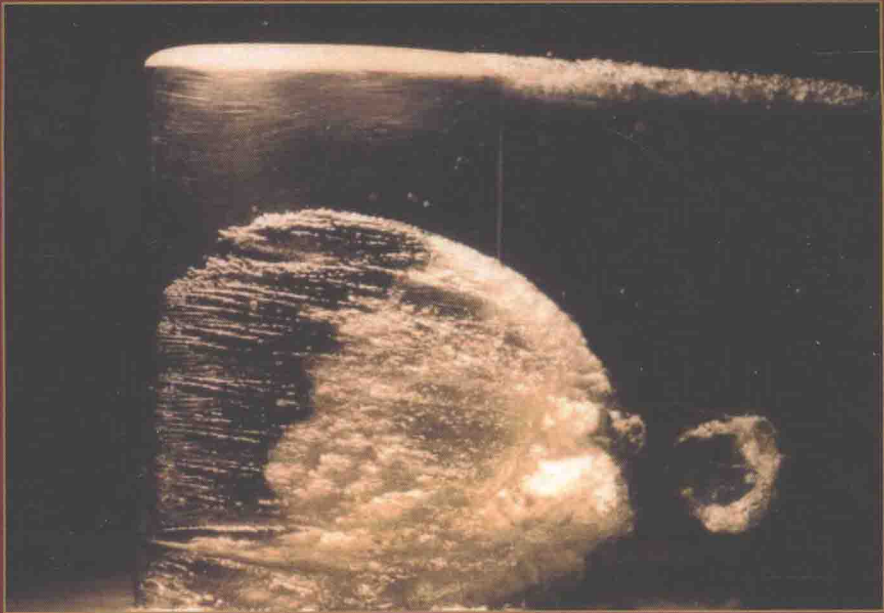


CHRISTOPHER EARLS BRENNEN



CAVITATION  
AND BUBBLE  
DYNAMICS

CAMBRIDGE

# CAVITATION AND BUBBLE DYNAMICS

**Christopher Earls Brennen**

California Institute of Technology



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## CAVITATION AND BUBBLE DYNAMICS

*Cavitation and Bubble Dynamics* deals with the fundamental physical processes of bubble dynamics and the phenomenon of cavitation. It is ideal for graduate students and research engineers and scientists; a basic knowledge of fluid flow and heat transfer is assumed. The analytical methods presented are developed from basic principles.

The book begins with a chapter on nucleation and describes both the theory and observations in flowing and non-flowing systems. Three chapters provide a systematic treatment of the dynamics and growth, collapse, or oscillation of individual bubbles in otherwise quiescent fluids. The following chapters summarize the motion of bubbles in liquids; describe some of the phenomena that occur in homogeneous bubbly flows, with emphasis on cloud cavitation; and summarize some of the experimental observations of cavitating flows. The last chapter provides a review of free streamline methods used to treat separated cavity flows with large attached cavities.

Christopher Earls Brennen is the Richard and Dorothy M. Hayman Professor of Mechanical Engineering Emeritus in the Faculty of Engineering and Applied Science at the California Institute of Technology. He has published more than 200 refereed articles and is especially well known for his research on cavitation, turbomachinery, and multiphase flows. He is the author of three textbooks – *Fundamentals of Multiphase Flows*, *Hydrodynamics of Pumps*, and *Cavitation and Bubble Dynamics* – and has edited several others. Among his many honors and positions are UN Consultant to India (1980); NASA New Technology Award (1980); ASME Centennial Medallion (1980); Chair ASME Fluids Engineering Division (1984–1985); ASME R. T. Knapp Award for Best Paper (1978 and 1981); Caltech Student Award for teaching excellence (1982, 1995, 1996); Christensen Fellow, St. Catherine's College, Oxford (1992); Fellow of the Japan Society for Promotion of Science (1993); JSME Fluids Engineering Award (2002); and Vice-President of Student Affairs at Caltech.

*“Where Alph, the sacred river, ran  
Through caverns measureless to man ...”*

Samuel Taylor Coleridge (1772–1834)

## Preface

This book is intended as a combination of a reference book for those who work with cavitation or bubble dynamics and as a monograph for advanced students interested in some of the basic problems associated with this category of multiphase flows. A book like this has many roots. It began many years ago when, as a young post-doctoral Fellow at the California Institute of Technology, I was asked to prepare a series of lectures on cavitation for a graduate course cum seminar series. It was truly a baptism by fire, for the audience included three of the great names in cavitation research, Milton Plesset, Allan Acosta, and Theodore Wu, none of whom readily accepted superficial explanations. For that, I am immensely grateful. The course and I survived, and it evolved into one part of a graduate program in multiphase flows.

There are many people to whom I owe a debt of gratitude for the roles they played in making this book possible. It was my great good fortune to have known and studied with six outstanding scholars, Les Woods, George Gadd, Milton Plesset, Allan Acosta, Ted Wu, and Rolf Sabersky. I benefited immensely from their scholarship and their friendship. I also owe much to my many colleagues in the American Society of Mechanical Engineers whose insights fill many of the pages of this monograph. The support of my research program by the Office of Naval Research is also greatly appreciated. And, of course, I feel honored to have worked with an outstanding group of graduate students at Caltech, including Sheung-Lip Ng, Kiam Oey, David Braisted, Luca d'Agostino, Steven Ceccio, Sanjay Kumar, Douglas Hart, Yan Kuhn de Chizelle, Beth McKenney, Zhenhuan Liu, Yi-Chun Wang, Ronald J. Franz, Tricia A. Waniewski, Fabrizio d'Auria, Garrett E. Reisman, Mark E. Duttweiler, and Keita Ando, all of whom studied aspects of cavitating flows.

The original edition of this book was published by Oxford University Press in 1995. This corrected edition is published by Cambridge University Press and I wish to express my sincerest thanks to Peter Gordon of Cambridge for his expert help and encouragement on this and other projects.

This edition is dedicated with great love and deep gratitude to my wife and lifelong friend, Barbara.

*Pasadena, Calif.  
January 2013*

C.E.B.

# Nomenclature

## Roman Letters

$a$	Amplitude of wave-like disturbance
$A$	Cross-sectional area or cloud radius
$b$	Body half-width
$B$	Tunnel half-width
$c$	Concentration of dissolved gas in liquid, speed of sound, chord
$c_k$	Phase velocity for wavenumber $k$
$c_p$	Specific heat at constant pressure
$C_D$	Drag coefficient
$C_{ij}$	Lift/drag coefficient matrix
$C_L$	Lift coefficient
$\tilde{C}_{Lh}, \tilde{C}_{Lp}$	Unsteady lift coefficients
$C_M$	Moment coefficient
$\tilde{C}_{Mh}, \tilde{C}_{Mp}$	Unsteady moment coefficients
$C_p$	Coefficient of pressure
$C_{pmin}$	Minimum coefficient of pressure
$d$	Cavity half-width, blade thickness to spacing ratio
$D$	Mass diffusivity
$f$	Frequency in Hz
$f$	Complex velocity potential, $\phi + i\psi$
$f_N$	A thermodynamic property of the phase or component, $N$
$Fr$	Froude number
$g$	Acceleration due to gravity
$g_N$	A thermodynamic property of the phase or component $N$
$g_x$	Component of the gravitational acceleration in direction $x$
$\mathcal{G}(f)$	Spectral density function of sound
$h$	Specific enthalpy, wetted surface elevation, blade tip spacing
$H$	Henry's law constant
$Hm$	Haberman-Morton number, normally $g\mu^4/\rho S^3$
$i, j, k$	Indices

$i$	Square root of $-1$ in free streamline analysis
$I$	Acoustic impulse
$I^*$	Dimensionless acoustic impulse, $4\pi I\mathcal{R}/\rho_L U_\infty R_H^2$
$I_{Ki}$	Kelvin impulse vector
$j$	Square root of $-1$
$k$	Boltzmann's constant, polytropic constant or wavenumber
$k_N$	Thermal conductivity or thermodynamic property of $N$
$K_G$	Gas constant
$K_{ij}$	Added mass coefficient matrix, $M_{ij}/\frac{4}{3}\rho\pi R^3$
$Kc$	Keulegan-Carpenter number
$Kn$	Knudsen number, $\lambda/2R$
$\ell$	Typical dimension in the flow, cavity half-length
$L$	Latent heat of vaporization
$m$	Mass
$m_G$	Mass of gas in bubble
$m_p$	Mass of particle
$M_{ij}$	Added mass matrix
$n$	Index used for harmonics or number of sites per unit area
$N(R)$	Number density distribution function of $R$
$\dot{N}_E$	Cavitation event rate
$Nu$	Nusselt number
$p$	Pressure
$p_a$	Radiated acoustic pressure
$p_s$	Root mean square sound pressure
$p_S$	A sound pressure level
$p_G$	Partial pressure of gas
$P$	Pseudo-pressure
$Pe$	Peclet number, usually $WR/\alpha_L$
$q$	Magnitude of velocity vector
$q_c$	Free surface velocity
$Q$	Source strength
$r$	Radial coordinate
$R$	Bubble radius
$R_B$	Equivalent volumetric radius, $[3\tau/4\pi]^{1/3}$
$R_H$	Headform radius
$R_M$	Maximum bubble radius
$R_N$	Cavitation nucleus radius
$R_P$	Nucleation site radius
$\mathcal{R}$	Distance to measurement point
$Re$	Reynolds number, usually $2WR/\nu_L$
$s$	Coordinate measured along a streamline or surface
$s$	Specific entropy
$S$	Surface tension



$St$	Strouhal number, $2fR/W$
$t$	Time
$t_R$	Relaxation time for relative motion
$t_*$	Dimensionless time, $t/t_R$
$T$	Temperature
$u, v, w$	Velocity components in Cartesian coordinates
$u_i$	Velocity vector
$u_r, u_\theta$	Velocity components in polar coordinates
$u'$	Perturbation velocity in x direction, $u - U_\infty$
$U, U_i$	Fluid velocity and velocity vector in absence of particle
$U_\infty$	Velocity of upstream uniform flow
$V, V_i$	Absolute velocity and velocity vector of particle
$w$	Complex conjugate velocity, $u - iv$
$w$	Dimensionless relative velocity, $W/W_\infty$
$W$	Relative velocity of particle
$W_\infty$	Terminal velocity of particle
$We$	Weber number, $2\rho W^2 R/S$
$z$	Complex position vector, $x + iy$

### Greek Letters

$\alpha$	Thermal diffusivity, volume fraction, angle of incidence
$\beta$	Cascade stagger angle, other local variables
$\gamma$	Ratio of specific heats of gas
$\Gamma$	Circulation, other local parameters
$\delta$	Boundary layer thickness or increment of frequency
$\delta_D$	Dissipation coefficient
$\delta_T$	Thermal boundary layer thickness
$\epsilon$	Fractional volume
$\zeta$	Complex variable, $\xi + i\eta$
$\eta$	Bubble population per unit liquid volume
$\eta$	Coordinate in $\zeta$ -plane
$\theta$	Angular coordinate or direction of velocity vector
$\kappa$	Bulk modulus of compressibility
$\lambda$	Mean free path of molecules or particles
$\Lambda$	Accommodation coefficient
$\mu$	Dynamic viscosity
$\nu$	Kinematic viscosity
$\xi$	Coordinate in $\zeta$ -plane
$w$	Logarithmic hodograph variable, $\chi + i\theta$
$\rho$	Density
$\sigma$	Cavitation number
$\sigma_c$	Choked cavitation number

$\sigma_{ij}$	Stress tensor
$\Sigma$	Thermal parameter in bubble growth
$\tau$	Volume of particle or bubble
$\phi$	Velocity potential
$\phi'$	Acceleration potential
$\varphi$	Fractional perturbation in bubble radius
$\Phi$	Potential energy
$\chi$	$\log(q_c/ w )$
$\psi$	Stream function
$\omega$	Radian frequency
$\omega^*$	Reduced frequency, $\omega c/U_\infty$

### Subscripts

On any variable,  $Q$ :

$Q_o$	Initial value, upstream value, or reservoir value
$Q_1, Q_2, Q_3$	Components of $Q$ in three Cartesian directions
$Q_1, Q_2$	Values upstream and downstream of a shock
$Q_\infty$	Value far from the bubble or in the upstream flow
$Q_B$	Value in the bubble
$Q_C$	Critical values and values at the critical point
$Q_E$	Equilibrium value or value on the saturated liquid/vapor line
$Q_G$	Value for the gas
$Q_i$	Components of vector $Q$
$Q_{ij}$	Components of tensor $Q$
$Q_L$	Saturated liquid value
$Q_n$	Harmonic of order $n$
$Q_P$	Peak value
$Q_S$	Value on the interface or at constant entropy
$Q_V$	Saturated vapor value
$Q^*$	Value at the throat

### Superscripts and Other Qualifiers

On any variable,  $Q$ :

$\bar{Q}$	Mean value of $Q$ or complex conjugate of $Q$
$\tilde{Q}$	Complex amplitude of oscillating $Q$
$\dot{Q}$	Time derivative of $Q$
$\ddot{Q}$	Second time derivative of $Q$
$\hat{Q}(s)$	Laplace transform of $Q(t)$
$\bar{Q}$	Coordinate with origin at image point
$Q^+, Q^-$	Values of $Q$ on either side of a cut in a complex plane

$\delta Q$	Small change in $Q$
$Re\{Q\}$	Real part of $Q$
$Im\{Q\}$	Imaginary part of $Q$

## Units

In most of this book, the emphasis is placed on the nondimensional parameters that govern the phenomenon being discussed. However, there are also circumstances in which we shall utilize dimensional thermodynamic and transport properties. In such cases the International System of Units is employed using the basic units of mass (kg), length (m), time (s), and absolute temperature (K); where it is particularly convenient units such as a joule ( $\text{kg m}^2/\text{s}^2$ ) are occasionally used.

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### **1.1 Introduction**

This first chapter will focus on the mechanisms of formation of two-phase mixtures of vapor and liquid. Particular attention will be given to the process of the creation of vapor bubbles in a liquid. In doing so we will attempt to meld together several overlapping areas of research activity. First, there are the studies of the fundamental physics of nucleation as epitomized by the books of Frenkel (1955) and Skripov (1974). These deal largely with very pure liquids and clean environments in order to isolate the behavior of pure liquids. On the other hand, most engineering systems are impure or contaminated in ways that have important effects on the process of nucleation. The later part of the chapter will deal with the physics of nucleation in such engineering environments. This engineering knowledge tends to be divided into two somewhat separate fields of interest, cavitation and boiling. A rough but useful way of distinguishing these two processes is to define cavitation as the process of nucleation in a liquid when the pressure falls below the vapor pressure, while boiling is the process of nucleation that occurs when the temperature is raised above the saturated vapor/liquid temperature. Of course, from a basic physical point of view, there is little difference between the two processes, and we shall attempt to review the two processes of nucleation simultaneously. The differences in the two processes occur because of the different complicating factors that occur in a cavitating flow on the one hand and in the temperature gradients and wall effects that occur in boiling on the other hand. The last sections of this first chapter will dwell on some of these complicating factors.

### **1.2 The Liquid State**

Any discussion of the process of phase change from liquid to gas or vice versa must necessarily be preceded by a discussion of the liquid state. Though simple kinetic theory understanding of the gaseous state is sufficient for our purposes, it is necessary to dwell somewhat longer on the nature of the liquid state. In doing so we shall follow Frenkel (1955), though it should also be noted that modern studies are usually couched in terms of statistical mechanics (for example, Carey 1992).



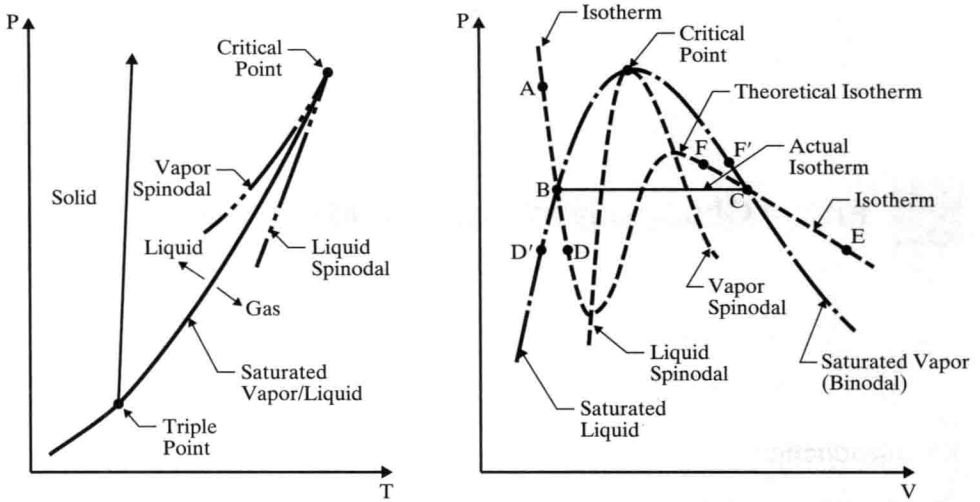


Figure 1.1 Typical phase diagrams.

Our discussion will begin with typical phase diagrams, which, though idealized, are relevant to many practical substances. Figure 1.1 shows typical graphs of pressure,  $p$ , temperature,  $T$ , and specific volume,  $V$ , in which the state of the substance is indicated. The triple point is that point in the phase diagram at which the solid, liquid, and vapor states coexist; that is to say the substance has three alternative stable states. The saturated liquid/vapor line (or binodal) extends from this point to the critical point. Thermodynamically it is defined by the fact that the chemical potentials of the two coexisting phases must be equal. On this line the vapor and liquid states represent two limiting forms of a single “amorphous” state, one of which can be obtained from the other by isothermal volumetric changes, leading through intermediate but unstable states. To quote Frenkel (1955), “Owing to this instability, the actual transition from the liquid state to the gaseous one and vice versa takes place *not* along a *theoretical* isotherm (dashed line, right, Figure 1.1), but along a horizontal isotherm (solid line), corresponding to the splitting up of the original homogeneous substance into two different coexisting phases...” The critical point is that point at which the maxima and minima in the theoretical isotherm vanish and the discontinuity disappears.

The line joining the maxima in the theoretical isotherms is called the vapor spinodal line; the line joining the minima is called the liquid spinodal line. Clearly both spinodals end at the critical point. The two regions between the spinodal lines and the saturated (or binodal) lines are of particular interest because the conditions represented by the theoretical isotherm within these regions can be realized in practice under certain special conditions. If, for example, a pure liquid at the state A (Figure 1.1) is depressurized at constant temperature, then several things may happen when the pressure is reduced below that of point B (the saturated vapor pressure). If sufficient numbers of nucleation sites of sufficient size are present (and this needs further discussion later) the liquid will become vapor as the state moves