HEP World's Classics

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热的解析理论

JOSEPH FOURIER

TRANSLATED, WITH NOTES, BY
ALEXANDER FREEMAN

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THE

ANALYTICAL THEORY OF HEAT

BY

JOSEPH FOURIER.

TRANSLATED, WITH NOTES,

BY

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FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

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PREFACE.

In preparing this version in English of Fourier's celebrated treatise on Heat, the translator has followed faithfully the French original. He has, however, appended brief foot-notes, in which will be found references to other writings of Fourier and modern authors on the subject: these are distinguished by the initials A. F. The notes marked R. L. E. are taken from pencil memoranda on the margin of a copy of the work that formerly belonged to the late Robert Leslie Ellis, Fellow of Trinity College, and is now in the possession of St John's College. It was the translator's hope to have been able to prefix to this treatise a Memoir of Fourier's life with some account of his writings; unforeseen circumstances have however prevented its completion in time to appear with the present work.

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or

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