept of stability decomposition.

Factually, there is a problem of how to apply above concept of stability decomposition of large scale systems proposed by Y.X. Qin to general theory of mathematics and to solving practical engineering problem.

In 1959 Wang Muqiu first achieved the stability decomposition of linear differential equations based on the criterion that characteristic roots of general linear steady systems have negative real parts, and on the Routh-Hurwitz principal subdeterminants (Ref.3,4).

In engineering techniques, it is often necessary to determine the stable domains of a nonlinear system. In 1959, when carrying out research on electric towing control system of San Xia ship-lifter, Liu Yongqing (Ref.6) first made an estimation of stable domain of two fifth-order (and two second-order) simultaneous nonlinear systems that could be solved by means of decompositing the systems stability into the stability of two isolated subsystems with lower dimensions. The procedure is as follows: to give out the Lyapunov function and its stability of each isolated subsystem, seperately; and to take the sum of Lyapunov function of the subsystems as a scalar function of these two simultaneous fifth-order (and two second -order) nonlinear differential equations; then to achieve the stability of two simultaneous fifth-order (and second-order) nonlinear systems by limiting the bound of interconnected terms of subsystems.

Later, Liu Yongqing defined the above approach as the decomposition method of Lyapunov function, namely, the method of the scalar sum of Lyapunov function. And in 1962, he applied this method to a synchronous servo-control system proposed by Shu Song-gui and Fan Ming-shi (Ref.7,8), and, furthermore, to the stability decomposition of general steady large scale systems (Ref.9).

In 1970, Thompson (Ref.10) also presented the similar method of scalar Lyapunov function.

On the basis of the concept of stability decomposition of large scale dynamic systems proposed by Y.X. Qin, and by combining two principles of large scale systems decomposition: physical and mathematical decompositions,some simple examples will be given in the following sections to describe the meanings and establishing procedure of decomposition method of Lyapunov function.

1.2 Decomposition Method of Lyapunov Function

Now we consider a simplest second-order compound linear steady system

$$\dot{x}_1=-a_{11}x_1+a_{12}x_2, \quad \dot{x}_2=a_{21}x_1-a_{22}x_2 \tag{1.4}$$

And its two isolated subsystems are

$$\dot{x}_1=-a_{11}x_1, \quad \dot{x}_2=-a_{22}x_2 \tag{1.5}$$

Example 5. Assume $a_{11}>0$, $a_{22}>0$. Construct the Lyapunov function of (1.5) as $V_i=x_i^2$ $(i=1,2)$. And taking the time derivative of V_i along the trajectories of (1.5), we have

$$\dot{V}_i=2x_i\dot{x}_i=-2a_{ii}x_i^2<0 \quad (i=1,2)$$

that is, the trivial solutions of subsystem (1.5) are asymptotically stable. Take $E_1=\max(|a_{12}|,|a_{21}|)$.

If we take the sum of Lyapunov function of two subsystems (1.5)

$$V=V_1+V_2=x_1^2+x_2^2 \tag{1.6}$$

as the Lyapunov function of compound system (1.4), and derive the derivative of V in terms of t along the trajectories of (1.4), then we have

$$\dot{V}_{(1.4)}=2x_1\dot{x}_1+2x_2\dot{x}_2=2x_1(-a_{11}x_1+a_{12}x_2)+2x_2(a_{21}x_1-a_{22}x_2)$$

$$\leq-2(a_{11}x_1^2+a_{22}x_2^2)+2|a_{12}||x_1||x_2|+2|a_{21}||x_1||x_2|$$

$$\leq -2(a_{11}x_1^2+a_{22}x_2^2)+2E_1(x_1^2+x_2^2)$$

When $E_1<\Delta_1=\min(/a_{11}/,/a_{22}/)$, $\dot{V}_{(1.4)}<0$. Therefore, we can prove that the trivial solutions of compound system (1.4) are also asymptotic ally stable.

Example 6. If $a_{11}<0$, $a_{22}<0$, prove trivial solutions of subsystem (1.5) are unstable. Since $V=x_1^2+x_2^2$, using similar calculation, we can get

$$\dot{V}_{(1.4)}\geq 2(-a_{11}x_1^2-a_{22}x_2^2)-2E_1(x_1^2+x_2^2)$$

When $E_1<\Delta_1=\min(/a_{11}/,/a_{22}/)$, $\dot{V}_{(1.4)}>0$. Then we can conclude that the trivial solutions of compound system (1.4) are unstable.

Summerizing above decomposition method of Lyapunov function for a general large scale system

$$\dot{x}=f(t,x)+g(t,x)=h(t,x), \qquad (A)$$

we suppose that it can be decomposited into m interconnected subsystems

$$\dot{x}_i=f_i(t,x_i)+g_i(t,x) \quad (i=1,2,\ldots,m) \qquad (A)_1$$

and its isolated subsystems are

$$\dot{x}_i=f_i(t,x_i) \quad (i=1,2,\ldots,m) \qquad (B)$$

where $g_i(t,x)$ is the interconnected term of the subsystems. Usually, the problem of stability decomposition of large scale system (A) will be solved according to following three steps.

Step (I) Analyze isolated subsystems. From one of Lyapunov function of subsystem (B) we can get the bound of an expression of $\dot{V}_{i(B)}$. In Example 5, $V_i=x_i^2>0$, $\dot{V}_i=-2a_{ii}x_i^2<0$ $(i=1,2)$.

Step (II) Analyze interconnected terms of subsystems to get the bound of an expression of $g_i(t,x)$; In Example 5, the bound of interconnected terms among subsystems is $2/a_{12}//x_1//x_2/+2/a_{21}//x_1//x_2/\leq 2E_1(x_1^2+x_2^2)$.

Step (III) One can be able to transform the stability of trivial solutions of subsystems into the stability of whole large scale system by means of combination. In fact this can be achieved only by constraining the bound of interconnected terms amongst subsystems. In Example 5, the bound of interconnected terms among subsystems is E_1. When $E_1<\Delta_1=\min(/a_{11}/,/a_{12}/)$ is constrained the asymptotic stability of trivial solutions