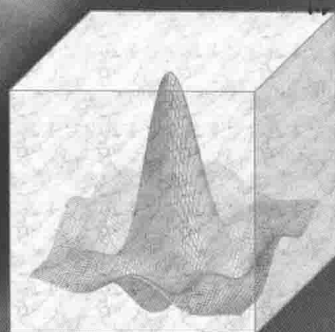


THEORY OF QUANTITATIVE MAGNETIC RESONANCE IMAGING

HERNÁN JARA



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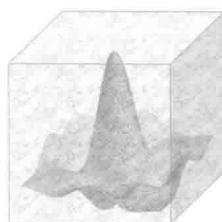
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THEORY OF
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RESONANCE IMAGING

*Para Sebastián y Carola,
mis compañeros de vida*

PREFACE

Quantitative Magnetic Resonance Imaging (qMRI) refers to a diverse collection of image acquisition techniques and image processing techniques that are used as matched pairs for mapping the spatial distributions of the physical quantities that influence MRI signals; such physical quantities are known as qMRI parameters. The many qMRI parameters include: 1) measures of the amount of the MR-active substance --e.g. the proton density--. 2) Measures of states of motion --i.e. kinetic properties such as molecular diffusion, perfusion, and flow--, and 3) measures of interactions between the MR-active substance and the molecular environment --e.g. relaxation times and magnetization exchange parameter-- as well as interactions between the patient and the MRI scanner.

Succinctly, qMRI is the science of mapping -- or imaging -- qMRI parameters, or equivalently, the science of quantifying tissue properties at the spatial scale of imaging volume element (voxel) as represented by the picture element (pixel). Accordingly, the central mathematical object of qMRI is the numerical value of every pixel --or pixel value-- and the main objective of qMRI is to generate scientific grade pixel values that bear scientific units of measurement and that therefore have a more absolute meaning than the pixel values of directly acquired MR images. Most MR images currently used in clinical practice consist of non-quantitative pixel values meaning that these do not bear scientific units and are not directly comparable to pixel values of other images of even the same patient. Such directly acquired images are weighted by qMRI parameters and the pixel values have meaning only in relation to other pixel values in that particular dataset.

qMRI is an evolving scientific discipline that has the potential of impacting all stages of clinical and research MRI practices, from image acquisition, to image processing, to image interpretation. To the best of my

knowledge, to date there are only two other books in this subject matter, specifically “Quantitative MRI of the Brain: Measuring Changes Caused by Disease”, Edited by Paul Tofts and “Quantitative MRI in Cancer”, Edited by William Hendee. These landmark books cover qMRI theory in a manner that is intertwined with medical applications. The purpose of the much shorter book herein is to provide a concise and unified theoretical description of qMRI theory only and is intended primarily as a textbook for a graduate level course potentially offered by academic departments such as Bioimaging, Biomedical Engineering, Computer Sciences, Mathematics, or Physics.

This textbook has its origin in lecture notes for an undergraduate course --Introduction to Medical Imaging-- that I have been teaching for the past eleven years as part of the curriculum of the Biomedical Engineering department at Boston University. It is through interactions with the students and with my colleagues of the Radiology Department at Boston University that I have come to understand the unifying principles of Medical Imaging, its implications to qMRI theory, and the value of quantification in medicine. I owe much to my research partners and colleagues Stephan Anderson, Joseph Ferrucci, Alexander Norbash, Naoko Saito, Osamu Sakai, Jorge Soto, and Memi Watanabe with whom I have collaborated for many years. I am also deeply grateful to Peter Joseph and Felix Wehrli who introduced me, in my formative years at the University of Pennsylvania, to the fields of MRI and qMRI. Finally, I wish to thank Stephan Anderson for editing this manuscript.

My dear father Alvaro Jara dedicated his academic life to research in the field of Quantitative History. He inculcated in me a deep appreciation for the power of quantification as a tool for understanding humankind as well as nature in its various dimensions. When he passed away some years ago, my dear friend and mentor Dr. Joseph Ferrucci told me that lost loved ones “reverberate” in time, and these kind words certainly reverberate in my mind as I finish this book.

HJ, 2013, Belmont, MA

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A. INTRODUCTION

A1. Historical Notes

The earliest known manmade images are paintings in cave walls representing various animal and human figures. Although precise dating of these primitive images is not without uncertainty, the earliest paintings are believed to date to the prehistoric times, some as old as 32,000 years ago. Considering that our earliest human ancestors date back several million years, this is very recent in the human evolutionary timeline, thus indicating that the practice of imaging as a human activity is a manifestation of very advanced intellectual functions.

The camera obscura, originally described around 1000-AD, is the first human invention that could generate images artificially in a manner bypassing the human brain. The principle of operation of the camera obscura is very similar to that of the eye consisting of a sealed box with a pinhole via which specific rays of light are selectively accepted into the box. With this setup, only the rays of light making a point-to-point correspondence between the object and the imaging plane are accepted into the camera obscura thus forming a high-fidelity image of an object positioned in front of the pinhole. It would take another 800 years until the invention of the photographic camera; the first imaging device that could not only create images artificially but that could also store those permanently using plates coated with a light-sensitive silver-halide emulsion.

It would take another hundred years until the discovery of x-rays by Röntgen (Roentgen, 1896), the first form of radiation that had the prodigious property of penetrating solid matter in general and human tissues in particular, thus permitting for the first time the investigation of the internal human body by nondestructive means. So began a new branch of medicine, namely radiology, which initially was limited to producing shadow projectional images of the human body.

During the classical period of medical imaging --from 1896 until the early 1970's-- , imaging scientists and pioneering imaging physicians investigated other penetrating radiations, contrast agents, and specialized scanning techniques and technologies thus widening the range of applications of this emerging medical imaging science. This classical period saw the advent of imaging with γ -rays emitted by radioactive nuclei (Cassen, Curtis, Reed, & Libby, 1951; Sweet, 1951; Wrenn Jr, Good, & Handler, 1951) and with ultrasonic radiation (Dussik, 1942; Woo, 2010); nuclear imaging and ultrasound imaging respectively.

In parallel to these scientific developments, and without an apparent connection to medical imaging, the field of nuclear magnetic resonance (NMR) emerged (Bloch, Hansen, & Packard, 1946; Purcell, 1946) and evolved into one of the most fruitful branches of physics and chemistry. The use of strong magnetic fields in conjunction with long-wavelength electromagnetic radiation (radiowaves) permitted nondestructively probing condensed matter *via* magnetic interactions with atomic nuclei, specifically with the magnetic moments of some atomic nuclei, and most notably with the ^1H -proton nucleus of the hydrogen atom the most abundant chemical element in nature and the primary building block of biologic matter.

The contemporary period of medical imaging begins with the invention in 1972 of computed tomography (CT) with γ -rays (Chesler, 1973) and x-rays (Cormack, 1963; Hounsfield, 1973). In the case of x-ray CT, the new imaging device combined a movable-x-ray-beam transmission apparatus that targeted a single thin axial slice, with a digital computer. The CT scanner would generate many geometrical projections of the targeted slice at different angles of tomographic projection, which were stored sequentially in the permanent memory of a computer thus generating a full data set in signal space and allowing further mathematical processing. A reconstruction program based on the mathematics of continuous geometric projections --developed by Johann Radon in 1917 (Radon, 1917) -- would then transform the signal space data set into a two dimensional representation of the axial slice in geometric or anatomic space. The ingenious strategy of probing internally an object with successive and systematically different radiation experiments for generating a complete representation of a thin slice in signal space --and to later transform it into geometric space-- was soon thereafter adopted for tomographic imaging with the nuclear induction experiment (Lauterbur, 1973). In this case, spectral projections in Fourier domain were obtained by reading time dependent NMR signals while applying a magnetic field gradient; thus marking the