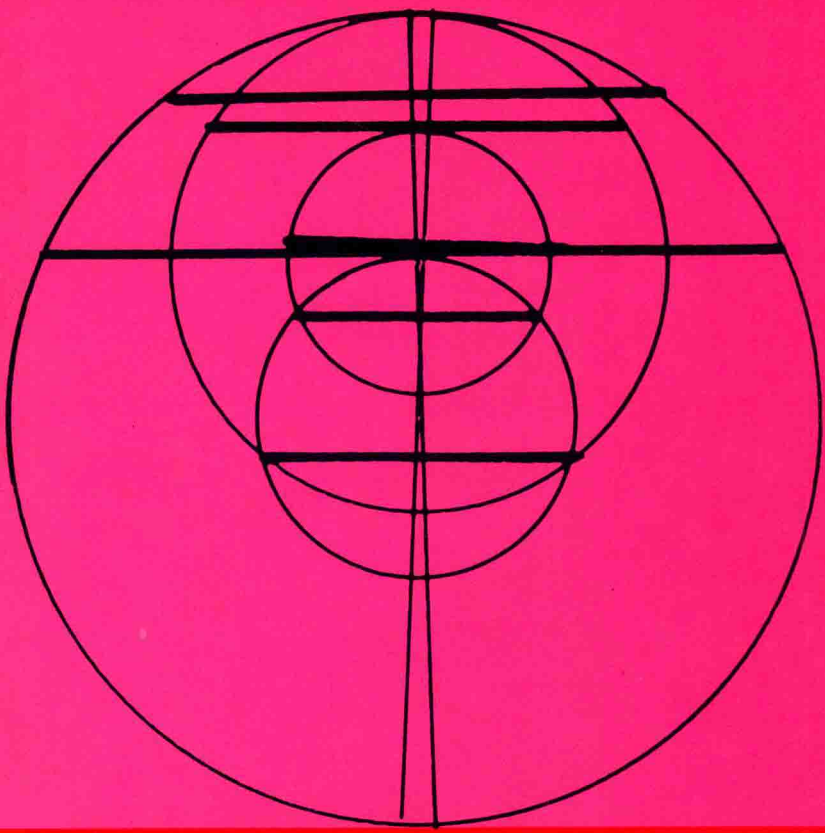
GEOMETRY

for the Practical Man



J. E. Thompson

MATHEMATICS FOR SELF STUDY SERIES

GEOMETRY
FOR THE PRACTICAL MAN

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MATHEMATICS FOR SELF-STUDY

A Group of Books That Make Easy
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Arithmetic *for the Practical Man*
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PREFACE TO THIRD EDITION

In this Third Edition of *Geometry for the Practical Man*, as in the revisions of the other volumes in this series, the authors have made every effort to retain the basic methods of presentation which have proven so successful in the earlier editions. However, new concepts have been introduced in the subject matter which, it is believed, will better prepare the student to study and to understand those parts of advanced mathematics in which he may be interested or in which he may feel the need to acquire competence. The non-Euclidean geometries play a major part in the intellectual and mathematical history of our time, and a chapter on them has been added to introduce the student to at least one of their basic postulates as well as to give him a modest appreciation of their significance. Moreover, in order that the student may further test his proficiency, a new section of Review Exercises has been added.

PREFACE TO FIRST AND SECOND EDITIONS

SIR ISAAC NEWTON, the English creator of the calculus and discoverer of the law of gravitation, studied geometry without the aid of a teacher and alone, but for most students it has long been considered a difficult subject and its teaching has been considered as belonging exclusively in the schools and organized classes. In recent years, however, there has been let loose on the general public a flood of books which attempt to tell the general reader "all about science," and a few such books on mathematics have timidly felt their way about in the flood. Most of these have been more about mathematics than of mathematics, and in particular they have not often tackled the ancient science of geometry. Furthermore, most books on so-called popular science and mathematics are either superficial and do not really teach, or they omit much of a subject because they use their available space in trying to delve into a few parts of the subject.

The first edition of this book, written as one of a series of similar treatments of other branches of mathematics, attempted to give a balanced treatment of the usual parts of elementary geometry in such form that it could be studied by any reader having a knowledge of arithmetic and a little elementary algebra, without the aid of a teacher. The very large sale which the publishers found for the book indicated either that its plan and method were successful or that a surprisingly large number of people were waiting for a book which they could use without a teacher. The publishers and the author have received a very large correspondence from readers which shows a great interest in and satisfaction with the book and indicates some small changes which seem necessary. In order to conform more nearly to the original plan some of the more formal proofs and calculations have been omitted in this edition and replaced by descriptive explanations and illustrations.

As in the other books of the series one of the aims here is to present the fundamentals of elementary geometry from the viewpoint of those who would wish to use it for practical purposes in the arts and industry

and in the study of other branches of mathematics. Thus, while not a book on so-called "practical geometry" as such, it is intended that the book shall be useful to those who wish to *apply* the *principles* in any way.

In conformity with the plan as stated the treatment has been made as informal as the nature of the subject will allow. Some of the traditional parts of the subject have been omitted, and several things have been included which are not usually found in books on elementary geometry. In order to indicate to the reader that the subject of geometry in its modern form is the result of a long and very human effort, rather than a gift from the gods in completed and perfect form, a fairly comprehensive sketch of its history is given in the first chapter. Because of the antiquity of the science of geometry, its fundamental importance in the study of other branches of mathematics, and its very great technical and practical importance, and also because it is the bright and shining example of the methods of logical procedure, the historical sketch is made long enough to be of some real value to the interested reader. References to this chapter and supplementary historical notes on important individual topics are given at appropriate places in the text in order to supply additional information or to make clear allusions found in mathematical literature.

The introduction to the main facts of geometry has been put into intuitive form and each new subject is introduced in the same manner or by means of its connection with one which has been previously discussed. A large number of illustrations and applications have been given; these are really more illustrations than applications, however. In order to understand and appreciate *applications* of mathematics one must have already some technical knowledge of the subject in which the application is made and such knowledge has not been assumed here, although this may be done in the higher branches of mathematics. A short *Foreword* has been included for readers who here approach the subject for the first time. Although the long and severely formal proofs have been omitted the simpler proofs are given fairly complete, though not in formal style, in order to illustrate and cultivate systematic reasoning and logical processes. The numerical parts of the subject, amounting to applications of some of the principles, are an extension and amplification of the work of arithmetic on measurement.

J. E. THOMPSON

Brooklyn, N. Y.
October, 1945

FOREWORD

THE question is often asked: Why should one study geometry? The answer depends on the questioner.

For the practical man it may be said that geometry has very wide application in the arts and trades, such as building and architecture, machine work, surveying, engineering, etc. This does not necessarily mean that the worker consciously applies the methods of geometry, rule by rule and principle by principle, but that the rules and methods which he does use in his work are based on or derived from the principles and methods of geometry.

For the student who intends to go farther in the study of mathematics and science and enter the higher branches it is one of the necessary foundations, and of an importance which requires no urging.

In this connection it may be said that after we learn to count and perform the simple operations of arithmetic, algebra and geometry form the joint foundations of all pure and applied mathematics. Algebra, as remarked elsewhere, supplies the "alphabet and grammar," so to speak, and geometry develops the forms and methods of reasoning used in mathematics. The one lays the foundations of the rules and methods of calculation, and the other the foundation of the rules and methods of measurement; and measurement and calculation are the heart of pure and applied mathematics.

When all is said and done, however, the fact remains that geometry has attracted and held the attention and interest of the human race from prehistoric times until the present, purely for its own sake. Geometry is at the same time a science and an art, mathematics and philosophy. It supplies us the only perfect system of logic and its beautiful completeness is not to be found in any other branch of knowledge. For one who loves logic, completeness, perfection, and beauty, geometry is a subject which is always fascinating, and which supplies its own reason for existence.

In this book we cannot set forth geometry in all its completeness and perfection, and in the procedure which we must follow it will un-

avoidably seem to lose some of its beauty. To one seeking to find something of its meaning and its usefulness, however, it can never lose its interest.

The method which is followed in this book is stated in the preface as being threefold: to make the study of geometry possible without a teacher, to present those parts of the subject which will be most directly useful in its applications and in further study, and to reveal something of its eternal interest. As an aid toward this last object a chapter on the history of geometry is given at the beginning of the book. It is hoped that this chapter will serve to open up to the reader approaching the study for the first time, the long road and vista down which geometry has come to us, beginning and progressing with the history of our race.

In the study of geometry many of the rules, methods and symbols of arithmetic and algebra are used and it will be assumed in this book that the reader has some knowledge of these subjects. In other wordy, in the usual course of study in mathematics, geometry comes after arithmetic and algebra.*

When studied as a compulsory subject and in the manner in which it is sometimes approached and presented geometry can be made distressingly dry and dull, but to one who begins its study voluntarily, and when it is properly approached and handled, geometry will be found to be simple and fascinating, and its mastery will provide the highest intellectual satisfaction and great practical utility.

In the statement of geometrical facts and in the proofs of new geometrical relations from those already established it is necessary and customary to refer to those previous relations, and this is done by giving each important result a number. This number is then cited as reference whenever it is used. In this way it comes about that any book on geometry will apparently require on the part of the reader frequent turning back to earlier results. This is not necessary in ordinary reading in this text, however, as most of these previous results are mentioned by quoting briefly their most important features. Thus the otherwise forbidding appearance of so many references need cause no trouble and these may be ignored except when specially needed or desired. In such cases the number of the statement and the article

* *Note.*—All the knowledge of algebra necessary for the study of this book may be obtained from the author's "Algebra for the Practical Man," published by D. Van Nostrand Company, Princeton, N. J.

in which it is found are available. In general, while some of the formality of strict logic is unavoidable, and occasional use of algebra is necessary, it is hoped that the reader will find this book easy and informal reading and at the same time a sufficiently complete treatment of the ancient and perennially fascinating subject of GEOMETRY.

CONTENTS

PART I

INTRODUCTION TO GEOMETRY

CHAPTER 1 ORIGIN AND DEVELOPMENT OF GEOMETRY

CHAPTER 2 SOME PRELIMINARY IDEAS AND METHODS OF GEOMETRY

PART II

PROPERTIES OF PLANE FIGURES

CHAPTER 3 SOME PROPERTIES OF STRAIGHT LINES AND ANGLES

CHAPTER 6 PROPORTION AND SIMILAR FIGURES

PART III

MEASUREMENT OF GEOMETRICAL FIGURES

CHAPTER 7 DIMENSIONS AND AREAS OF PLANE FIGURES

CHAPTER 8 REGULAR POLYGONS AND MEASUREMENT OF THE CIRCLE

CHAPTER 9 SURFACE AREAS AND VOLUMES OF SOLIDS HAVING PLANE SURFACES

Part I

INTRODUCTION TO GEOMETRY