# EMIL WOLF

EDITOR



VOLUME 45

# CONTRIBUTORS

V. Baby, N. Bokor, H. Cao, N. Davidson, W. Gao, D.J. Gauthier, G. Gbur, I. Glesk, P.R. Prucnal, B.C. Wang, L. Xu, J. Yin, Y. Zhu

# PROGRESS IN OPTICS

# **VOLUME 45**

EDITED BY

## E. Wolf

University of Rochester, N.Y., U.S.A.

#### Contributors

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**VOLUME 45** 

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# **Preface**

This volume contains six review articles on diverse topics that have become of particular interest to optical scientists and optical engineers in recent years.

The first article, by N. Davidson and N. Bokor, reviews researches on the anamorphic shaping of laser beams and of diffuse light whose input or output is substantially elongated along one direction. Such elongated beams have come to be of special interest in recent years with the appearance of high-power laser diode bars, whose excellent properties are offset by their poor beam quality and by the fact that the output beam profile is highly anamorphic (with typical aspect ratio 1000:1) and hence unsuitable for many applications. Several techniques have been developed to collimate and shape the output beams of such laser diode bars into symmetric spots. Anamorphic beam shaping has also been used to concentrate symmetrical fields such as solar radiation into very narrow lines, for use in heating water pipes, for side-pumping solar lasers, and in optical metrology (e.g. for improving resolution in surface profile measurement and high-resolution spectrometry). The article presents a review of the main reflective, refractive, diffractive, and adiabatic techniques for anamorphic beam characterization.

The second article, by I. Glesk, B.C. Wang, L. Xu, V. Baby and P.R. Prucnal, presents a review of recent progress in the development of ultra-fast all-optical switching devices with various applications for future optical networks. The operation principle and performance of different all-optical switches based on nonlinearities in optical fiber semiconductor optical amplifiers (SOA) and passive waveguides are discussed. Special attention is paid to interferometric SOA-based all-optical switches. Several testbed demonstrations are described.

The next article, by J. Yin, W. Gao and Y. Zhu, is concerned with the generation of dark hollow beams and their applications. Such beams have been used to form optical pipes, optical tweezers, atomic pipes, atomic tweezers, atomic refrigerators, and atomic motors. They can be applied in the accurate, non-contact manipulation and control of microscopic particles, such as biological cells, neutral atoms and molecules. The principles and experimental methods for generating various types of dark hollow beams are discussed. Applications of such beams in optical traps for microscopic particles including biological cells

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are also discussed, as are recent studies of dark hollow beams in atom optics and coherent matter-wave optics.

The fourth article, by D.J. Gauthier, presents a review of a new type of quantum oscillator known as two-photon laser. Such devices are based on two-photon stimulated emission processes whereby two photons incident on an atom induce it to drop to a lower energy state and four photons are scattered. This kind of laser has been realized only relatively recently by combining cavity quantum electrodynamics experimental techniques with novel nonlinear optical interactions. Research on two-photon quantum processes, leading to the development and characterization of two-photon masers and lasers, is discussed. The unusual quantum-statistical and nonlinear dynamical properties predicted for the device are also reviewed.

The subsequent article, by G. Gbur, discusses a rather old but poorly understood subject, the so-called non-radiating sources and the related concept of non-visible objects. These are certain extended charge-current distributions that may oscillate without generating radiation. Such sources have many intriguing mathematical and physical properties whose existence is intimately related to nonuniqueness of the solution of the so-called inverse source problem. The current state of understanding of such sources is discussed and they are compared with other classes of "invisible objects".

The concluding article, by H. Cao, is concerned with random lasers. These are unconventional lasers whose feedback is provided by disorder-induced scattering. Random lasers may be separated into two categories: those with coherent feedback and those with incoherent feedback. In this article both types are discussed, as well as measurements of a variety of properties of such devices, such as the lasing threshold, lasing spectra, emission pattern, dynamical response, photon statistics and speckle patterns. Furthermore, investigations regarding the relevant length scales are described. Large disorder in the lasing material leads to spatial confinement of the lasing modes, which is the foundation for the micro-random laser. Some theoretical models of random lasers with coherent feedback are briefly introduced. Such research helps the understanding of the interplay between light localization and coherent amplification.

In view of the broad coverage presented in this volume, it is hoped that many readers will find in it something that is of particular interest to them.

**Emil Wolf** 

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April 2003

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# Chapter 1

# Anamorphic beam shaping for laser and diffuse light

by

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## § 1. Introduction

Beam shaping constitutes a large and important field in optics. For many applications the beam must be shaped in one transverse direction substantially differently than in the other transverse direction. In what follows, the term anamorphic beam shaping designates techniques in which the one-dimensional beam quality in one transverse direction is improved at the expense of reducing the one-dimensional beam quality in the orthogonal direction. In a broader sense, techniques for one-dimensional beam shaping would include onedimensional concentration of diffuse light with a curved diffractive element (Bokor, Shechter, Friesem and Davidson [2001], Bokor and Davidson [2001c, 2002b]) and one-dimensional diffuse beam shaping using a single reflection on a curved step mirror (Bokor and Davidson [2001a]). The simplest element capable of one-dimensional beam shaping is a cylindrical lens. In the examples listed above the one-dimensional beam-quality factors in the two orthogonal transverse directions remain the same. In this article we will not consider such techniques. Comprehensive reviews on concentration of diffuse beams, with special emphasis on non-imaging concentrators and solar energy applications, have been provided by Winston and Welford [1989] and Bassett, Welford and Winston [1989].

# 1.1. Diffuse light and its phase-space representation

We first define a number of concepts related to diffuse light. The term diffuse light refers to beams for which the diffraction-limited angles and spot sizes are much smaller than the diffusive ones. This means that the transverse (spatial) coherence length of the beam is much smaller than its size, and that we can use the geometrical optics approximation and ray-tracing techniques to describe the beam propagation. The longitudinal (temporal) coherence is less relevant to the scope of this chapter, except when a broad wavelength range may cause considerable chromatic aberrations, in particular for diffractive optical elements. An effectively spatially incoherent, diffuse light can be formed even for laser experiments, by sending the laser beam through a rotating diffuser.

Following Winston and Welford [1989], we define the four-dimensional phase-space volume (PSV) (often referred to as "étendue") of a diffuse beam as

$$PSV = \int_{x} \int_{y} \int_{\sin \alpha_x \sin \alpha_y} dx' \, dy' \, d(\sin \alpha_x') \, d(\sin \alpha_y'), \qquad (1.1)$$

where x and y are the sizes, and  $\alpha_x$  and  $\alpha_y$  the diffusive angles, of the beam in two orthogonal directions (x and y are perpendicular to the direction of beam propagation). For a beam with uniform and space-invariant diffusivity (explained below) we define the phase-space areas (PSAs) in the x- and y-directions as follows:

$$PSA_x = \sqrt{\pi} x \sin \alpha_x, \qquad PSA_y = \sqrt{\pi} y \sin \alpha_y. \tag{1.2}$$

Note that, in general, a diffuse beam is represented by a non-uniform distribution function  $I(x, y, \alpha_x, \alpha_y)$  in four-dimensional phase-space. Space-invariant diffusivity means that the distribution function can be written as a product of two distributions:  $I_1(x,y) \cdot I_2(\alpha_x, \alpha_y)$ . In this chapter we will mostly assume uniform distributions with Cartesian symmetry that are thus characterized by the four quantities  $x, y, \alpha_x$  and  $\alpha_y$ . This largely simplifies the notation and captures most of the basic effects and ideas that we will describe. For the extension to non-uniform distributions (which are quite common in practice), the distribution as well as the conservation laws should be described in different terms than in the uniform case, e.g., by the RMS sizes of the distribution function.

Optical brightness is defined as:

$$B = \frac{P}{\text{PSV}},\tag{1.3}$$

where P is the optical power transmitted by the beam. A fundamental conservation law – closely related to the second Law of thermodynamics – states that for spatially incoherent light and passive optical transformations B cannot increase. In the optimal case brightness is conserved. For lossless transformations P is constant and the conservation of brightness implies a conservation of PSV. This so-called étendue invariance (Winston and Welford [1989]) is the main guiding principle in the design of diffuse beam-shaping techniques.

In the paraxial approximation [ $\sin \alpha_x \ll 1$ ;  $\sin \alpha_y \ll 1$ ] the PSV can be written as

$$PSV \approx A\Omega, \tag{1.4}$$

where  $\Omega$  is the solid angle of divergence and A is the cross-sectional area of the beam. Recently it was proposed that the notation  $M^4$  should be used to describe

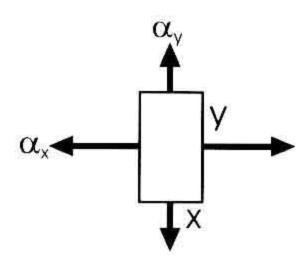


Fig. 1.1. Phase-space representation of a space-invariant, uniform, rectangular diffuse beam: the size of the rectangle represents the spatial dimensions of the beam, and the length of the arrow for each direction represents the diffuse angle.

the two-dimensional beam-quality factor, defined in the paraxial case as (Graf and Balmer [1996]):

$$M^4 = M_x^2 M_y^2 = \frac{A\Omega}{\lambda^2},\tag{1.5}$$

where – in the practical case of quasi-monochromatic beams –  $\lambda$  is the center wavelength, and  $M_x^2$  and  $M_y^2$  are the one-dimensional beam-quality values in the x- and y-directions, respectively, defined as:

$$M_x^2 = \frac{x \cdot \sin \alpha_x}{\lambda}, \qquad M_y^2 = \frac{y \cdot \sin \alpha_y}{\lambda}.$$
 (1.6)

It is obvious from eqs. (1.2) and (1.4)–(1.6) that in the paraxial case of a monochromatic beam the beam-quality factor and the phase-space volume are equivalent descriptions of the beam. Even in the non-paraxial case the symmetry condition  $M_x^2 = M_y^2$  is equivalent to the symmetry condition  $PSA_x = PSA_y$ . Since all the techniques described below contain these symmetry requirements either at the input or at the output, we will often use the concepts of  $M_x^2$  and  $M_y^2$  in the text – because of their wide acceptance – even for beams with large diffusive angles. For diffraction-limited beams – e.g., in the x-direction –  $M_x^2$  takes its optimum value:  $M_x^2 = 1$ . For diffuse beams  $M_x^2 \gg 1$ . Note that the beam quality  $M^4$  is related to the focusability of the beam, i.e. it gives the minimum spot size to which it can be focused, in units of the diffraction-limited area  $\lambda^2$ .

Following Davidson and Khaykovich [1999], we now introduce a graphical method to depict the four-dimensional PSV of a diffuse light distribution. A beam with a space-invariant diffusivity can be represented with a simple diagram, as shown in fig. 1.1. The spatial dimensions are represented by the size of the rectangle, and the length of the arrow for each direction represents the diffuse angle. The beam represented in fig. 1.1, for example, has larger dimensions in the y-direction than in the x-direction, however, its diffusivity is larger in the x-direction than in the y-direction. We will use this type of phase-space representations – called phase-space diagrams – throughout the text.