Precalculus

SECOND EDITION

ENHANCED WITH GRAPHING UTILITIES

Michael Sullivan ◆ Michael Sullivan, III

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Enhanced with Graphing Utilities

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For Michael S., Kevin and Marissa (Sullivan)
Shannon, Patrick, and Ryan (Murphy)
and Kaleigh (O'Hara)
The Next Generation

❧

PREFACE TO THE INSTRUCTOR

As Professors, one at an urban public university and the other at a community college, Michael Sullivan and Michael Sullivan III are aware of the varied needs of Precalculus students, ranging from those who have little mathematical background and fear of mathematics courses to those who have had a strong mathematical education and are highly motivated. For some of these students, this will be their last course in mathematics, still many others will be going on to calculus and beyond. This text is written for both groups. As the author of precalculus, engineering calculus, finite math, and business calculus texts, and as a teacher, Michael understands what students must know if they are to be focussed and successful in upper level math courses. However, as a father of four, including the co-author, he also understands the realities of college life. Michael Sullivan III believes passionately in the value of technology as a tool for learning that enhances understanding without sacrificing important skills. Both authors have taken great pains to insure that the text contains solid, student friendly examples and problems, as well as a clear, seamless writing style.

In the Second Edition

The second edition builds upon a strong foundation by integrating new features and techniques that further enhance student interest and involvement. The elements of the previous edition that have proven successful remain, while many changes, some obvious, others subtle, have been made. The text has been streamlined to increase accessibility. A huge benefit of authoring a successful series is the broad-based feedback upon which improvements and additions are ultimately based. Virtually every change in this edition is the result of thoughtful comments and suggestions made by colleagues and students who have used the previous edition. This feedback has proved invaluable and has been used to make changes that improve the flow and usability of the text. For example, some topics have been moved to better reflect the way teachers approach the course. In other places, problems have been added where more practice was needed. The supplements package has been enhanced through upgrading traditional supplements and adding innovative media components.

Changes to the Second Edition

Specific Organizational Changes

- Chapters One, Three and certain topics from the Appendix have been rewritten and reorganized so they now appear in Chapter One. This format allows more efficient use of the text and will improve the flow. Equations are solved in Chapter One from both and algebraic and graphing approach. The graphing approach makes extensive use of the ZERO (ROOT) feature and INTERSECT. The line is discussed in a single section (this includes parallel and perpendicular lines). In addition, circles are presented in their own section. Once Chapter One is completed, Chapters 2–5 provided an uninterrupted discussion of functions.

- New to Chapter Two is a discussion of Linear Functions and Models (Section 2.2). The topics of Linear Curve Fitting and Variation are discussed in the context of a linear function. Notice the topic "One-to-One Functions; Inverse Functions" has been delayed until Chapter Four where it is needed.
- Chapter Three discusses polynomial and rational functions. The chapter has been reorganized so the discussion of polynomials in uninterrupted. Section 3.6 has been completely rewritten. Notice Polynomial and Rational Inequalities is now in Section 3.8.
- Chapter Four discusses the exponential and logarithmic functions. New to this section is the topic "One-to-One Functions; Inverse Functions". In addition, a discussion of logistic functions and curve fitting is presented.
- Chapter Five introduces the Trigonometric Functions and their graphs. Sinusoidal Curve fitting is a new topic to this edition. Students model temperature, tide, and length of day data. The discussion of Inverse Trigonometric Functions is delayed until the section preceding Trig Equations.
- Chapter Six discusses Analytic Trigonometry. New to the section is Inverse Trigonometric Functions. In addition, Trig Equations are discussed in two sections.
- Chapter Seven is a complete discussion of applications.
- Chapter Eight introduces Polar Coordinates and Vectors.
- Chapter 10 is Systems of Equations and Inequalities. The old Section 11.1 has been rewritten into two sections; Section 10.1 is Systems of Linear Equations Containing Two Variables and Section 10.2 is Systems of Linear Equations Containing Three Variables. Systems of Inequalities and Linear Programming have been combined into one section.
- Chapter 12 in the old edition has been divided into two chapters. Chapter 11 is Sequences; Induction; The Binomial Theorem. Chapter 12 presents counting and probability. There are now two sections on probability. Section 12.3 discusses probability using counting techniques, while Section 12.4 discusses probability using relative frequency.

Specific Content Changes

- Emphasis is placed on the role of modeling in Precalculus. To this end, dedicated sections appear on Linear Functions and Models, Quadratic Functions and Models, Power Functions and Models, Polynomial Functions and Models, Exponential, Logarithmic and Logistic Models, and Sinusoidal Models.
- Section 1.5 is new. Inequalities are discussed in more detail, rather than treating the topic in the appendix.
- Section 3.6 on complex zeros has been completely rewritten. Only polynomials with real coefficients are discussed.
- Section 4.8 now has logistic curve fitting.
- Section 5.6 (Sinusoidal graphs) discusses sinusoidal curve fitting (both by hand and using a graphing utility).
- Section 7.5 (Simple Harmonic Motion; Damped Motion) has been rewritten to address damped motion as it is done in Physics.
- Section 8.6 (Vectors in Space) is new to this edition.

- Section 9.7 (Parametric Curves and Parametric Equations) has been rewritten in order to utilize the power of graphing utilities. Students model projectile motion using their graphing utilities.
- Section 11.3 now discusses geometric series with annuity applications, including annuities using recursive relations.
- A section on univariate data and obtaining probability from data is new to this edition.
- Chapter 13 is new to this edition. It presents an introduction to limits and the derivative.
- The use of technology has been updated to make more extensive use of TABLES, ZERO(ROOT) and INTERSECT.

Acknowledgments

Textbooks are written by authors, but evolve from an idea into final form through the efforts of many people. Special thanks to Don Dellen, who first suggested this book and the other books in this series. Don's extensive contributions to publishing and mathematics are well known; we all miss him dearly.

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Michael Sullivan
Michael Sullivan, III

PREFACE TO THE STUDENT

As you begin your study of Precalculus, you may feel overwhelmed by the numbers of theorems, definitions, procedures, and equations that confront you. You may even wonder whether or not you can learn all of this material in the time allotted. These concerns are normal. Do not become frustrated and say "This is impossible." It is possible! Work hard, concentrate, use all the resources available to you, and, above all, don't give up.

For many of you, this may be your last math course; for others, it is just the first in a series of many. Either way, this text was written to help you, the student, master the terminology and basic concepts of Precalculus. These aims have helped to shape every aspect of the book. Many learning aids are built into the format of the text to make your study of the material easier and more rewarding. This book is meant to be a "machine for learning," that can help you focus your efforts and get the most from the time and energy you invest.

About the authors

Michael Sullivan has taught Precalculus courses for over thirty years. He is also the father of four college graduates, including this text's co-author, all of whom called home, from time to time, frustrated and with questions. He knows what you're going through. Michael Sullivan, III, as a fairly recent graduate, experienced, first-hand, learning math using graphing calculators and computer algebra systems. He recognizes that, while this technology has many benefits, it can only be used effectively with an understanding of the underlying mathematics. Having earned advanced degrees in both economics and mathematics, he sees the importance of mathematics in applications.

So we, father and son, have written a text that doesn't overwhelm, or unnecessarily complicate, Precalculus, but at the same time provides you with the skills and practice you need to be successful. Please do not hesitate to contact us through Prentice Hall with any suggestions or comments that would improve this text.

Best Wishes!
Michael Sullivan
Michael Sullivan, III

OVERVIEW

CHAPTER OPENERS AND CHAPTER PROJECTS

Chapter openers use current articles to set up **Chapter Projects.** Many of the concepts that you encounter in this course relate directly to today's headlines and issues. These chapter projects are designed to give you a chance to use math to better understand the world. The list of topics for review will help you in two major ways, …First, it allows you to review basic concepts immediately before using them in context. Second, it emphasizes the natural building of mathematical concepts throughout the course.

CHAPTER

2 Functions and Models

Wednesday February 10, 1999
***The Oregonian* "Ship awaits salvage effort"**

COOS BAY—Cleanup crews combed the oil-scarred south coast Tuesday as authorities raced to finish plans to refloat a 639-foot cargo ship mired for six days 150 yards off one of Oregon's most biologically rich beaches.

All day Tuesday, streaks of oil oozed from the cracked hull of the bulk cargo carrier *New Carissa* and spread over six miles of beach. Despite the breached hull, authorities believe they had a better chance of pulling the stricken ship out of beach sands than pumping nearly 400,000 gallons of oil off the ship in the winter surf.

See Chapter Project 1.

Preparing for This Chapter

Before getting started on this chapter, review the following concepts:

- **Intercepts (pp. 18–19)**
- **Intervals (pp. 51–53)**
- **Slope–Intercept Form of a Line (pp. 70–71)**
- **Scatter Diagrams (pp. 9–10)**
- **Slope of a Line (pp. 63–65)**
- **Tests for Symmetry of an Equation (p. 21)**
- **Graphs of Certain Equations (Example 2, p. 15; Example 8, p. 21;**

Outline

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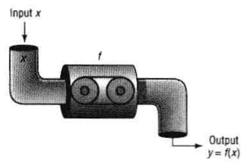
CHAPTER PROJECTS

1. **Oil Spill** An oil tanker strikes a sand bar that rips a hole in the hull of the ship. Oil begins leaking out of the tanker, with the spilled oil forming a circle around the tanker. The radius of the circle is increasing at the rate of 2.2 feet per hour.
   (a) Write the area of the circle as a function of the radius r.
   (b) Write the radius of the circle as a function of time t.
   (c) What is the radius of the circle after 2 hours? What is the radius of the circle after 2.5 hours?
   (d) Use the result of part (c) to determine the area of the circle after 2 hours and 2.5 hours.
   (e) Determine a function that represents area as a function of time t.
   (f) Use the result of part (e) to determine the area of the circle after 2 hours and 2.5 hours.
   (g) Compute the average rate of change of the area of the circle from 2 hours to 2.5 hours.
   (h) Compute the average rate of change of the area of the circle from 3 hours to 3.5 hours.
   (i) Based on the results obtained in parts (g) and (h), what is happening to the average rate of change of the area of the circle as time passes?
   (j) If the oil tanker is 150 yards from shore, when will the oil spill first reach the shoreline? (1 yard = 3 feet)
   (k) How long will it be until 6 miles of shoreline is contaminated with oil? (1 mile = 5280 feet)

CLEAR WRITING STYLE

Sullivan's **accessible writing style** is apparent throughout, often utilizing various approaches to the same concept. An author who writes clearly makes potentially difficult concepts intuitive, making class time more productive.

Figure 3



Sometimes it is helpful to think of a function f as a machine that receives as input a number from the domain, manipulates it, and outputs the value. See Figure 3.

The restrictions on this input/output machine are as follows:

1. It only accepts numbers from the domain of the function.
2. For each input, there is exactly one output (which may be repeated for different inputs).

For a function $y = f(x)$, the variable x is called the **independent variable**, because it can be assigned any of the permissible numbers from the domain. The variable y is called the **dependent variable**, because its value depends on x.

Any symbol can be used to represent the independent and dependent variables. For example, if f is the *cube function*, then f can be defined by $f(x) = x^3$ or $f(t) = t^3$ or $f(z) = z^3$. All three functions are the same: Each tells us to cube the independent variable. In practice, the symbols used for the independent and dependent variables are based on common usage, such as using C for cost in business.

2 The variable x is also called the **argument** of the function. Thinking of the independent variable as an argument can sometimes make it easier to find the value of a function. For example, if f is the function defined by $f(x) = x^3$, then f tells us to cube the argument. Thus, $f(2)$ means to cube 2, $f(a)$ means to cube the number a, and $f(x + h)$ means to cube the quantity $x + h$.

4.7 GROWTH AND DECAY

1 Find Equations of Populations That Obey the Law of Uninhibited Growth
2 Find Equations of Populations That Obey the Law of Decay
3 Use Newton's Law of Cooling
4 Use Logistic Growth Models

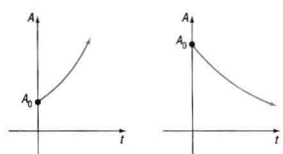
1 Many natural phenomena have been found to follow the law that an amount A varies with time t according to

$$A = A_0 e^{kt} \qquad (1)$$

where A_0 is the original amount $(t = 0)$ and $k \neq 0$ is a constant.

If $k > 0$, then equation (1) states that the amount A is increasing over time; if $k < 0$, the amount A is decreasing over time. In either case, when an amount A varies over time according to equation (1), it is said to follow the **exponential law** or the **law of uninhibited growth** $(k > 0)$ **or decay** $(k < 0)$. See Figure 40.

Figure 40



(a) $A(t) = A_0e^{kt}$, $k >$

LEARNING OBJECTIVES

Begin each section by reading the **Learning Objectives.** These objectives will help you organize your studies and prepare for class.

Section 6.1 Trigonometric Identities 455

EXAMPLE 5 Establishing an Identity

Establish the identity: $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\csc\theta$

Solution The left side is more complicated, so we start with it and proceed to add.

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)(\sin\theta)} \qquad \text{Add the quotients.}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)} \qquad \text{Remove parentheses in numerator.}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{(1+\cos\theta)(\sin\theta)} \qquad \text{Regroup.}$$

$$= \frac{2 + 2\cos\theta}{(1+\cos\theta)(\sin\theta)} \qquad \text{Pythagorean Identity.}$$

$$= \frac{2(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)} \qquad \text{Factor and cancel.}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\csc\theta \qquad \text{Reciprocal Identity.}$$

Sometimes it helps to write one side in terms of sines and cosines only.

STEP-BY-STEP EXAMPLES

Step-by-step examples insure that you follow the entire solution process and give you an opportunity to check your understanding of each step.

REAL-WORLD DATA

Real-world data is incorporated into examples and exercise sets to emphasize that mathematics is a tool used to understand the world around us. As you encounter these problems and examples, you will see the relevance and utility of the skills being covered.

27. **Federal Income Tax** Two 1998 Tax Rate Schedules are given in the accompanying table. If x equals the amount on Form 1040, line 37, and y equals the tax due, construct a function f for each schedule.

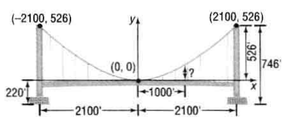
1998 Tax Rate Schedules

SCHEDULE X—IF YOUR FILING STATUS IS SINGLE				SCHEDULE Y-1—USE IF YOUR FILING STATUS IS MARRIED FILING JOINTLY OR QUALIFYING WIDOW(ER)			
If the amount on Form 1040, line 37, is: Over—	But not over—	Enter on Form 1040, line 38	of the amount over—	If the amount on Form 1040, line 37, is: Over—	But not over—	Enter on Form 1040, line 38	of the amount over—
$0	$25,750	____15%	$0	$0	$43,050	____15%	$0
25,750	62,450	$3,862.50 + 28%	25,750	43,050	104,050	$6,457.50 + 28%	43,050
62,450	130,250	14,138.50 + 31%	62,450	104,050	158,550	23,537.50 + 31%	104,050
130,250	283,150	35,156.50 + 36%	130,250	158,550	283,150	40,432.50 + 36%	158,550
283,150	____	90,200.50 + 39.6%	283,150	283,150	____	85,288.50 + 39.6%	283,150

Section 3.1 Quadratic Functions and Models 189

Solution We begin by choosing the placement of the coordinate axes so that the x-axis coincides with the road surface and the origin coincides with the center of the bridge. As a result, the twin towers will be vertical (height $746 - 220 = 526$ feet above the road) and located 2100 feet from the center. Also, the cable, which has the shape of a parabola, will extend from the towers, open up, and have its vertex at $(0, 0)$. As illustrated in Figure 14, the choice of placement of the axes enables us to identify the equation of the parabola as $y = ax^2$, $a > 0$. We can also see that the points $(-2100, 526)$ and $(2100, 526)$ are on the graph.

Figure 14



Based on these facts, we can find the value of a in $y = ax^2$.

$$y = ax^2$$
$$526 = a(2100)^2$$
$$a = \frac{526}{(2100)^2}$$

The equation of the parabola is therefore

$$y = \frac{526}{(2100)^2}x^2$$

The height of the cable when $x = 1000$ is

$$y = \frac{526}{(2100)^2}(1000)^2 \approx 119.3 \text{ feet}$$

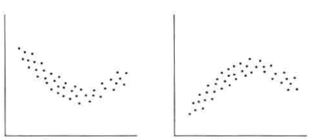
The cable is 119.3 feet high at a distance of 1000 feet from the center of the bridge.

NOW WORK PROBLEM 77.

Fitting a Quadratic Function to Data

5 In Section 2.2 we found the line of best fit for data that appeared to be linearly related. It was noted that data may also follow a nonlinear relation. Figures 15(a) and (b) show scatter diagrams of data that follow a quadratic relation.

Figure 15



$y = ax^2 + bx + c, a > 0$ (a) $y = ax^2 + bx + c, a < 0$ (b)

"NOW WORK" PROBLEMS

Many examples end with **"Now Work Problems."** The problems suggested here are similar to the corresponding examples and provide a great way to check your understanding as you work through the chapter. The solutions to all "Now Work" problems can be found in the back of the text as well as the *Student Solutions Manual.*

77. **Suspension Bridge** A suspension bridge with weight uniformly distributed along its length has twin towers that extend 75 meters above the road surface and are 400 meters apart. The cables are parabolic in shape and are suspended from the tops of the towers. The cables touch the road surface at the center of the bridge. Find the height of the cables at a point 100 meters from the center. (Assume that the road is level.)

PROCEDURES

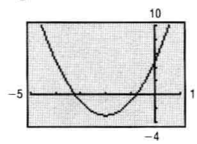
Procedures, both algebraic and technical, are clearly expressed throughout the text.

EXAMPLE 1 Graphing a Quadratic Function

Graph the function $f(x) = 2x^2 + 8x + 5$. Find the vertex and axis of symmetry.

Graphing Solution Before graphing, notice that the leading coefficient, 2, is positive and therefore the graph will open up and the vertex will be the lowest point. Now graph $Y_1 = f(x) = 2x^2 + 8x + 5$. See Figure 6. We observe that the graph does in fact open up. To estimate the vertex of the parabola, we use the MINIMUM command. The vertex is the point $(-2, -3)$ and the axis of symmetry is the line $x = -2$.

Figure 6



Algebraic Solution We begin by completing the square[†] on the right side.

$$\begin{aligned} f(x) &= 2x^2 + 8x + 5 \\ &= 2(x^2 + 4x) + 5 && \text{Factor out the 2 from } 2x^2 + 8x \\ &= 2(x^2 + 4x + 4) + 5 - 8 && \text{Complete the square of } 2(x^2 + 4x). \quad (2) \\ &= 2(x + 2)^2 - 3 && \text{Notice that the factor of 2 requires that 8 be added and subtracted.} \end{aligned}$$

The graph of f can be obtained in three stages, as shown in Figure 7. Now compare this graph to the graph in Figure 5(a). The graph of $f(x) = 2x^2 + 8x + 5$ is a parabola that opens up and has its vertex (lowest point) at $(-2, -3)$. Its axis of symmetry is the line $x = -2$.

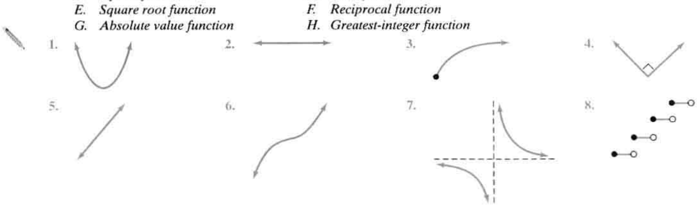
* We shall study parabolas using a geometric definition later in this book.
[†] Refer to the Appendix, Section 2, for a review.

134 Chapter 2 Functions and Models

2.3 EXERCISES

In Problems 1–8, match each graph to the function listed whose graph most resembles the one given.

A. Constant function *B. Linear function*
C. Square function *D. Cube function*
E. Square root function *F. Reciprocal function*
G. Absolute value function *H. Greatest-integer function*



END-OF-SECTION EXERCISES

Sullivan's exercises are unparalleled in terms of thorough coverage and accuracy. Each **end-of-section exercise** set begins with visual and concept based problems, starting you out with the basics of the section. Well-thought-out exercises better prepare you for exams.

◀ **EXAMPLE 7 A Cubic Function of Best Fit**

Table 5

Year, x	Refined Oil Products (1000 Barrels per Day), R
1977, 1	2193
1978, 2	2008
1979, 3	1937
1980, 4	1646
1981, 5	1599
1982, 6	1625
1983, 7	1722
1984, 8	2011
1985, 9	1866
1986, 10	2045
1987, 11	2004
1988, 12	2295
1989, 13	2217
1990, 14	2123
1991, 15	1844
1992, 16	1805

Source: U.S. Energy Information Administration, *Monthly Energy Review*, February, 1995.

The data in Table 5 represent the number of barrels of refined oil products imported into the United States for 1977–1992, where 1 represents 1977, 2 represents 1978, and so on.

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.

(b) The cubic function of best fit is

$$R(x) = -2.4117x^3 + 63.4297x^2 - 453.7614x + 2648.0027$$

where x represents the year and R represents the number of barrels of refined oil products imported. Draw the cubic function of best fit on your scatter diagram.

(c) Use this function to predict the number of barrels of refined oil products imported in 1993 ($x = 17$).

(d) Verify that the function given in part (b) is the cubic function of best fit.

(e) Do you think the function given in part (b) will be useful in predicting the number of barrels of refined oil products for the year 1999? Why?

MODELING

Many examples and exercises connect real-world situations to mathematical concepts. Learning to work with **models** is a skill that transfers to many disciplines.

◀ **EXAMPLE 9 Using a Calculator to Approximate the Value of Trigonometric Functions**

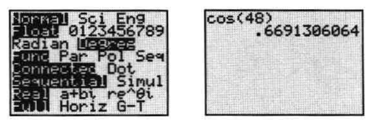
Use a calculator to find the approximate value of:

(a) $\cos 48°$ (b) $\csc 21°$ (c) $\tan \frac{\pi}{12}$

Express your answer rounded to two decimal places.

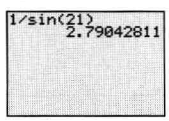
Solution (a) First, we set the MODE to receive degrees. See Figure 29(a). Figure 29(b) shows the solution using a TI-83 graphing calculator.

Figure 29



(a) (b)

Figure 30



Then

$$\cos 48° = 0.67$$

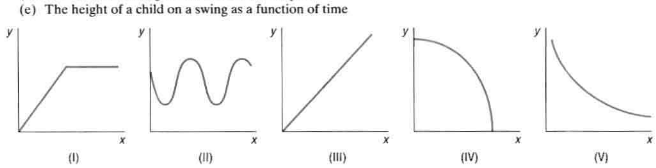
rounded to two decimal places.

(b) Most calculators do not have a csc key. The manufacturers assume that the user knows some trigonometry. To find the value of $\csc 21°$, we use the fact that $\csc 21° = 1/(\sin 21°)$. Figure 30 shows the solution using a TI-83 graphing calculator. Then

GRAPHING UTILITIES AND TECHNIQUES

Increase your understanding, visualize, discover, explore, and solve problems using a **graphing utility.** Sullivan uses the graphing utility to further your understanding of concepts not to circumvent essential math skills.

63. Match each of the following functions with the graphs that best describe the situation.
(a) The cost of building a house as a function of its square footage
(b) The height of an egg dropped from a 300-foot building as a function of time
(c) The height of a human as a function of time
(d) The demand for Big Macs as a function of price
(e) The height of a child on a swing as a function of time



DISCUSSION WRITING AND READING PROBLEMS

These **problems** are designed to get you to "think outside the box," therefore fostering an intuitive understanding of key mathematical concepts. In this example, matching the graph to the functions insures that you understand functions at a fundamental level.

CHAPTER REVIEW

The **Chapter Review** helps check your understanding of the chapter materials in several ways. **"Things to Know"** gives a general overview of review topics. The **"How To"** section provides a concept-by-concept listing of the operations you are expected to perform. The **"Review Exercises"** then serve as a chance to practice the concepts presented within the chapter. The review materials are designed to make you, the student, confident in your knowledge of the chapter material.

CHAPTER REVIEW

LIBRARY OF FUNCTIONS

Linear function
$f(x) = mx + b$ Graph is a line with slope m and y-intercept b.

Constant function
$f(x) = b$ Graph is a horizontal line with y-intercept b (see Figure 27).

Identity function
$f(x) = x$ Graph is a line with slope 1 and y-intercept 0 (see Figure 28).

Square function
$f(x) = x^2$ Graph is a parabola with intercept at (0, 0) (see Figure 29).

Cube function
$f(x) = x^3$ See Figure 30.

Square root function
$f(x) = \sqrt{x}$ See Figure 31.

Reciprocal function
$f(x) = 1/x$ See Figure 32.

Absolute value function
$f(x) = |x|$ See Figure 33.

THINGS TO KNOW

Function A relation between two sets of real numbers so that each number x in the first set, the domain, has corresponding to it exactly one number y in the second set. The range is the set of y values of the funtion for the x values in the domain.
x is the independent variable; y is the dependent variable.
A function f may be defined implicitly by an equation involving x and y or explicitly by writing $y = f(x)$.
A function can also be characterized as a set of ordered pairs (x, y) or $(x, f(x))$ in which no two distinct pairs have the same first element.

Function notation $y = f(x)$
f is a symbol for the function.
x is the argument, or independent variable.
y is the dependent variable.
$f(x)$ is the value of the function at x, or the image of x.

HOW TO

Determine whether a relation represents a function
Find the domain and range of a function from its graph
Find the domain of a function given its equation
Find the equation of the line of best fit
Solve direct variation problems
Find the average rate of change of a function
Determine algebraically whether a function is even or odd without graphing it
Use a graphing utility to find the local maxima and local minima of a function
Use a graphing utility to determine where a function is increasing or decreasing
Graph certain functions by shifting, compressing, stretching, and/or reflecting (see Table 10)
Perform operations on a function
Find the composite of two functions
Construct functions in applications, including piecewise-defined functions

FILL-IN-THE-BLANK ITEMS

1. If f is a function defined by the equation $y = f(x)$, then x is called the ______________ variable and y is the ______________ variable.
2. A set of points in the xy-plane is the graph of a function if and only if every ______________ line intersects the graph in at most one point.
3. For the linear function $f(x) = mx + b$, m is the ______________ and b is the ______________ of the graph.
4. The average rate of change of a function equals the ______________ of the secant line containing two points on its graph.
5. A(n) ______________ function f is one for which $f(-x) = f(x)$ for every x in the domain of f; a(n) ______________ function f is one for which $f(-x) = -f(x)$ for every x in the domain of f.

TRUE/FALSE ITEMS

T F 1. Every relation is a function.
T F 2. Vertical lines intersect the graph of a function in no more than one point.
T F 3. The y-intercept of the graph of the function $y = f(x)$ whose domain is all real numbers is $f(0)$.
T F 4. A function f is decreasing on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.
T F 5. Even functions have graphs that are symmetric with respect to the origin.
T F 6. The graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$.
T F 7. $f(g(x)) = f(x) \cdot g(x)$.
T F 8. The domain of the composite function $(f \circ g)(x)$ is the same as that of $g(x)$.

REVIEW EXERCISES

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

1. Given that f is a linear function, $f(4) = -5$ and $f(0) = 3$, write the equation that defines f.
2. Given that g is a linear function with slope $= -4$ and $g(-2) = 2$, write the equation that defines g.
3. A function f is defined by

$$f(x) = \frac{Ax + 5}{6x - 2}$$

4. A function g is defined by

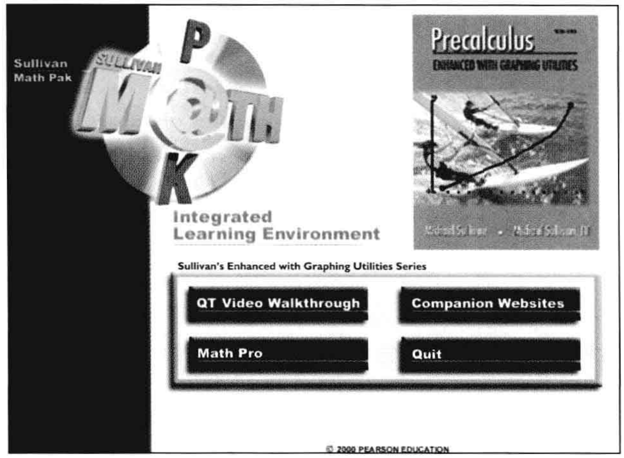
$$g(x) = \frac{A}{x} + \frac{8}{x^2}$$

MULTIMEDIA

Sullivan M@thP@k

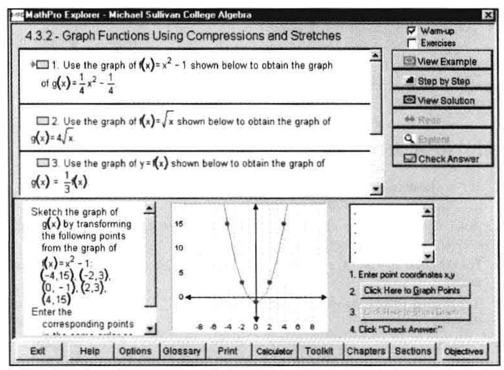
An Integrated Learning Environment

Today's textbooks offer a wide variety of ancillary materials to students, from solutions manuals to tutorial software to text-specific Websites. Making the most of all of these resources can be difficult. Sullivan M@thP@k helps students get it together. M@thP@k seamlessly integrates the following key products into an **integrated learning environment.**



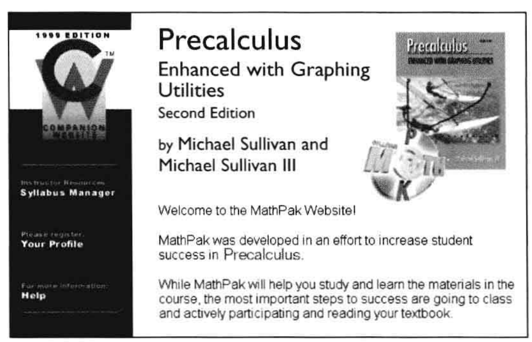
MathPro Explorer 4.0

This hands-on tutorial software reinforces the college algebra content. More than 100 video clips, unlimited practice problems, and interactive step-by-step examples review college algebra materials. The exploratory component allows students to experiment with mathematical principles on their own in addition to the prescribed exercises.



The Sullivan M@thP@k Website

This robust pass-code protected site features quizzes, homework starters, live animated examples, graphing calculator manuals, and much more. It offers the student many, many ways to test and reinforce their understanding of the course material.



Student Solutions Manual

Written by Michael Sullivan III, co-author of both Sullivan series, and by Katy Murphy, the *Student Solutions Manual* offers thorough solutions that are consistent with the precise mathematics found in the text.

Sullivan M@thP@k.

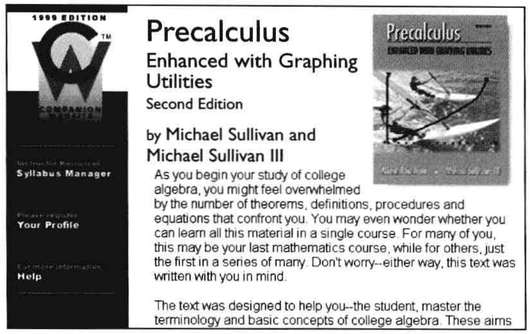
Helping students *Get it Together.*

ADDITIONAL MEDIA

Sullivan Companion Website

www.prenhall.com/sullivan

This text-specific website beautifully complements the text. Here students can find chapter tests, section-specific links, and PowerPoint downloads in addition to other helpful features.



TestPro 4.0

This algorithmically generated testing software features an equation editor and a graphing utility, and offers online testing capabilities. Windows/Macintosh CD-ROM.

SUPPLEMENTS

Student Supplements

Student Solutions Manual

Worked solutions to all odd-numbered exercises from the text and complete solutions for chapter review problems and chapter tests. ISBN: 0-13-022677-7

Lecture Videos

The instructional tapes, in a lecture format, feature worked-out examples and exercises, taken from each section of the text. ISBN: 0-13-022504-5

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Free guide providing a brief history of the Internet, discussing the use of the World Wide Web, and describing how to find your way within the Internet and how to find others on it.

Instructor Supplements

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Contains complete step-by-step worked-out solutions to all even-numbered exercises in the textbook. Also included are strategies for using the Collaborative Learning projects found in each chapter. ISBN: 0-13-022671-8

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