Models for Public Systems Analysis

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Preface

The past several years have been witness to a new interest in the use of mathematical models for solving urgent problems in the public sector. In this book, several classes of such models are reviewed with particular attention centered on questions of how to improve the delivery of urban service systems (sanitation, fire, police, ambulances). The emphasis throughout is on model formulation, by which we mean the translation of significant societal problems into a mathematical framework. Each framework idealizes reality to a lesser or greater degree according to the complexity of the problem. It is important therefore that one discuss the limitations and pitfalls of modeling, and give an appraisal of when its use is appropriate.

In contrast to model formulation little will be said about solution methods. On occasion, however, a particular method is discussed when we feel that it is not adequately covered in standard texts. This particularly applies to our treatment of heuristic algorithms in the last chapter. In an attempt to make the book reasonably self-contained, a set of short appendixes is provided dealing with a review of specific techniques. These give the background for a discussion of the basic models that are interwoven throughout the text in one form or other (notably fixed charge plant location, multiple set covering, and multiservor queueing).

The contents of this book are based on lectures given in the last few years to students in applied mathematics and policy sciences at the State University of New York at Stony Brook, as well as on a number of lectures given elsewhere. By and large the material is standard fare in operations research except that the choice of applications is decidably not standard. Indeed, many of the references are scattered about in a number of unpublished reports which are not easily accessible to the general reader. Moreover, with the exception of the journal *Urban Analysis*, there appears to be no single scholarly publication that is devoted entirely to the material that is the subject of this book, although a number of important papers are published from time to time in *Operations Research* and *Management Science*.

We have kept the level of mathematics deliberately elementary in order to reach a wide audience of people who include, in addition to operations analysts and applied mathematicians, urban and regional planners, specialists in public administration, and consultants in industrial management. Prerequisite is an ability to work comfortably with quantitatively oriented ideas. First year calculus and some probability and statistics should be all the formal training that is needed to follow most, perhaps all, of the text. We believe the book is useful for self-study. To assist in a review of the material and as an aid to understanding, a few fairly uncomplicated exercises are appended to each chapter. If the book is to be used in a classroom environment, it is recommended that students be asked to code and run some of the material discussed, using standard computer packages such as MPS-X, in order to acquire a feel for the kind of numbers one gets in using such models. This is in fact the way we conducted our own classroom exercises at Stony Brook.

Now for a quick review of the contents. Chapter 1 is essentially a treatment of plant location and siting questions, with a brief overview of water resource modeling. The Federal Water Pollution Control Act of 1972 has brought these ideas to the forefront of public interest. Also included is a formulation of recent models developed at the Brookhaven National Laboratory relating to energy supply and distribution on a regional and national scale. Chapter 2 develops set-covering models for manpower scheduling as a direct outgrowth of the author's experience with the Sanitation Department in New York City. The third and fourth chapters deal with the delivery of emergency services, particularly with models of congestion and delay and of optimal deployment. Here the analysis is probabilistic in nature since both the spatial and the temporal patterns of demand are intrinsically uncertain. This is in contrast to the nonemergency systems dealt with in the other chapters, where deterministic modeling appears to be more appropriate. The tools used are queueing theory and geometric probability. In the last chapter network optimization methods are employed, mainly to explore questions of vehicle routing and scheduling. Such problems occur in a variety of contexts in the public sector. Following this are a few comments on large-scale models of urban growth, these being generally more familiar to the regional planner then to the operations analyst.

Most of the work discussed in the book is based on studies conducted in an operational environment and much, if not all, of it has been implemented with varying degrees of success in a number of municipalities. Thus, for example, the models of Chapters 3 and 4 have been applied in parts of the greater New York City and Boston areas, while the second and fifth chapters relate to work carried out principally (but not exclusively) in New York City. A number of other cities have also benefited from these studies, ranging from Washington, D.C. and Denver to Milan and Zurich. Unfortunately not all this work has been adequately documented and so an overall assessment of what has, and is, being done in the area of public systems analysis is difficult to attain. The purpose of this book is to give an early overview of some of the ideas that have already proven their usefulness.

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INTRODUCTION

Some Thoughts on Mathematics and Public Policy

Moral: A Word to the Wise Is Not Sufficient If It Doesn't Make Any Sense

JAMES THURBER
Further Fables for Our Time

In this book interest is focused on problems related to the delivery of municipal services. Most of the uniformed services, such as sanitation, fire, police, and transit, are all labor-intensive activities. Rising labor costs and increasing demand have in recent years put pressure on the cities to meet or, as it often happens, to close the gap between service levels and actual need. A growing percentage of the municipal budget has correspondingly been allocated to provide for these needs and, to cite the case of New York City, over a quarter of a billion dollars annually is currently budgeted to each of the departments of Sanitation and Fire.

In addition to rapidly increasing labor costs and soaring demand (some of it abusive, as witnessed by the high incidence of false alarms), the delivery of services has been hampered by such factors as a physical armature which has not kept pace with demographic shifts within the city and the existence of powerful and sometimes militant unions who tend to resist changes in long standing but often obsolete labor practices.

As much as 90% of the costs of the services is due to wages and benefits for the workers and therefore one way to effect an improvement in their delivery is to induce changes in manpower utilization so as to *increase productivity for the same level of dollar cost*. These changes are not so much in the direction of new and better equipment, desirable as such capital investments may be, but rather in altering the operational configuration in which both men and equipment are deployed. Admittedly this is a short term tactic, but one which often can relieve visable and pressing needs.

There is a role for analysis here and in this book we briefly survey some of the operational models which have been developed in the last few years for this purpose. Since my own background and experience is principally in New York, I will restrict myself largely to describing work performed there, almost all of it during the recent administration of Mayor John V. Lindsay.

Although our review focuses on some of the modeling questions, it would be a discredit to this kind of analysis not to emphasize that *the real goal is change*. Therefore, the extent to which analysis becomes a tool for this purpose is to be regarded as a measure of its success.

It appears that much well-intentioned work by analysts is relegated to dusty shelves because it does not (and, in all fairness, often cannot) bridge what has been referred to as the "missing chapter" between a set of recommendations and an implemented change. It is characteristic of many otherwise well-trained people to want to believe in a rational approach to problem solving. Unfortunately, urgent city problems often resist the application of "obvious" solutions simply because such solutions fail to account for the institutional mechanisms through which action takes place. Ideas which are acceptable at the upper echelons of urban management either have no impact at all on policy or often reappear in altered guise after percolating down to the day to day operating level. This can happen for a number of reasons. Any change, for example, that threatens long established traditions of the workers or that in any way tends, however inadvertently, to foul them up is an open invitation to low compliance. I recall, to cite an instance, how a negotiated rescheduling of sanitationmen nearly fell through because it ignored the fact that the change could disrupt worker carpools.

We see then that unlike military or even industrial systems, urban systems achieve their goals by satisfying the needs of many different interest groups. Every proposed "optimal solution" to a perceived problem is ultimately altered and modified to suit political, fiscal, and even social expediency. Moreover, even if all parties agree on certain innovations it may turn out that some of the actions are not worth carrying out. Often such pitfalls in the way analysis is transferred is not anticipated until the recommended "optimal" policy is taken out of the hands of the analyst and into the streets.

What this suggests is that in dealing with public sector problems what one generally faces is less a matter of optimization than of accommodation among feasible alternatives, some of which are more or less acceptable than others. It also suggests that successful implementation of quantitative analysis requires a sensitivity to how public institutions actually work and not only of how they should work. For an applied mathematician to be credible in this area he or she should be willing to invest some time and effort working with public agencies on their own turf and playing by their rules. In this way one infiltrates, as it were, the municipal system and becomes an agent for change. Moreover, for the kind of short term operational questions that we are dealing with here the analysis should be action oriented. One thinks in terms of daily, sometimes violent, street terms: ambulances responding to incidents, barges loading garbage at downtown piers. Perhaps I run the risk of overstatement. Surely not all involvement of scientists with the cities requires such an unremitting struggle but I think it important that the pitfalls which lie between mathematical modeling and an effective urban policy not be underestimated.

In the remainder of this book the emphasis is on some of the mathematical questions which were harvested from a few actual case experiences. In discussing these we generally obscure how the analysis was translated into action but let me state my conviction that at least several implementable and, very likely, beneficial results regarding municipal services can be traced to the mathematical arguments that are reviewed here. Although at first sight the organizational, legal, and fiscal constraints which constitute the urban environment would appear to preclude any formal treatment of such questions, mathematical models have in fact sometimes yielded penetrating insights into the operation of these services.

REFERENCES

There are a number of papers which explore the role of analysis in the public arena. To list a few that relate to the study of municipal service systems in New York City let us mention but three:

R. Archibald and R. Hoffman, "Introducing Technological Change in a Bureaucratic Structure," Rand Rep. P-4025 (1969);

P. Szanton, "Systems Problems in the City," Rand Rep. P-4821 (1972);

B. Gifford, "Quantitative Policy Analysis in the City: Limits to Growth," IEEE Section on Systems, Man and Cybernetics, Meeting, Boston (Nov. 1973);

see also

"Making Ideas Work," Introduction to *Third Annual Report*, New York City Rand Inst. (1973).

CHAPTER

1

Plant Location and Optimal Distribution

1.1 A WASTE DISPOSAL PROBLEM

The following material outlines the essential details of a study done for New York City about 1971. It was part of the problem to develop a strategy for the city on waste disposal. Because of antipollution legislation the question examined here is the tradeoff between new incinerator technology and the more traditional landfill disposal methods. We begin with some background material.

New York City disposes of over 25,000 tons of solid waste daily or about 7 million tons per year. The refuse is collected by truck and is either taken to incinerators, or to one of a half dozen sanitary landfills on the periphery of the city, or to one of the transfer stations located on the waters' edge. From the transfer stations, where trucks unload onto waiting barges, the refuse is then hauled by tugboat to a remote landfill known as Fresh Kills (Staten Island, New York).

On-site incinerators in apartments and other buildings as well as from municipal incinerators accounted for about 70% of particulate matter pumped into the atmosphere in New York City in 1965. In the following year an air pollution control bill, known as local law 14, was passed and signed into law.

With regard to municipal incinerators where, at that time, about 30% of all refuse was disposed (or over 2 million tons per year) compliance with the antipollution law meant that New York City would be faced with the prospect of either shutting down or upgrading its low efficiency and high pollution incinerators. The upgrading had been estimated to cost several millions of dollars, a considerable sum even for New York, and in addition would be technologically risky. The alternative, then, would appear to be the shutdown of at least some of these facilities.

The removal of some of the City's disposal locations has the effect of increasing the cost of collection since the trucks now have to travel longer distances to dump their loads at the remaining sites and, since these locations are likely to be more congested than usual, the collection cost (which already accounts for about 80% of the overall budget for collection–disposal) is increased further.

Moreover, most of the landfills were nearly exhausted, with essentially no new sites available for development as sanitary landfills, which meant that although the then current disposal cost at a fill was low, the most feasible alternative to landfill—rail haul to disposal sites outside the city—would cost about five times as much in the future. For this reason early depletion of the fills will also result in higher disposal costs. In addition the capacity of the fills and transfer stations is limited to a maximum daily throughput. For example, a typical marine transfer facility can handle no more than about 1800 tons daily.

It is clear then, that in order to comply with local law 14, at least part of the roughly six–seven thousand tons per day processed by the municipal incinerators would have to go to the remaining facilities. Since these locations are physically limited in the amount of additional load they can absorb, and since the economic drain on both collection and disposal costs is substantial, one is faced with a difficulty. The problem was to develop a strategy for disposal which reduced the impact of these obstacles.

One possibility suggested at that time was to build a series of small incinerators in the 150–300 tons/day range (the usual size was about 1000 tons/day) which have the effect of dispersing the air pollution over a larger area and in smaller quantities. Indeed in this study we want to evaluate the cost–effectiveness of using a large number of smaller high efficiency pyrolytic incinerators. Such incinerators burn the refuse at extremely high temperatures in the absence of air and they may have a number of useful by-products: notably heavy oil, magnetic metals such as iron fillings, and steam (which can be used for space heating). Moreover the residue from these incinerators is essentially negligible (as we assume in this study) and air pollution is low.

It should be mentioned, however, that such new facilities cannot be located anywhere. The public is generally reluctant to accept them in their own neighborhoods and political pressure is often sufficient to discourage city administrators from considering certain otherwise desirable sites as potential locations. A community can reject a site because of real or perceived risks to their health, loss of neighborhood status, increase in noise and traffic, and other hazards. However, this possible political pitfall will not be considered further in our study and it is assumed that selected sites are available for construction of new facilities.

The operating costs of these "mini"-incinerators (or, as we shall call them, MI) were estimated to be \$10/ton. However, there is the possibility of recycling metal for \$2/ton saving and selling steam for another \$2/ton. Metal recycling looked more promising at that time than the selling of the steam.

Capital costs of an MI were estimated by the vendors to be in the range of ten to fifteen thousand dollars a day per ton of capacity. Capital costs can be made part of operating costs by amortising repayment of capital costs over a number of years. Assuming 15 years of amortisation at 7% interest, MI capital costs would be between \$3.46 and \$5.18/ton. Therefore the total per ton cost of an MI was in the range of \$9.46–\$15.18.

Because of the possibility of adding MIs to the disposal system it is important to carry out an analysis of whether MIs are economically feasible. Note that what is not being discussed is technological feasibility which is a problem of meeting engineering requirements and environmental constraints.

In order to focus the discussion, we have chosen an area in the borough of Queens (New York) which is presently served by one marine transfer station (MTS) as shown in Fig. 1.1. That is, all the refuse generated within the five

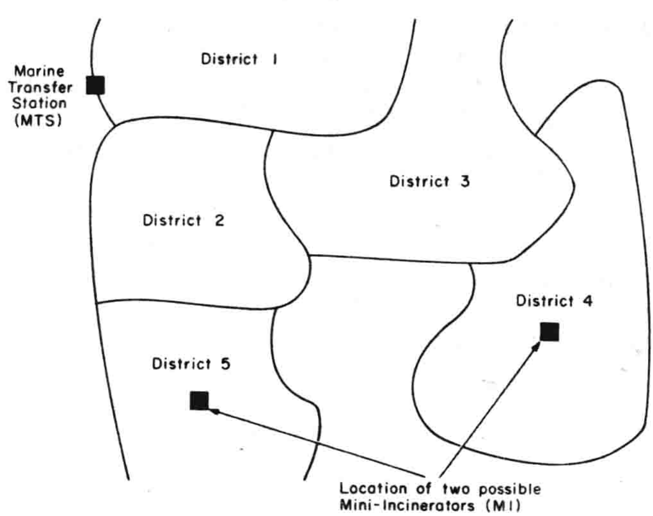

FIG. 1.1 Collection districts in the borough of Queens, New York.

districts of this area is sent to the MTS by trucks and is taken from there by barge to the Fresh Kills landfill.

The intuitive feeling about the economic effects of an MI system is that although the operational–capital cost of an MI will be higher than that of an MTS, having MIs located in certain districts has two potential advantages:

1. More facilities reduce collection costs since roundtrip travel for the trucks between the sites they service and disposal locations will now be less.
2. Disposal costs are reduced by virtue of the fact that the present landfill supply can be maintained somewhat longer than if MIs were not available. As we noted above, any alternative to landfill will inevitably cost more.

The question of the economic acceptability of this disposal alternative then becomes an analysis of whether truck travel costs and landfill costs are reduced more than the increase in disposal costs.

One technique to find the cost savings of an MI disposal system is to first calculate what the MI cost per ton would have to be to make this system as costly as the present MTS system.

This *breakeven cost* is calculated below but first it is necessary to give more cost figures (based on 1971 estimates). To begin with, the MTS incurs an operational expense which is calculated to be \$4.33/ton, which includes landfill cost at Fresh Kills. Secondly, the future cost of disposal when present landfill capacity is depleted is roughly taken to be \$10/ton. This figure is based on considering various alternatives to the exhausted landfill supply. The savings that accrue by diverting refuse from the fills to the incinerators is \$10 − \$4.33 = \$5.67/ton, since this prolongs the life of the existing fills. However, this savings will not be realized until later years and so its effect on present cost is discounted (see discussion in the notes at the end of this chapter). This results in a best guess of \$3.32 as the savings per ton of refuse not sent to landfill. The actual saving could lie in a range of between two and five dollars, a point which is taken up again later.

Now consider a ton of refuse diverted from the MTS and sent to an MI. How much more does it cost? Suppose y is the combined operating and capital expense of the MI. Then y − \$4.33 is the additional expense. Now consider the savings which result from diverting this same ton. Landfill saving is \$3.32, and there is an additional gain of x dollars because of lower hauling costs. Thus the breakeven cost for y is defined by

$$y - 4.33 = x + 3.22 \qquad [1.1.1]$$

The quantity x is found like this: consider how much it costs to send all refuse daily to the MTS alone. Call this amount α. Now compute how much it costs to dispose refuse when the MIs are available. Call this β. Then $\alpha - \beta$