

Presents the basic concepts
of structures, the stability and determinacy
of structures, and the analysis
of both statically determinate and
indeterminate structures

ELEMENTARY THEORY OF STRUCTURES

YUAN-YU HSIEH

PRENTICE-HALL
INTERNATIONAL
SERIES
IN DYNAMICS

Y. C. FUNG
Editor

ELEMENTARY THEORY OF
STRUCTURES

YUAN-YU HSIEH

Professor of Civil Engineering
National Taiwan University

PRENTICE-HALL, INC.
Englewood Cliffs, New Jersey

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA, PTY. LTD., *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LTD., *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*

© 1970 by Prentice-Hall, Inc., Englewood Cliffs, N. J.
All rights reserved. No part of this book may be
reproduced in any form or by any means without per-
mission in writing from the publisher. Library of
Congress Catalog Card Number 77-85801.

13-261552-5

Current printing (last digit):

10 9 8

Printed in the United States of America

ELEMENTARY THEORY OF
STRUCTURES

Prentice-Hall Series in Dynamics

Y. C. Fung, *Editor*

PRENTICE-HALL, INC.
PRENTICE-HALL INTERNATIONAL, INC., UNITED KINGDOM AND EIRE
PRENTICE-HALL OF CANADA, LTD., CANADA

Elementary Theory of Structures by Yuan-Yu Hsieh
Foundations of Solid Mechanics by Y. C. Fung
Principles of Dynamics by Donald T. Greenwood

To
My Parents

PREFACE

This book is intended for elementary courses in the structural theory of civil engineering. In preparing material for it, the author has assumed that the reader is not familiar with the subject. The first seven chapters contain the basic concepts of structure and an analysis of statically determinate structures. Chapter 8 deals with elastic deformations. Chapters 9 through 15 are concerned with statically indeterminate structures, including the method of consistent deformations, least work, slope deflection, and moment distribution. Chapter 16 is devoted to an introductory discussion of matrix algebra. The scope is wide enough to provide adequate background for reading the remainder of the book. Finally, the last two chapters present a unified treatment of structures by matrix methods based on the finite-element approach. Since this is an elementary treatment, emphasis is on the development of general theory in terms of matrix operations rather than on the particular details of computer programs.

The elementary theory of structures is not difficult, involving as it does only a limited amount of higher mathematics. For most of the book, the only preparation needed is a knowledge of arithmetic and high school algebra. We may also say, however, that this theory is difficult, in that it generally requires careful study in order to achieve a thorough understanding of its basic philosophy. The author has endeavored to present clearly and lucidly the fundamentals of structural theory. These fundamentals are arranged in a systematic order and are supplemented by examples illustrating their application in some of the common structures—namely, beams, trusses, and rigid frames.

While written primarily for use as a textbook in the classroom, this book can also be of help to structural and architectural engineers in their independent study.

The author wishes to express his appreciation to those who have assisted in the preparation of this book. He is especially grateful to Dr. Y. C. Fung, professor at the University of California, San Diego, for his enthusiastic encouragement throughout the writing of this volume and for his constructive criticism of the manuscript. Particular acknowledgment is also due Dr. Z. A. Lu of the University of California, Berkeley, who made many valuable sug-





gestions for improving the contents, and Dr. John Yao, who read part of the manuscript. Thanks also are due Pamela Fischer and Cordelia Thomas of Prentice-Hall, who offered their competent knowledge in producing and editing this book. Finally, the author is indebted to his wife, Nelly, who typed all of the manuscript.

Y. Y. H.

ELEMENTARY THEORY OF
STRUCTURES

CONTENTS

1 INTRODUCTION	1
1-1. Engineering Structures	1
1-2. Theory of Structures Defined	2
1-3. Theories of Structures Classified	3
1-4. Actual and Ideal Structures	4
1-5. Scope of This Book	4
2 STABILITY AND DETERMINACY OF STRUCTURES	6
2-1. Equations of Equilibrium for a Coplanar Force System	6
2-2. Support Reactions	9
2-3. Internal Forces at a Cut Section of a Structure	12
2-4. Equations of Condition or Construction	14
2-5. Stability and Determinacy of a Structure with Respect to Supports When the Structure Is Considered a Monolithic Rigid Body	14
2-6. General Stability and Determinacy of Structures	18
3 STATICALLY DETERMINATE BEAMS	31
3-1. General	31
3-2. Analysis of Statically Determinate Beams	32
3-3. Principle of Virtual Work and Beam Analysis	41
3-4. Relationships Between Load, Shear, and Bending Moment	49
3-5. Numerical Examples	53
4 STATICALLY DETERMINATE TRUSSES	59
4-1. General	59
4-2. Analysis of Simple Trusses: Method of Joint and Method of Section	63

4-3. Analysis of Compound Trusses: Mixed Method	68
4-4. Analysis of Complex Trusses: Method of Substitute Member	73
4-5. General Description of Bridge and Roof Truss Frameworks	77
4-6. Conventional Types of Bridge and Roof Trusses	79
5 STATICALLY DETERMINATE RIGID FRAMES AND COMPOSITE STRUCTURES	86
5-1. Rigid Joint	86
5-2. Analysis of Statically Determinate Rigid Frames	86
5-3. Analysis of Statically Determinate Composite Structures	92
5-4. Approximate Analysis for Statically Indeterminate Structures	95
6 INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES	102
6-1. The Concept of Influence Line	102
6-2. Use of the Influence Line	105
6-3. Influence Lines for Statically Determinate Beams	107
6-4. Beam Influence Lines by Virtual Work	113
6-5. Stringer and Floor-Beam System	119
6-6. Influence Lines for Girders with Floor Systems	123
6-7. Influence Lines for Statically Determinate Bridge Trusses	124
7 MOVING CONCENTRATED LOADS: CRITERIA FOR MAXIMA	134
7-1. General	134
7-2. Criterion for the Maximum Value of a Function Having the Influence Diagram 	134
7-3. Criterion for the Maximum Value of a Function Having the Influence Diagram 	135
7-4. Criterion for the Maximum Value of a Function Having the Influence Diagram 	139
7-5. Criterion for the Maximum Value of a Function Having the Influence Diagram 	142
7-6. Absolute Maximum Bending Moment	143

8 ELASTIC DEFORMATIONS OF STRUCTURES	147
8-1. General	147
8-2. Curvature of Elastic Line	147
8-3. External Work and Internal Work	150
8-4. Method of Virtual Work (Unit-Load Method)	152
8-5. Castigliano's First Theorem	166
8-6. Conjugate-Beam Method	174
9 ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE METHOD OF CONSISTENT DEFORMATIONS	183
9-1. General	183
9-2. Analysis of Statically Indeterminate Beams by the Method of Consistent Deformations	185
9-3. Analysis of Statically Indeterminate Rigid Frames by the Method of Consistent Deformations	195
9-4. Analysis of Statically Indeterminate Trusses by the Method of Consistent Deformations	197
10 ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE METHOD OF LEAST WORK	205
10-1. General	205
10-2. Analysis of Statically Indeterminate Beams by the Method of Least Work	206
10-3. Analysis of Statically Indeterminate Rigid Frames by the Method of Least Work	211
10-4. Analysis of Statically Indeterminate Trusses by the Method of Least Work	215
10-5. Analysis of Statically Indeterminate Composite Structures by the Method of Least Work	217
11 INFLUENCE LINES FOR STATICALLY INDETERMINATE STRUCTURES	222
11-1. Maxwell's Law of Reciprocal Deflections	222
11-2. Betti's Law	223
11-3. Influence Lines as Deflected Structures: The Müller-Breslau Principle	224

11-4. Sketch of Influence Lines	226
11-5. Numerical Examples	230
11-6. Solving Influence Ordinates for Statically Indeterminate Structures by Consistent Deformations	237
12 ANALYSIS OF STATICALLY INDETERMINATE BEAMS AND RIGID FRAMES BY THE SLOPE-DEFLECTION METHOD	240
12-1. General	240
12-2. Basic Slope-Deflection Equations	240
12-3. Procedure of Analysis by the Slope-Deflection Method	245
12-4. Analysis of Statically Indeterminate Beams by the Slope-Deflection Method	250
12-5. Analysis of Statically Indeterminate Rigid Frames Without Joint Translation by the Slope-Deflection Method	253
12-6. Analysis of Statically Indeterminate Rigid Frames with One Degree of Freedom of Joint Translation by the Slope-Deflection Method	255
12-7. Analysis of Statically Indeterminate Rigid Frames with Two Degrees of Freedom of Joint Translation by the Slope-Deflection Method	260
12-8. Analysis of Statically Indeterminate Rigid Frames with Several Degrees of Freedom of Joint Translation by the Slope-Deflection Method	266
13 AN INTRODUCTION TO MOMENT DISTRIBUTION: MOMENT DISTRIBUTION WITHOUT JOINT TRANSLATION	270
13-1. General	270
13-2. Fixed-End Moment	271
13-3. Stiffness, Distribution Factor, and Distribution of External Moment Applied to a Joint	271
13-4. Carry-Over Factor and Carry-Over Moment	274
13-5. The Process of Locking and Unlocking: One Joint	276
13-6. The Process of Locking and Unlocking: Two or More Joints	280
13-7. Modified Stiffnesses	285
14 MOMENT DISTRIBUTION WITH JOINT TRANSLATIONS	291
14-1. General	291
14-2. Fixed-End Moment Due to Joint Translation	294

14-3. Analysis of Statically Indeterminate Rigid Frames with One Degree of Freedom of Joint Translation by Moment Distribution	295
14-4. Analysis of Statically Indeterminate Rigid Frames with Two Degrees of Freedom of Joint Translation by Moment Distribution	303
14-5. Analysis of Statically Indeterminate Rigid Frames with Several Degrees of Freedom of Joint Translation by Moment Distribution	310
14-6. Influence Lines by Moment Distribution	312
14-7. Secondary Stresses in Trusses Solved by Moment Distribution	314
15 ANALYSIS OF STATICALLY INDETERMINATE BEAMS AND RIGID FRAMES COMPOSED OF NONPRISMATIC MEMBERS	318
15-1. General	318
15-2. Integral Expressions for Fixed-End Moments, Stiffness, and Carry-Over Factors	319
15-3. Fixed-End Moments, Stiffness, and Carry-Over Factors Solved by the Conjugate-Beam Method; Numerical Solution	322
15-4. Relationship of Stiffness and Carry-Over Factors	326
15-5. Charts and Tables	328
15-6. Modified Stiffness	329
15-7. Fixed-End Moments Due to Joint Translation	332
15-8. Numerical Examples	333
15-9. The Generalized Slope-Deflection Equations	338
16 MATRIX ALGEBRA FOR STRUCTURAL ENGINEERS	342
16-1. Introduction	342
16-2. Matrix Definitions and Notation	342
16-3. Equality, Addition, Subtraction, and Scalar Multiplication	345
16-4. Matrix Multiplication	347
16-5. Matrix Partitioning	350
16-6. Transpose of Product	351
16-7. Inversion of Matrix	352
16-8. Inversion by Successive Transformations	356
16-9. Solution of Linear Simultaneous Equations	357
17 MATRIX ANALYSIS OF STRUCTURES BY THE FINITE ELEMENT METHOD PART I: THE FORCE METHOD	360
17-1. General	360
17-2. Basic Concepts of Structures	361

17-3. Equilibrium, Force Transformation Matrix	363
17-4. Compatibility	365
17-5. Force-Displacement Relationship, Flexibility Coefficient, Flexibility Matrix	365
17-6. Analysis of Statically Determinate Structures by the Force Method	370
17-7. Analysis of Statically Indeterminate Structures by the Force Method	375
17-8. The Treatment of Distributed Load	381
17-9. On the Notion of Primary Structure	385

**18 MATRIX ANALYSIS OF STRUCTURES BY THE FINITE
ELEMENT METHOD PART II: THE DISPLACEMENT
METHOD**

392

18-1. General	392
18-2. Compatibility, Displacement Transformation Matrix	392
18-3. Force-Displacement Relationship, Stiffness Coefficient, Stiffness Matrix	394
18-4. Equilibrium	398
18-5. Analysis of Structures by the Displacement Method	398
18-6. Use of the Modified Member Stiffness Matrix	409
18-7. The Direct Stiffness Method	414
18-8. Comparison of the Force Method and the Displacement Method	416

SELECTED REFERENCES 419

ANSWERS TO SELECTED PROBLEMS 421

INDEX 427

1-2. THEORY OF STRUCTURES DEFINED

The complete design of a structure is contained in the following stages:

1. Establishing the general layout. The general layout for a structure is selected from many possible alternatives. The primary concern is the performance requirement of the structure whether it is to house, to convey, or to support in space. Many secondary considerations should also be examined including the economic, aesthetic, legal, and financial aspects.

2. Investigating the loadings. General information about the loadings imposed on a structure is usually given in the specifications and codes. Basically it is part of the designer's responsibility to specify the loading conditions and to take care of exceptional cases. Loadings based on *static* consideration may be classified as:

a. Dead load. Dead load is the weight of the structure itself and is regarded as fixed in magnitude and location. Since the dead load must be assumed before the structure is designed, the original data are only tentative. Revision must be made if the estimate is not satisfactory.

b. Live load. Live load may be classified into movable load and moving load. *Movable loads* are loads that may be transported from one location to another on a structure, for example, people and furniture on a building floor. *Moving loads* are loads that move continuously over the structure, such as railway trains or trucks on a bridge.

c. Impact load. The effects of impact are usually associated with moving live loads. In structural design the impact load is treated as an increase in the live load if the live load is taken as a gradually applied static load.

3. Stress analysis. Once the external loads are defined, a *stress analysis* must be made to determine the internal forces, sometimes referred to as *stresses*, that will be produced in the various members. When live loads are involved, emphasis should be put on the maximum possible stresses in each member under consideration. To obtain this we must know not only what loading is imposed but also where it is placed.

4. Selection of members. The choice of materials and proportioning of the sizes for structure members are based on the results of Stage 3 together with the design provisions given by the specifications or codes.

5. Drawing and detailing. After the make-up of each part of the structure is determined, the final stage, drawing and detailing, provides the necessary information for construction.

The subject matter of the theory of structures is stress analysis with occasional reference to loadings. The emphasis of structural theory is usually on the fundamentals rather than on the details of design.

1-3. THEORIES OF STRUCTURES CLASSIFIED

Structural theories may be classified from various points of view. For convenience, we shall characterize them by the following aspects:

1. Statics versus dynamics. Ordinary structures are usually designed under static loads. Dead load and snow load are static loads that cause no dynamic effect on structures. Some live loads, such as trucks and locomotives moving on bridges, are also assumed as concentrated static load systems. They do cause impact on structures; however, the dynamic effects are treated as a fraction of the moving loads to simplify the design.

The particular, specialized branch that deals with the dynamic effects on structures of accelerated moving loads, earthquake loads, wind gusts, or bomb blasts is *structural dynamics*.

2. Plane versus space. No structure is really planar. However, structural analyses for beams, trussed bridges, or rigid frame buildings are usually treated as plane problems, although they are never two-dimensional in themselves. On the other hand, in some structures, such as towers and framing for domes, the stresses are interrelated between members not lying in a plane in such a way that the analysis cannot be simplified on the basis of component planar structures. Such structures must be considered as space frameworks under non-coplanar force system.

3. Linear versus nonlinear structures. Linear structure means that a linear relationship is assumed to exist between the applied loads and the resulting displacements in a structure. This assumption is based on the following conditions:

a. The material of the structure is elastic and obeys Hooke's law at all points and throughout the range of loading considered.

b. The changes in the geometry of the structure are so small that they can be neglected when the stresses are calculated.

Note that if the principle of superposition is to apply, a linear relationship must exist, or be assumed to exist, between loads and displacements.

A nonlinear relationship between the applied actions and the resulting displacements exists under either of two conditions:

a. The material of the structure is inelastic.

b. The material is within the elastic range, but the geometry of the structure changes significantly during the application of loads.

The study of nonlinear behavior of structures includes *plastic analysis of structures* and *buckling of structures*.

4. Statically determinate versus statically indeterminate structures. The term *statically determinate structure* means that structural analysis can be