String Theory and Its Applications

From meV to the Planck Scale

Michael Dine • Thomas Banks • Subir Sachdev

editors

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Preface

Theoretical particle physics is a rapidly developing subject. Students planning to work in the field need intense exposure to a broad array of topics. Since 1984, the Theoretical Advanced Study Institute (TASI) has provided a four week program of pedagogical lectures aimed at advanced Graduate Students in elementary particle physics. Since 1989, the school has been located at the University of Colorado at Boulder. Each year, a theme for the school and directors are selected by the Scientific Advisory Board. In 2010, it was decided that the school should have a string theory emphasis.

Two developments guided the structure of the program. First, the school took place just as the Large Hadron collider was beginning to ramp up. So lectures included experimental and theoretical reviews of LHC physics, while many of the string and supersymmetry talks had a phenomenological emphasis. Second, the past few years have seen striking progress in the application of gauge/gravity dualities to questions in condensed matter and high temperature/density physics (heavy ion physics). About 1/3 of the lectures were aimed at providing students with the tools to understand these developments. Given these themes, we selected the title "String Theory and its Applications: from the meV to the Planck scale". The result was an eclectic, intense but coherent program.

Almost all of the lectures are reproduced in this volume. The exceptions are those by Nathan Seiberg and Krishna Rajagopal. The lectures by Seiberg introduced important recent developments in our understanding of four dimensional supersymmetric theories. Interested readers might wish to consult his review with K. Intriligator, "Lectures on Supersymmetry Breaking", in *Classical and Quantum Gravity* **24**, BS741 (2007), [arXiv:hep-ph/0702069]. Rajagopal focused on heavy ion physics and its understanding within QCD, including, but not limited to, connections with duality. The review article, "Gauge/String Duality, Hot QCD and Heavy Ion Collisions" by H. Liu, D. Mateos, K. Rajagoal and U.A. Wiedemann, arXiv:1101.0618 provides a pedagogical introduction to the material in these lectures. We

thank Cambridge University Press for permission to publish the lectures by Minwalla et al., which will appear also in the volume *Black Holes in Higher Dimensions* (G. Horowitz, ed.).

We are very appreciative of the efforts of the lecturers, both in their presentations and for their contributions to this volume. We are grateful to all of the work of K.T. Mahanthappa in making this school work. His wisdom ranged from physics issues to the day to day details of the school. Efforts of other Colorado faculty, especially Tom Degrand (who, as always, organized mountain adventures for the students), Shanta DeAlwis and Oliver De Wolfe, were greatly appreciated. Susan Spika provided vital support before, during and after the school. TASI received partial support from the National Science Foundation, the Department of Energy, and the University of Colorado.

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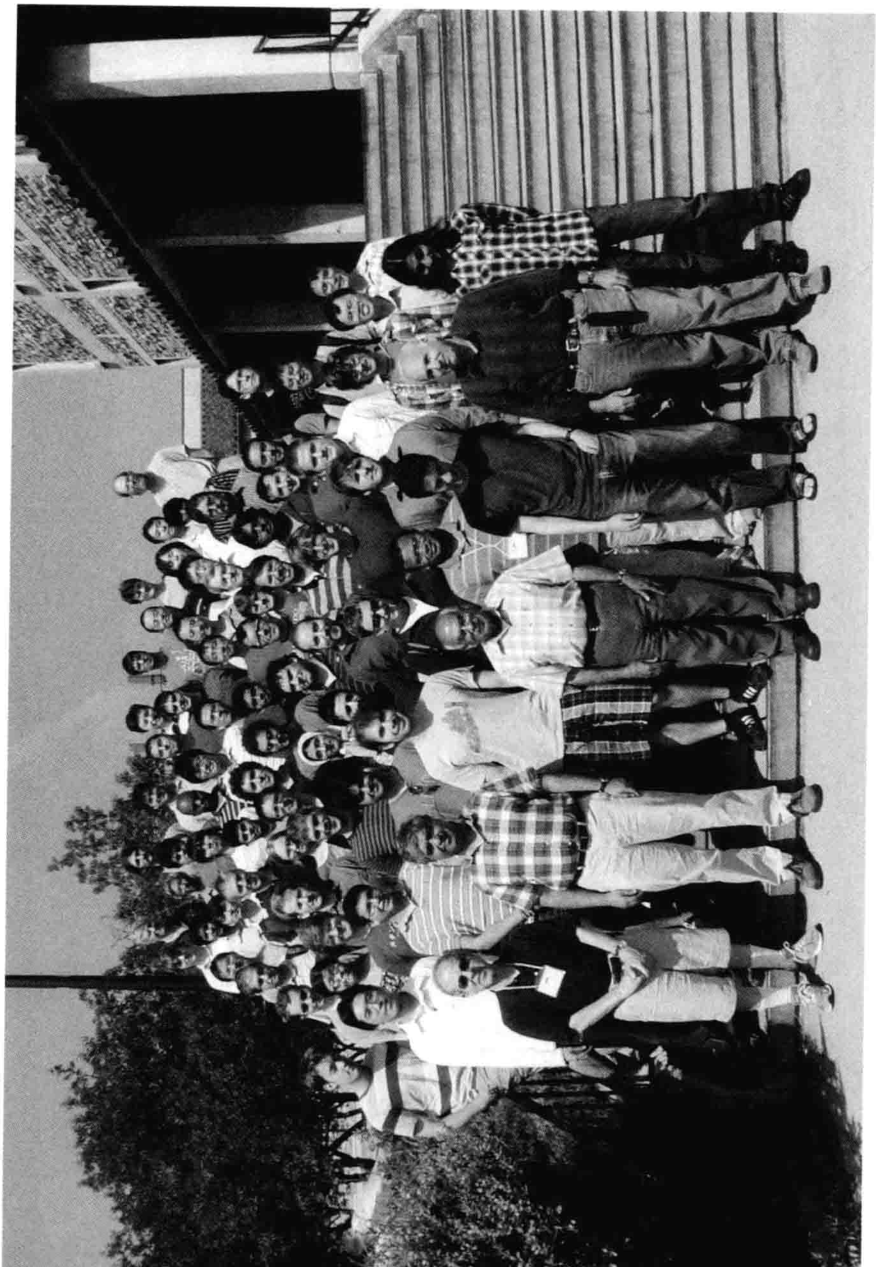

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Chapter 1

Introduction to Gauge/Gravity Duality

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These lectures are an introduction to gauge/gravity duality. The first three sections present the basics, focusing on $AdS_5 \times S^5$. The last section surveys a variety of ways to generate duals of reduced symmetry.

1.1. Generalities

1.1.1. *The greatest equation*

A few years back, Physics World magazine had a reader poll to determine the Greatest Equation Ever, and came up with a two-way tie between Maxwell's equations

$$d * F = j \,, \quad dF = 0 \,, \tag{1.1}$$

and Euler's equation

$$e^{i\pi} + 1 = 0 \,. \tag{1.2}$$

The remarkable appeal of Euler's equation is that it contains in such a compact form the five most important numbers, $0, 1, i, \pi, e$, and the three basic operations, $+, \times,$ ^. But my own choice would have been Maldacena's equation

$$\mathrm{AdS} = \mathrm{CFT} \,, \tag{1.3}$$

because this contains all the central concepts of fundamental physics: Maxwell's equations, to start with, and their non-Abelian extension, plus the Dirac and Klein-Gordon equations, quantum mechanics, quantum field theory and general relativity. Moreover, in addition to these known principles of nature, it contains several more that theorists have found appealing:

supersymmetry, string theory, and extra dimensions, and it ties these all together in an irreducible way. Also, while Euler's equation is a bit of an oddity, the relation AdS = CFT is just the tip of a large iceberg, it can be deformed into a much large set of gauge/gravity dualities.

So I get to teach you about this, trying to focus on things that you will need in the many upcoming lectures which will use the left-hand side of the equation, string theory and gravity, to learn more about the the right-had side, quantum field theory. Of course it's also interesting to use the equation in the other direction, and maybe some of that will sneak in. Roughly speaking, today's lecture will be a conceptual overview, lecture 2 will give some essential details about the two sides of the duality, lectures 3 and 4 will work out the dictionary between the two sides focussing on the familiar $AdS_5 \times S^5$ example, and lecture 5 will discuss generalizations in many directions.

Let me start by noting a few other reviews. The early MAGOO review[1] contains a detailed summary of the early literature, in which many of the basic ideas are worked out. The 2001 TASI lectures by d'Hoker and Freedman[2] are thorough and detailed, particularly with regard to the constraints from supersymmetry and the conformal algebra, and the calculation of correlation functions. McGreevy's course notes[3] are similar in approach to my lectures.

1.1.2. *A hand-waving derivation*

I am going to first motivate the duality in a somewhat unconventional way, but I like it because it connects the two sides, gauge theory and gravity, without going directly through string theory (as do many of the applications), and it allows us to introduce many ideas that will be important later on. So let me start with the question, is it possible to make the spin-2 graviton as a bound state of two spin-1 gauge bosons? With the benefit of generous hindsight, we are going to make this idea work. To start off, there is a powerful no-go theorem that actually forbids it.[4] Theories without gravity have more observables than theories with gravity (local operators, in particular, since there is no invariant local way to specify the position in general relativity), and this leads to a contradiction. Specifically, Weinberg and Witten show that if there is a massless spin-2 particle in the spectrum, then the matrix element

$$\langle \text{massless spin 2}, k|T_{\mu\nu}|\text{massless spin 2}, k'\rangle \qquad (1.4)$$

of the energy momentum tensor (which exists as a local observable in the gauge theory) has impossible properties.

Of course, to prove a no-go theorem one must make assumptions about the framework, and it often proves to be the case that there is some assumption that is so natural that one doesn't even think about it, but which turns out to be the weak link. The Coleman-Mandula theorem, classifying all possible symmetries of the S-matrix,[5] is a classic example. This paper played an important role in its time, ruling out a class of ideas in which spin and flavor were unified in $SU(6)$. However, it made the unstated assumption that the symmetry generators had to be bosonic,[a] which was sufficient for the immediate purposes but missed the possibility of supersymmetry. The more powerful a no-go theorem, the deeper its counterexamples.[b]

The reason for going through this is that the no-go theorem is indeed wrong, but to violate it we have to recognize a deep property of quantum gravity, the holographic principle.[6,7] The entropy of a black hole is proportional to its area in Planck units, and this is the largest possible entropy for a system with given surface area. This suggests that quantum gravity in any volume is naturally formulated in terms of degrees of freedom on its surface, one per Planck area. Thus we see the hidden assumption, that the graviton bound state moves in the same spacetime as its gauge boson constituents; rather, it should move in one additional dimension. Of course, there might be other ways to violate the theorem that will turn up in the future.

So how is the two-gauge boson state in four dimensions supposed to correspond to a graviton in five dimensions? With the benefit of hindsight, there are several places in QCD phenomenology where the size of a gluon dipole, the magnitude z of the separation, behaves like a spacetime coordinate. In color transparency,[8] and in the BFKL analysis of Regge scattering,[9] interactions are approximately local in z and the pair wavefunction satisfies a five-dimensional wave equation. So when you look at the gluon pair you picture it as a graviton four of whose coordinates are the center of mass of the pair, and the fifth is the separation.

We need just two more ingredients to make this idea work, but first we will make an excursion and discuss the shape of the five dimensional spacetime. Quantum field theory is nicest when it applies over a wide

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[a]Rereading Ref. 5, this assumption seems to have made its entrance at the point where the symmetry generators are diagonalized, which can't be done for a nilpotent operator.
[b]Perhaps this is what Bohr meant when he said "It is the hallmark of any deep truth that its negation is also a deep truth."

range of scales, so it is natural to consider scale-invariant theories first. If we rescale the system, the c.m. coordinates x and the separations z scale together. The most general metric respecting this symmetry and the symmetries of the four-dimensional spacetime is

$$ds^2 = \frac{L^2 dz^2 + L'^2 \eta_{\mu\nu} dx^\mu dx^\nu}{z^2} \to L^2 \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}, \tag{1.5}$$

where I have rescaled z in the last form so as to emphasize that there is only one scale R.

This is the metric of anti-de Sitter space, in Poincaré coordinates. If we replace the z with t in the denominator we get de Sitter space, the approximate geometry of our own accelerating spacetime. Certainly one of the major frontiers in gauge/gravity duality, though not one that I will focus on, is to figure out how to interchange z and t in the dual field theory: it appears to require great new concepts. Anti-de Sitter spacetime is not expanding but warped: clocks that run at the same rate in inertial coordinates run at different rates in terms of x^0, depending on where they are in z.

To finish off our 'derivation' of the duality, we need two more ingredients. The first is a large number of fields. We want the AdS scale L to be large compared to the Planck length L_P, so that we can use Einstein gravity. This means that we can fit a large black hole into the space, one with many Planckian pixels and so a large entropy, and so the field theory had better have a correspondingly large number of degreees of freedom. As we will discuss in more detail later, the number of fields is a power of L/L_P, depending on the example. The other ingredient we need is strong coupling, so that the gauge boson pair behaves like a graviton and not like a pair of gauge bosons.[c] I should say VERY strong coupling, much larger than one, to get a limit in which the gravitational description is quantitative.

Thus we have two necessary conditions for the duality, many fields and very strong coupling. The second condition can actually be made a bit stricter, as we will see: we need that most operators get parametrically large anomalous dimensions. In this form, these necessary conditions are actually likely to be sufficient, as shown in part in Ref. 10. Large anomalous dimensions clearly require very strong coupling, but do not necessarily follow from it as we will see in an example later.

These conditions of many fields and strong coupling will reappear at various points in these lectures. Clearly they play a controlling role in the

[c] Of course, it will then mix with states having more constituents, but one can still retain a bit of the basic idea that the graviton spin comes from two 'valence' gauge bosons.

applications, as to whether we have a quantitive description or merely a 'spherical cuprate,' a solvable model that captures the qualitative physics that we wish to understand. For example, in the application to heavy ion physics, the coupling seems to be of order one, midway between weak and strong. The strong coupling picture is not quantitative in a precise way, but seems to do better than perturbation theory on many qualitative properties. Naively the number of fields is $N^2 - 1 = 8$ for the gluon states, which is a modestly large number, though it has been noted that the parameter N/N_{flavor} is only unity, and there should be corrections of this order. Fortunately the large-N approximation seems to be fairly robust even at small values.

In the condensed matter applications, again the relevant couplings are of order one at a nontrivial fixed point, where a strong coupling expansion has a chance to capture things that a weak coupling expansion cannot. There is no large N, but condensed matter theorists in the past have not been above introducing a large-N vector index in order to get a tractable system. The large-N vector is a mean-field approximation; this is true for the large-N matrix limit as well, though in a more subtle and perhaps more flexible way:[11] expectations of products of color singlets factorize, but there is a very large number of color singlet fields.

Notice that I have not yet mentioned supersymmetry. However, it tends to enter necessarily, through the requirement of very strong coupling. Quantum field theories tend to become unstable at strong coupling, through the production of pairs whose negative potential energy exceeds their kinetic energy. In continuum theories this can happen at all scales, and the theory ceases to exist. Supersymmetry protects against this: schematically the Hamiltonian is the sum of the squares of Hermitian supercharges, $H = \sum_i Q_i^2$, so the energy is bounded below. Thus, we can break the supersymmetry softly and still have a duality, but if we break it at high energy we lose the theory. We may have to be satisfied with metastability, but that is OK, we probably live with that in our own vacuum.

We have also not mentioned strings, we seem to have found a theory of quantum gravity that uses only gauge theory as a starting point, but in a range of parameters for N and coupling that is not so familiar. But of course what happens is that when we get gravity in this way we get everything else as well, the strings, branes, extra dimensions and so on. Again this was anticipated by 't Hooft,[12] who argued that the planar structure of large-N gauge theory made it equivalent to a theory of strings.