Wavelets In Physics

Edited by: Li-Zhi Fang & Robert L. Thews

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Preface

In the spring of 1996, the late Peter Carruthers proposed the initiation of a new course in the Department of Physics at the University of Arizona, entitled "Wavelets in Physics". His draft course syllabus contained the following assessment:

> *Wavelet analysis has become prominent of late, possessing and surpassing the merits of Fourier analysis and fractal thinking, being simultaneously useful in many fields and pretty mathematically.*

The topics to have been discussed in this course included applications of wavelets in various areas, such as pattern analysis, statistical physics, field theory, computational physics and large scale structures of the universe. We were invited by Peter to lecture on some of these topics in his class. However, this proposal was never realized. Peter met with a serious automobile accident in the fall of 1996, and passed away August 3, 1997.

Peter Carruthers was our dear friend and an enthusiastic colleague. It is not our intention to include here all of his contributions to physics and non-physics issues, but to survey only one of his pursuits of physics: he was a vigorous proponent of the applications of wavelet analysis in theoretical physics. Since 1992 when discrete wavelet analysis, i.e. the wavelet transformation via orthogonal and complete bases, was first shown to be possible, Peter's influence convinced several of his colleagues to enter and develop this new field. He had planned to write or edit a book on wavelets in physics subsequent to the initial offering of his course. Unfortunately, Peter was not afforded the time required to complete this task.

It is now possible to provide for our physics colleagues and students an introduction to the techniques and present status of wavelet applications in physics, through a compilation of initial results in various areas. This is our motivation for editing this book. Much of the material presented here actually originated from our draft notes intended for lectures in Peter's proposed class. Therefore, it is fitting and appropriate that we dedicate this work to the memory of our friend Peter Carruthers.

We are grateful to Mrs. Lucy Carruthers for providing Peter's painting "faculty meeting" for the cover of this book.

Li-Zhi Fang
Robert L. Thews
Tucson, August 21, 1998

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Contents

Peter Carruthers' Notes on Wavelet Analysis

A. Wavelet Analysis of Music, Art and Poetry

The wonderful achievements in music, art and poetry have attracted a great deal of scholarly attention but rarely in a unified form. Neither have these analyses been quantitative. Yet for along time it has been suspected that a strong similarity exists in the structure, and even of creative process, among these subjects. Kandinsky stressed similarities between art and music [1]; recently the Yale Poet John Hollander published a book [2] which couples well known paintings with analogues poems. Another, more controversial, effort on these lines is the book by Robert Hofstadter with the provocative title Goedel, Escher and Bach [3].

Recent developments in applied mathematics known as "wavelet analysis" have attracted a wide following. Although there exist various antecedents, the modern results originated in earthquake analysis and were subsequently developed in the mathematics community.

In order to present the problem addressed by wavelets, consider the common experience of listening to music. At each instant of time our ear/brain system decomposes the signal according to its frequency composition. However the famous Fourier analysis of the signal does not allow the simultaneous determination of the conjugate variables frequency and time. In contrast, the description of a signal by a superposition of well-chosen wavelets provides an optimal mathematical description.

A similar improvement occurs in image analysis, where massive data compression is possible. Consider the representation of a photo. It might take one thousand Fourier coefficients to get a good rendition of a face, but about ten for wavelets. In addition the wavelets are quite good at analyzing contrast.

Our first goal is to obtain the wavelet correlations for music, art and poetry, and to look for common patterns. One can imagine pulling up the Mona Lisa on the web, and applying a two-dimensional wavelet transform to it, comparing the results with those from a Bach sonata. The case of poetry needs study due to the variety of rhythmic and tonal elements that occur.

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B. Moment and Wavelet Analysis of Correlations in Multihadron and Galaxy Distribution

1 Introduction

Here we consider recent developments in the analysis of textures of systems composed of many "points". For clarity we focus on two important examples, for which experimental progress has been decisive. The first is that of multihadron production [1], in which a large collection of final states in the momentum phase space can be prepared having identical initial conditions. The second is that of galaxy distributions, in which the points live in ordinary space-time. In this case there is no ensemble since there is only one specimen, presumably created by the big bang. Although the system is very large, it possesses correlations of very long range, making the usual procedure of creating a fake ensemble by partitioning the system into arbitrarily chosen subsystems suspect as a method of deriving correlation functions.

For simplicity we consider systems composed of one species of particle. Typically one considers charged hadrons without regard to sign. However recent experiments have found a strong and important effect for like-sign charges for high resolution, which must be taken into account. In the case of galaxies we do not discriminate among the different types, spiral, elliptic, etc. At the expense of writing more complicated, it is possible to handle such details.

The classic approach to texture analysis is by means of correlation functions [2]. Unfortunately they are hard to measure beyond second order. However it is now possible to measure factorial moments of the count distributions to fifth order and high resolution in the hadronic case. These moments are found in principle by integrating the density correlation functions $\rho_p(x_1...x_p)$ over some patch of phase space called Ω. Defining a sequence of densities of which the first two are

$$\rho_1(x,s) = \sum_i \delta(x - x_i(s)) \tag{1}$$

$$\rho_2(x, x', s) = \sum_{i+j} \delta(x - x_i(s))\delta(x' - x_j(s)) \tag{2}$$

and performing suitable averages leads to

$$\langle n \rangle_\Omega = \int_\Omega \rho_1(x) d^3x \tag{3}$$

$$\langle n(n-1) \rangle_\Omega = \int_\Omega d^3x \int_\Omega d^3x' \rho_2(x, x') \tag{4}$$

Another formulation for densities in particle physics uses the sequence of inclusive differential cross sections. These formulations are equivalent; for uniformity of presentation we shall use the delta function definition of densities.

The dependence of moments on the location, shape and size of Ω tells us much about the correlation function. Of particular interest has been the search for domains (and appropriate variables) in which scaling might occur. This development has been much influenced by the emergence of fractal thinking in many fields.

By now there is a detailed technology on this topic. This allows the model-independent description of data in a form of considerable utility. We shall refer to earlier papers for technical details.

In the past decade a powerful variant of the Fourier transform – "wavelet analysis" – has shown great promise for the analysis of textures and also great data compression capability. Unlike the Fourier method, wavelets allow simultaneous localization in conjugate variables. For example, when listening to music we make somehow a frequency analysis at each moment in time. The vast possibilities of this approach have already led to new insights into many subjects, such as signal analysis, pattern recognition, etc. In physical systems we expect advances both in phenomenology and the reformulation of dynamical problems. We briefly examine the behavior of correlations in simple cascade models using the simplest wavelet, the Haar basis.

2 The situation for multihadron production

Ten years ago the field of multiparticle production was rejuvenated by the suggestion of Bialas and Peschanski [3], to study the dependence of "bin-averaged" factorial moments as a function of rapidity bin-width δy. Recall that the rapidity $y = (1/2)\ln[(E+p_z)/(E-p_z)]$ is additive under change of inertial frames and is a natural variable for longitudinal kinematics. When true momentum is not measured it is well approximated by pseudorapidity $\eta = -\ln\tan\theta/2$, with θ the angle of the final particle with respect to the collision axis. From eqs. (3) and (4) and their generalization to higher order we write the factorial moment $F_p(\delta y)$ as

$$F_p(\delta y) = \frac{1}{M}\sum_{i=1}^{M}\frac{\langle n_i(n_i-1)...(n_i+p+1)\rangle}{\langle n_i\rangle^p} \tag{5}$$

We have chosen the Ω to be M adjacent bins of width δy. Note that for Poissonian count statistics the F_p are unity. Typically one expects dynamical correlations due to resonance decays and statistical correlations among like sign particles, (such as $\pi^-\pi^-$, etc) usually called Bose-Einstein correlations.

In the case of the second moment we can measure $\rho_2(y, y')$ and directly integrate eq.(4) to get $F_2(\delta y)$, as shown in Fig.1 of [4]. The bending of the curve is best attributed to the existence of a correlation length [4]. Much attention has been given to the possibility of scaling for very small y. To the accuracy of the data there is no conclusive evidence for scaling. However recent data show good scaling for like sign pions which can dominate for small phase space cells (in this case the best variable seems to be the invariant momentum transfer (Q^2) between particle pairs.)

The density correlations and the corresponding moments inevitably contain contributions of lower order. The systematic way to remove these is by going over to cumulants C_p, and the factorial cumulant moments, as we have discussed elsewhere [5].

In summary we can mention some salient results

1. The moments F_p (or K_p) typically increase with order p and with collision energy. They also increase (that is fluctuations increase) with shrinking bin size.

2. For complex targets (involving nuclei, typically) there are essentially no true (cumulant) correlations beyond second order [6].

3. For most multiparticle production processes the p-th order cumulant can be built from (symmetrized) linked products of two particle cumulants

$$C_p(1,2,...p) = \frac{Ap}{(p/2)} \sum_{perm} C_2(1,2)C_2(2,3)...C_2(p-1,p) \quad (6)$$

with the C's normalized to the single particle densities $\rho_1(1)...\rho_1(p)$.

4. If A_p is properly chosen [7] as $(p-1)!$ eq.(6) integrates to give the factorial cumulant moments characteristic of the negative binomial distribution (NBD), which provides a good description of charged particle counts in much of phase space. For later reference we write the NBD distribution as

$$P_n^k = \frac{(n+k-1)!}{(k-1)!n!} \frac{(\bar{n}/k)^n}{(1+\bar{n}/k)^{n+k}} \quad (7)$$

The parameter k can be any positive real number. It depends on the reaction, on the energy, and the size of the phase space volume in which the counting takes place. In terms of correlations, $1/k$ is given by the integral of the two particle cumulant correlation C_2 over the domain Ω.

Although a variety of dynamical models (typically cascade processes) suggest the NBD, more work needs to be done. As in the case of the Maxwell distribution for molecular velocities, the form of eq.(7) seems to be independent of dynamical details, a situation that has both good and bad features.

5. An interesting "statistic" is $P_0(\Omega)$, the probability of finding nothing [8] in Ω. This is usually called the "rapidity gap" probability. In the case of galaxies it is the void probability, discussed in the next section. In each case the data are well described by negative binomial count statistics.

6. Some important technical modifications of the bin-averaging approach have been developed by the Tucson group. To appreciate these, first note that for the one-dimensional case F_2 is given by the integral over adjacent boxes [9] of side δy, centered on the diagonal $y_1 = y_2$ in the $y_1 - y_2$ plane. (For higher order one has hypercubes.) Two points close by in this plane but not in the same bin do not contribute to the moment. For highly clumped events the problem is serious because the moments jump around when the bin size is reduced. As a consequence these rare but important events give rise to spurious statistical fluctuations in the domain of greatest interest.

The situation is much improved if the integration over boxes is replaced by a strip. Originally [4] the strip domain was used as a convenient approximation, but later we realized [5, 8] that the strip domain gives a general approach, close numerically to the box method but without the spurious fluctuations mentioned above [10]. The strip with width ϵ now provides the resolution scale; all pairs of points closer than ϵ contribute to the strip moment on an equal footing [8, 9]. This approach is now widely used in multihadron data analysis. Note that should scaling occur, the method is numerically close to that used to define correlation dimensions, in which case *all* pairs are less than a shrinking value ϵ. In addition to the Grassberger-Hentschel-Procaccia algorithm [11] one can define [10] a "star integral", which has the additional merit of reducing computational needs, allowing the analysis of events having very high multiplicity, as expected in the planned accelerators LHC and RHIC.

For details about these developments consult the references.

3 The situation for galaxy counts and correlations

Although we reviewed this topic fairly recently [8], new results and techniques are constantly emerging in this popular field, often known as "large scale structure". The classic reference is the book by Peebles [12]. The method of correlation functions is developed in detail, and a conjecture about the structure of correlation known as the "hierarchical model" is put forward. In fact it is basically the same as the "linked