



普通高等教育“十三五”规划教材

PUTONG GAODENG JIAOYU “13·5” GUIHUA JIAOCAI

A Lecture Note on Theoretical Mechanics

(Simplified Edition)

Jianlin Liu

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2016

内 容 提 要

This textbook mainly includes three sections, i. e. Statics, Kinematics and Kinetics. It has filled the gaps towards the foreign students, which can provide more basic trainings on the elementary knowledge of mechanics. The architecture is well organized, and the hierarchy on the force, motion, and dynamics is clearly demonstrated. More examples from our daily life and industries are displayed in this book.

This book is suitable to the international students, whose education experience is different from that of Chinese students. It can also be adaptable to a broad range of majors, including aerospace, mechanical engineering, petroleum engineering, automobiles, civil engineering, material science and engineering, etc.

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Preface

As we all know, the word “mechanics” is quite similar to the word “mechanical”, but we can see that it has a special letter “s”, which is the abbreviation of “science”. This indicates that, mechanics is an independent subject. In fact, mechanics was closely related with mechanical engineering in the ancient time, meaning “manufacture” and “design”; but it also has the theoretical characteristics of rigorous logistic derivation. Then a critical question is “What’s mechanics?” Generally speaking, mechanics is a subject to investigate the laws of force and motion. Engineering mechanics is aimed to solve engineering design problems in engineering areas, such as aircrafts in aerospace, automobiles, civil engineering, petroleum engineering, material preparation, mechanical engineering, and architecture engineering.

The origins of mechanics are lost in antiquity. There were numerous famous scholars who made contributions to engineering mechanics. For example, Aristotle (384 BC-322 BC), a scholar of encyclopedia, gave the first thinking to motion, velocity and force. The Great scientist Archimedes (287 BC-212 BC), an encyclopedic scholar, proposed a lot of physical laws and mechanical equipments on mechanics. For instance, the famous Archimedes law of buoyant force, Archimedes lever rule, and calculation methods of volume, area and

center of mass on geometric bodies were all given by Archimedes. Actually he was regarded as one of the four greatest mathematicians in history, for he gave the definition of limitation in mathematics. Moreover, Simon Stevin (1548 ~ 1620) proposed the principle of virtual work, and he was viewed as the father of Statics. He also gave the parallelogram law on force summation. In addition, the mathematician Johannes Kepler (1571 ~ 1630) proposed the three motion laws of planet. Another encyclopedic scientist is Leonardo Da Vinci (1452 ~ 1519), who drew a lot of aircrafts (as shown in Fig. 1.1), automobiles, bicycles and some other mechanical equipment in his drafts, which were not published in his era. Da Vinci also explored the strength of materials, and pointed out the importance of flaws in materials. Certainly he was also a great painter with a lot of famous works, such as *Mona Lisa* and *the Last Supper*. Shortly after Da Vinci, the great mechanics scholar, Galileo Galilei (1564 ~ 1642) was born. He was viewed as the father of classical mechanics and experiments, who introduced the new concept of acceleration, which depicts the variation ratio of the velocity. As a consequence, he was thought of as the father of dynamics. Galileo was also one of the predecessors who explored the

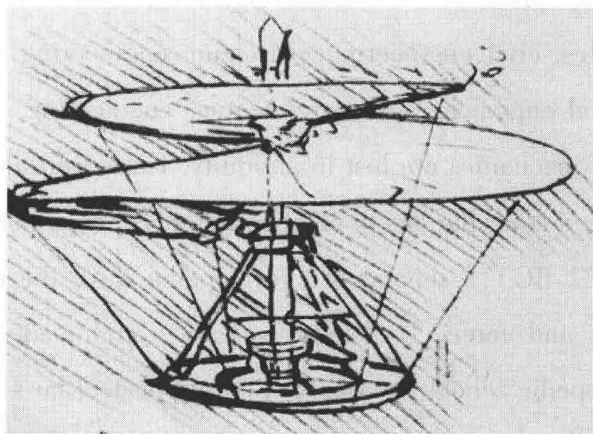


Fig. 1.1 Aircraft designed by Da Vinci

secret of material strength. He performed the first experiment all over the world to examine the strength of a cantilever, which was schematized in Fig. 1.2. Based upon the experimental results, he proposed the so-called “first strength theory”, still adopted in the textbook of today. In what follows, several French scientists were eager to advance the development of mechanics. Among others, René Descartes (1596 ~ 1650) who was also a philosopher, proposed the concept of “momentum” and “theorem of momentum”. One of his important contributions is the introduction of Cartesian coordinate system, which laid the foundation of analytical geometry. He was also concerned with optics, universe evolution, and biological system of human body. One of his peers, Blaise Pascal (1623 ~ 1662), who was also a genius, proposed the law of Pascal pressure. Moreover, he was the grandfather of computer and probability theory in mathematics. Especially, he was a famous philosopher and a prose writer. Nearly at the same time, a neglected scientist Pierre-Louis Moreau de Maupertuis (1698 ~ 1759) proposed the principle of least action, which was used to analyze the transmission of light, then extended to mechanics. Following these giants, an epochal scholar, Isaac Newton (1643 ~ 1727)



Fig. 1.2 The first cantilever experiment by Galileo

(Fig. 1.3) was born. He concluded the three Newton's laws of motion, Newton's law of universal gravity, Newton optics, Calculus, etc. He was regarded as one of the greatest scientists in the human history. He was so famous in the scientific world that he was awarded many honors before he died. In fact, this great giant was fighting with his competitors all through his life, among which including Robert Hooke (1635 ~ 1703), Gottfried Wilhelm Leibniz (1646 ~ 1716), and Christiaan Huygens (1629 ~ 1695). The elaborate Hooke's law was named by Robert Hooke, but it has been verified that it was first proposed by a Chinese scholar Xuan Zheng (郑玄, 127 ~ 200) in the Han Dynasty. Leibnitz was another founder of calculus as the biggest competitor of Newton, and he also proposed the theorem of kinetic energy. He designed the first machine to perform the multiplication operation, which had a great influence on computer. He was so learned that he knew law, diplomacy, metallurgy, chemistry, physics, and philosophy. After Newton, a lot of familiar names appeared, including Leonhard Euler (1707 ~ 1783), J. L. Lagrange (1736 ~ 1813), J. C. F. Gauss (1777 ~ 1855), Augustin L. Cauchy (1789 ~ 1857), P. S. Laplace (1749 ~ 1827), Simeon D. Poisson (1781 ~ 1840), William R.



Fig. 1.3 Newton's apple

Hamilton (1805 ~ 1865), Claude-Louis Navier (1785 ~ 1836), Saint Venant (1797 ~ 1886), George G. Stokes (1819 ~ 1903), etc. These scholars further laid the foundations of the mechanics building. For example, Euler proposed the stability theory of slender structures and motion equation of rigid bodies (as shown in Fig. 1.4), Lagrange set up the Lagrange equation, which paved a new way to analytical mechanics, and Gauss proposed the principle of least constraint and advanced differential geometry. Some other theorems and laws are also related with these names, such as Bernoulli equation, Cauchy stress, Cauchy's theorem, Poisson's ratio, Poisson's distribution, Laplace equation, Hamilton principle, Navier-Stokes equation, principle of Saint Venant. In this period, the fundamental theory of continuum mechanics was built based upon these pioneering contributions.

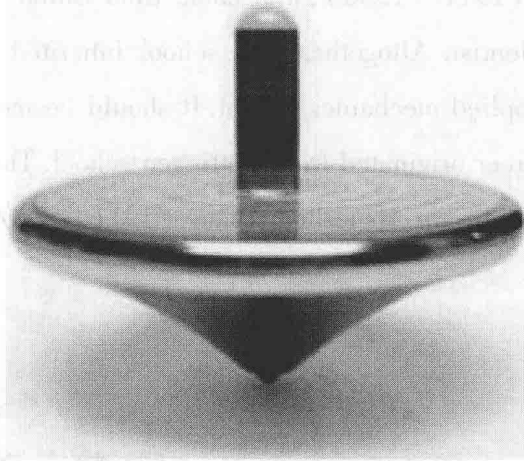


Fig. 1.4 Rotation of a gyroscope

The work mentioned above was actually the elementary theory of applied mechanics. During World War I, a lot of scientists had some new understandings on the importance of mechanics to weapons and industries. The famous mathematician Felix Klein (1849 ~ 1925), who was the mathematics

leader in Göttingen University, proposed the spirit of applying mathematics to solve engineering problems. He made the applied mechanics scholar Ludwig Prandtl (1875 ~ 1953) move to Göttingen to design one of the earliest wind tunnels in the world. Prandtl established a research institute in Göttingen, aiming to investigate applied mechanics issues in engineering. His most famous theory, the boundary layer theory made the Navier-Stokes equation simplified, which was the basic theory in the aircraft design. Moreover, Prandtl cultivated a lot of scholars in mechanics, such as S. Timoshenko (1878 ~ 1971), who wrote a series of popular textbooks which were still adopted nowadays; Theodore von Kármán (1881 ~ 1963), who later went to Caltech to build another center of applied mechanics; William Prager (1903 ~ 1980), who was an applied mathematician in Brown University. Prandtl had only one female student, Shijia Lu (1911 ~ 1986), who came from China and later also became a famous scientist. Altogether, this school inherited from Prandtl was called Göttingen applied mechanics school. It should be mentioned that, China's mechanics career originated from Göttingen school. The founders of Chinese mechanics career are Hsueshen Tsien (1911 ~ 2009) (Fig. 1.5),



Fig. 1.5 Prandtl, Hsueshen Tsien and von Kármán

Peiyuan Zhou (1902 ~ 1993), Weichang Qian (1912 ~ 2010) and Yonghuai Guo (1909 ~ 1968). Peiyuan Zhou was the student of Werner K. Heisenberg (1901 ~ 1976) and Albert Einstein (1879 ~ 1955), whose majors are turbulence and theory of relativity. Except Prof. Zhou, the other three scholars were all students of Prof. von Kármán in Caltech. When they returned China, they led the rocket, satellite and atomic bomb careers, and also laid the foundations of mechanics education and research in China. These four scholars were also the founders of the Chinese Society of Theoretical and Applied Mechanics.

It can be seen that mechanics is not far away from us; in fact it exists widely in nature, industry and our daily life. We always face so many questions on the secret of nature: Why does the bird fly in the sky freely, and why is it so difficult for an airplane to fly? Why can the fish swim fluently in water, and why is it so hard for a ship to swim? Why can some creatures jump on the liquid surface and why is it impossible for a human to move on water? How to design a bridge or a building in prevention of wind, snow and earthquake? If there is a crack in a machine or a ship, can it still work? If can, how long will it last? How to protect the passengers in a car when crashed? How to design a structure with least materials and lowest cost? All these questions are related with mechanics. In the current education system of China, the class of Engineering Mechanics includes two portions, namely Theoretical Mechanics and Mechanics of Materials (or Strength). Mechanics of materials is aimed to investigate the deformation and stress of structures, which will be introduced after the current task. We mainly concentrate on Theoretical Mechanics in this course, which deals with the force equilibrium and motion laws on rigid bodies.

A rigid body is only a perfect model, which is assumed to be without expe-

riencing deformation under the action of external forces. In other words, the distance of two arbitrary points in the rigid body never changes, and the shape of the whole rigid body does not alter. In the practical world, it is impossible to find a real rigid body. However, when the deformation is not important to the property of the problem, it can be neglected and the object can be thought of as a rigid body.

The class of Theoretical Mechanics embraces three sections, namely, *Statics*, *Kinematics* and *Kinetics*. The latter two sections are often named as *Dynamics*. Due to so colorful contents of theoretical mechanics, it is not easy to introduce all the knowledge in this short course, including only 48 lessons. Another fact we should face is that not all the students are in the same level. Therefore, we will give the fundamental knowledge in the second chapter, in order to make the students recall the past equations and formulas. We then simplify most knowledge of Theoretical Mechanics, but it still includes three sections, i. e. Statics, Kinematics and Kinetics. From this textbook, the author expects the students to master the skill of force analysis, writing out the equilibrium equations, velocity analysis of rigid body, and using the fundamental laws in dynamics.

In Statics, we only focus on the force analysis, force group simplification and equilibrium conditions of the rigid body. In Kinematics, we analyze the motion laws of the particle and rigid body from the viewpoint of geometry, not considering forces. In the last portion, i. e. Kinetics, we investigate the relations between the motion and external force by introducing the theorems in dynamics, i. e., Newton's second law, theorem of momentum, theorem of angular momentum, and theorem of kinetic energy.

We thank the useful comments from Dr. Dongying Liu, Dr. Jing Li, Miss

Caixia Hu, Miss Xian Han, and the graduate students with the names of Jing Sun, Pingcheng Zuo, Gaofeng Cao, Shanpeng Li and Yulong Gong. Although we have tried our best to write this lecture note, there may still be some errors. We are expecting the comments from all the readers, and then we hope to give a better version of this lecture note in the near future.

With my best regards,

Jianlin Liu

Department of Engineering Mechanics

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January, 2016

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Preliminary Knowledge

2.1 Trigonometric function

We have already learned the trigonometric functions in the middle school stage, and here let's first have a brief overview about these kernel definitions. We can see that in the forthcoming content of this curriculum, it is quite necessary to master these definitions and calculations very fluently.

We consider a right triangle with an acute triangle, where its three sides are a , b and c , as schematized in Fig. 2.1. The angle between b and c is θ .

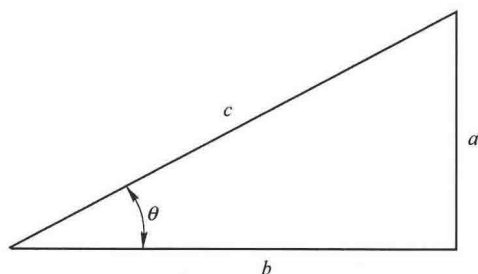


Fig. 2.1 A right triangle

According to Fig. 2.1, the most common trigonometric functions and corre-

lated relations are formulated as

$$\sin\theta = \frac{1}{\csc\theta} = \frac{a}{c},$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{b}{c},$$

$$\tan\theta = \frac{a}{b} = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta},$$

$$\sin^2\theta + \cos^2\theta = 1.$$

In practice, if the side length c or b is already known, we normally use the following formats:

$$a = c\sin\theta,$$

$$b = c\cos\theta,$$

$$a = b\tan\theta.$$

Furthermore, we should remember the actual values of the trigonometric functions about the special angles, such as 0° , 30° , 45° , 60° , 90° , 120° , 150° , and 180° . The detailed relations are listed as follows:

$$\sin 30^\circ = \cos 60^\circ = \sin 150^\circ = \frac{1}{2},$$

$$\cos 30^\circ = \sin 60^\circ = \sin 120^\circ = \frac{\sqrt{3}}{2},$$

$$\sin 45^\circ = \cos 45^\circ = \sin 135^\circ = \frac{\sqrt{2}}{2},$$

$$\sin 0^\circ = \cos 90^\circ = \sin 180^\circ = 0,$$