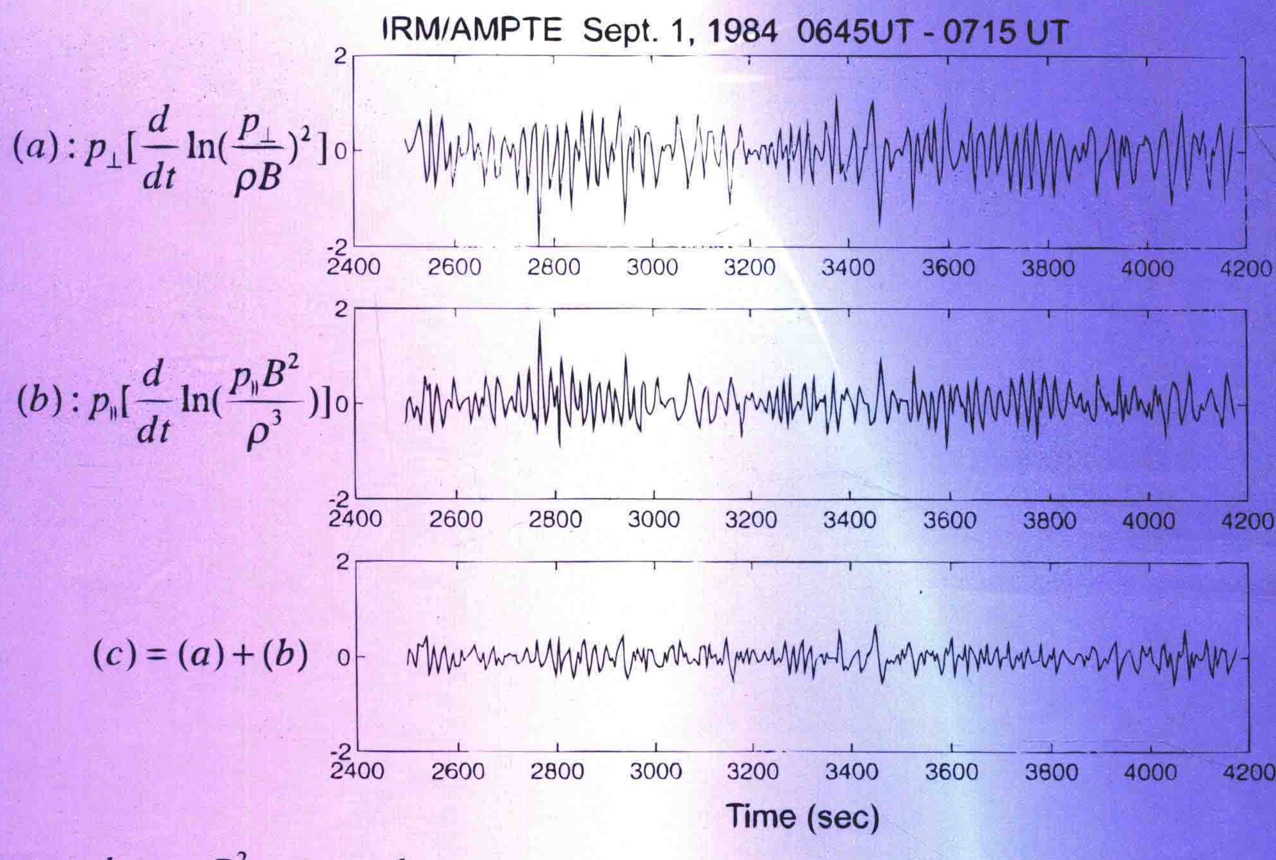
本書通過中央大學出版中心學術審查

基礎太空電漿物理學

Elementary Space Plasma Physics



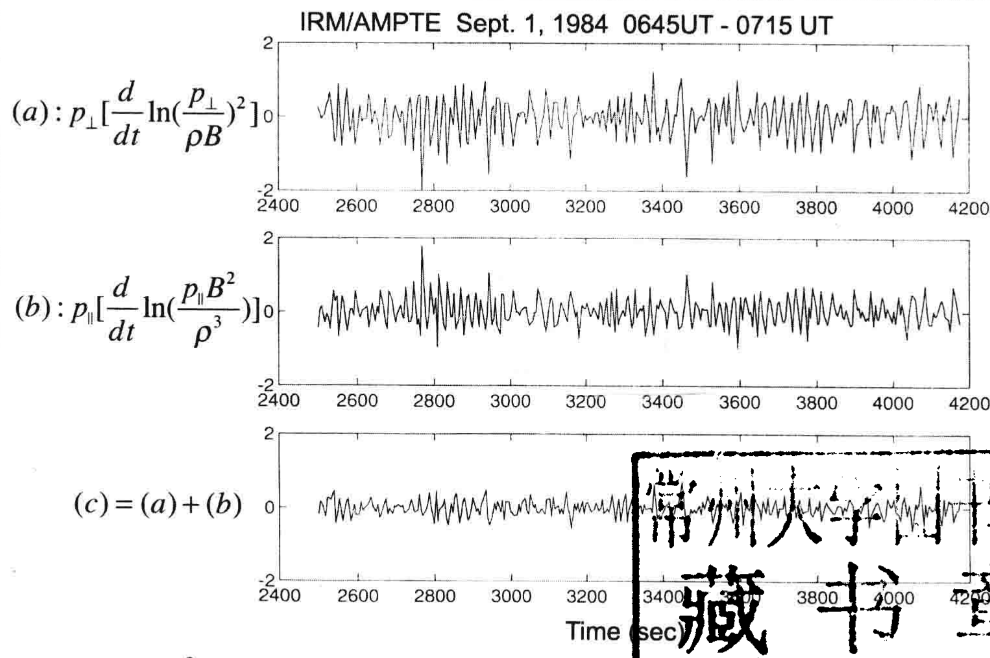
$$p_\parallel[\frac{d}{dt}\ln(\frac{p_\parallel B^2}{\rho^3})] + p_\perp[\frac{d}{dt}\ln(\frac{p_\perp}{\rho B})^2] = 0$$

呂凌霄 著

Ling-Hsiao Lyu

基礎太空電漿物理學

Elementary Space Plasma Physics



$$p_{\parallel}[\frac{d}{dt}\ln(\frac{p_{\parallel}B^2}{\rho^3})] + p_{\perp}[\frac{d}{dt}\ln(\frac{p_{\perp}}{\rho B})^2] = 0$$

呂凌霄 著

Ling-Hsiao Lyu

中大出版中心
NCU Press
airiti press

Elementary Space Plasma Physics

基礎太空電漿物理學

Author: Ling-Hsiao Lyu 呂凌霄
Publisher: Dr. Wei-Ling Chiang
Publisher: National Central University Press & Airiti Press Inc.

Editor-in-Chief: Eliza Chang
Editor: Judie Ku, Marcia Cheng
Assistant Editor: Yichun Hsu
Cover Design: Yayu Wu
Publishing:

National Central University Press
3F, Kwoh-Ting Library, No.300, Jhongda Rd., Jhongli City, Taoyuan County 32001, Taiwan
Tel: +886-3-4227151#34832 / Fax: +886-3-4275020
E-mail: ncu34832@ncu.edu.tw

Airiti Press Inc.
18 F, No. 80, Sec. 1, Chenggong Rd. Yonghe City, Taipei 234, Taiwan
Tel: +886-2-2926-6006 / Fax: +886-2-2231-7711
E-mail: press@airiti.com
Official Site: http://www.airitipress.com/

ISBN: 978-986-82709-5-4
GPN: 1009904729
Publishing Date: December 2010
Price: NT$ 450 / USD$ 18

Preface

The characteristic scale lengths of various space plasma phenomena range from the electron inertial length to the magnetohydrodynamic (MHD) scale length. Just like the geostrophic-wind approximation in the atmospheric physics, the MHD approximation in the space plasma physics has limitation in its applications. One of the important goals of this book is to show the students how scientists obtain the governing equations of a given plasma model and what assumptions have been made to obtain the set of governing equations shown in the literatures. We believe that, unless the students know how to derive the governing equations and how to obtain the wave mode from a simplified linear dispersion relation, it will be difficult for the students to fully understand the limitations of a given plasma model and how to choose the correct model to explain the observed phenomena.

This book is written based on three textbooks: Krall & Trivelpiece (1973), Nicholson (1983), Chen (1984), which are the most popular textbooks when I was a student, as well as the lectures I learned from Professor J. K. Chao when I was an undergraduate student in National Central University, and the lectures I learned from Professor J. R. Kan when I was a graduate student in University of Alaska Fairbanks. I later found out that the extra information I learned from Professor Chao can be found in the following books or articles: Chao (1970), Rossi & Olbert (1970), and Kantrowitz & Petschek (1966). The extra information I learned from Professor Kan are based on the classical book by Stix (1962). Of course, I have also add additional information based on my own research experiences in various subjects.

Many textbooks and theoretical papers written before 1980s or even 1990s are based on the Gaussian units. Since magnetic field and electric field have the same dimension in the Gaussian units, it is easy for theorists to check the correctness of their theoretical derivations. But all instruments are designed based on the SI units, thus, it is hard to apply the theoretical results (in the Gaussian units) to the space observations (in the SI units). Scientists in our space community have tried very hard to change Gaussian units to SI units in all the new textbooks and scientific papers. Change of the units makes the old textbooks and the classical papers hard to follow by the readers of the new generations. We use the SI units in most of the derivations presented in this book. To help the students to read the classical papers in the literatures, we also present the basic equations in both units in Chapters 1 and 3. A brief summary of each chapter is given below:

Chapter 1 gives a brief introduction to the plasma physics. In addition to introduce the characteristic frequencies, characteristic scale lengths, and collision frequencies, I also review the SI units and the Gaussian units in Chapter 1.

Chapter 2 derives the Vlasov equation based on the Klimontovich equation, which I learned from Nicholson (1983). They are many different ways to obtain the Vlasov equation (e.g., Nicholson, 1983). I preferred to derive the Vlasov equation from the Klimontovich equation, for the reason that the approaches used in the Klimontovich equations are very similar to the approaches used in the particle-code simulation.

Chapter 3 derives the two-fluid equations and the one-fluid equations from the Vlasov equation. This chapter is written based on what I learned from Professor J. K. Chao (Chao, 1970; Rossi & Olbert, 1970) and with some extensions based on my own experiences in these subjects. The fluid equations in the flux conservation forms shown in this chapter are particularly useful in analyzing the space plasma data and in designing the fluid simulation models.

Chapter 4 shows the classical way to obtain the Boltzmann equation from the Liouville equation. Since the way to reduce a 6N-dimensional system to a 6-dimensional system is very similar to the way to reduce a 6-dimensional system to a 3-dimensional system, I decided to put the Chapter 4 after the derivation of fluid equations from Vlasov equations in Chapter 3.

Chapter 5 derives the dispersion relation of the linear waves in the ion-electron two-fluid plasmas. The mobility-tensor approach makes the complete linear wave analysis in the ion-electron two-fluid plasma an easy task. I learned the mobility-tensor approach from Professor J. R. Kan. According to Professor Kan, his lecture was based on the classical book by Stix (1962). For advanced study, I recommend the students to find the numerical solutions $\omega(\mathbf{k})$ and plot the Friedrich's diagrams of all the wave modes based on the linear dispersion relation obtained in this chapter.

Chapter 6 derives the dispersion relation of the linear waves in the MHD plasmas. This chapter is written based on the classical paper by Kantrowitz & Petschek (1966). I leave the Friedrich's diagrams of the MHD waves as an exercise to the students. What they leaned from the Friedrich's diagrams of the MHD waves can also help them to study the linear waves in Chapter 5.

Chapter 7 and Appendices D & E introduce the single particle drift motions, the diamagnetic current, and the ponderomotive force. This chapter is written based on what I learned from Nicholson (1983) and with some extensions based on my own experiences in these subjects. The multiple-time-scale analysis shown in this chapter can be generalized to study wave-wave interactions at different time scales. It can also help the students to develop or to understand the gyro-kinetic simulation model. Since the gyro-kinetic model is a model in between the fluid model and the kinetic model, I put this chapter between the discussion of fluid models in Chapters 5 & 6 and the discussion of kinetic models in Chapters 8-11. Indeed, I think most of the single particle drift motions are strongly related to the kinetic plasma physics, except the diamagnetic current and the polarization drift. Therefore, I decide to put this chapter before the section of kinetic plasma physics, rather than at the beginning of this book. Examples discussed in this chapter are exclusively from space plasma physics. Instructors from different field can ignore these examples and use their own examples to demonstrate the drift motions and their consequences.

Chapter 8 shows the possible nonlinear equilibrium solutions of the plasma distribution function with given background electric field and magnetic field. This chapter is written based on what I learned from the books by Krall & Trivelpiece (1973) and with some extensions based on my own experiences in this topic. The nonlinear equilibrium solutions discussed in this chapter are essential for studying linear waves in the kinetic plasma as discussed later in Chapters 9 and 11.

Chapter 9 shows the importance of Landau contour in studying linear waves in the kinetic plasma and discusses the possible unstable distribution function in a spatially uniform plasma. This chapter is written based on what I learned from the books by Krall & Trivelpiece (1973) and Nicholson (1983) and with some extensions and detailed derivations based on my own experiences in this subject.

Chapter 10 briefly describes the two-stream instability based on the classical approaches discussed in the textbooks Krall & Trivelpiece (1973) and Chen (1984). The two-stream instability is a good example of the unstable waves that could occur in a uniform medium as discussed in Chapter 9. I do not discuss the nonlinear evolution of the two-stream instability in this chapter. I recommend the students to learn the nonlinear evolution of the two-stream instability from a kinetic simulation of the two-stream instability.

Chapter 11 derives the dispersion relation of the linear waves in unmagnetized and magnetized kinetic plasma. This chapter is written based on what I learned from the books by Krall & Trivelpiece (1973) and with some extensions and detailed derivations based on my own experiences in this subject.

Note that, for simplicity, all the linear waves discussed in this book are assumed to propagate in a uniform equilibrium medium. A brief discussion has been made at the beginning of Chapter 5 to show the students how to linearize the governing equations when the background medium is non-uniform. Since studying the linear waves in a non-uniform medium requires the knowledge of non-uniform equilibrium solutions, which are highly nonlinear solutions, the study of linear waves in a non-uniform medium is beyond the scope of this book. It will be discussed in the advanced course of nonlinear plasma physics in the future.

This book is a collection of the lecture notes I put on my web site during the past 10 years. The lecture notes are written for a two-semester graduate course, which I taught for nearly 20 years in National Central University. Initially, I put the lecture notes on the web so that I do not need to carry the lecture notes or the textbooks with me everywhere I go, particularly when I visit another institute or attend a conference abroad. Since these notes are initially written for myself, there are limited discussions on the physical meanings of the governing equations and the wave modes in these notes. When I finally converted them into lecture notes, I found that it is a good idea to give the students a chance to think and to find out the physical meaning of different plasma processes all by themselves. As a result, this book will provide both the students and the instructors enough freedom to use their own examples to interpret the wave modes or the governing equations without the interference from the author. Another type of information that is missing from this book is the historical review of the discovery of different wave modes. This type of information can help students to learn the importance of each wave mode. Fortunately, nowadays, the students can easily learn the historical information by the keyword searching on the internet-based encyclopedia.

This book is written for a two-semester graduate course. It contains only the fundamental subjects in the plasma physics. Thus, an instructor can easily cover the entire book in two semesters. I usually taught the Chapters 1-4, 6, and the first half of the Chapters 5 and 7 during the first semester. It took me about two third of the second semester to finish the Chapters 8-11. For the rest of the second semester, depending on the research background of the students in the class, I will either go back to Chapters 5 and 7 or teach some nonlinear plasma physics, which are not included in this book but can be found in textbooks elsewhere.

In acknowledgment, I would like to thank Professor J. K. Chao of National Central University, Professor K. R. Chu of National Taiwan University (previously a professor of

National Tsing Hua University for nearly 30 years), and Professor J. R. Kan of University of Alaska Fairbanks, who have shown me the beauty of the plasma physics in my early studies. I am also grateful to the students for their corrections and comments over the years. Without their help, it will be very difficult for me to find out all the typos hiding in my lecture notes.

Ling-Hsiao Lyu
National Central University
December 10, 2010

Table of Contents

Chapter 1. Introduction

Topics or concepts to learn in Chapter 1:

1. What is plasma?
2. The two systems of units that are commonly used in the literatures of plasma physics: The Gaussian units and the SI units (Also: The basic equations, the dimension analysis, and the scale analysis)
3. What is the difference between 10^6 °K electrons and 86 eV thermal electrons? Understand the temperature, thermal pressure, and the kinetic thermal energy of a plasma. Understand how special the 0.5 MeV electron is.
4. What is Boltzmann relation?
5. What is Debye shielding? How to determine the Debye length of a plasma?
6. What is plasma parameter?
7. What is the plasma oscillation frequency in a un-magnetized plasma?
8. What are the gyro frequency and the gyro radius (or Larmor radius) of a magnetized charge particle?
9. What is the definition of "collision" in the plasma physics?

Suggested Readings:

(1) Chapters 1 and 9 in Nicholson (1983)

(2) Sections 1.1~1.5, 1.8 and Appendix III in Krall and Trivelpiece (1973)

(3) Chapter 1 in F. F. Chen (1984)

1.1. Definition of Plasma

Plasma is the fourth state of matter. Heating can transfer matter from *solid state* to *liquate state*, then to *gas state*, and then to *plasma state*. Plasma is a fully ionized gas or a partially ionized gas. Gas with only 1% ionization can be considered as plasma. Therefore, 99% of the matter in the universe is in the plasma state.

Plasma is usually a *high-temperature* and *low-density* ionized gas. High temperature and low density are the favorite conditions for *ionization* but not for *recombination*. Without recombination, ionized particles can remain ionized so that the ions (with positive charge) and the electrons will not be recombined into neutral gas.

Plasma can be considered as a *fluid* even though sometimes *it does not reach to thermal dynamic equilibrium state*. It is the collective behavior that makes the plasma behave more like a fluid than independent particles. Like a fluid, there must be a large number of ionized gas particles (N>>1) in a plasma system, so that the number density and the thermal pressure of plasma can be statistically meaningful. Due to low-density nature, the basic scale length of plasma must be large enough in order to contain enough numbers of ionized particles. We shall show that this characteristic scale length in the plasma is roughly the Debye length.

Before we introduce the concept of Debye length, Debye shielding, and plasma parameter, we shall first briefly review the differences between the SI (MKS) units and the Gaussian units. Both of them are commonly used in the plasma research community. We shall also introduce two different units of temperature. Both of them are commonly used in the space plasma observations.

1.2. The SI Units and The Gaussian Units

The SI nits are the standard units today for all scientific communities around the world. But the expressions in Gaussian units have also been used for more than 50 years. Many textbooks and theoretical papers written before 1980s are based on the Gaussian units. Since magnetic field and electric field have the same dimension in the Gaussian units, it is easy for theorists to check the correctness of their theoretical derivations. But all instruments are designed based on the SI units, as a result, it is hard to apply the theoretical results (in the Gaussian units) to the space observations (in the SI units). Thus, scientists in space community have tried very hard to change this old habit and try to use SI units in all new textbooks and scientific papers of space plasma physics. Change of the units can make the readers of the new generation hard to follow the contents in the old textbooks and the classical papers written in the Gaussian units. We shall use the SI units in most of the derivations presented in this book. To help the students to read early literatures in plasma physics, we will present the basic equations in both units in Chapters 1 and 3.

Table 1.1 and Table 1.2 list some of the commonly used equations and physical terms in both units, where c is the speed of light, ρ_c and $\mathbf{J}$ are the charge density and the electric current density, respectively. Note that the charge density and the current density appeared in these equations include both free and bounded components. They are in contrast to the Maxwell's equations (in SI units) listed below, in which ρ_{cf} and $\mathbf{J}_f$ are the free charge density and the free current density, respectively.

$$\nabla \cdot \mathbf{D} = \rho_{cf}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

Here, the "free" means the distributions of the charge density and the current density will not be affected by the applied electric field and the magnetic field. In most space plasmas, we have $\rho_{cf} = 0$ and $\mathbf{J}_f = 0$. Namely, the charge density and the electric current density in the space plasma will change according to the electric field and the magnetic field.

Additional information on these units can be found in the Dimensions and Units section of *NRL Plasma Formulary* (on page 10). URL address of the NRL Plasma Formulary web site is http://wwwppd.nrl.navy.mil/nrlformulary

Table 1.1 Maxwell's Equations in the SI Units and in the Gaussian Units

SI Units	Gaussian Units
$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_0}$	$\nabla \cdot \mathbf{E} = 4\pi\rho_c$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} (= \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t})$	$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$

Table 1.2 Force, energy, and frequency in the SI Units and in the Gaussian Units

	SI Units	Gaussian Units
Magnetic force of a charged particle	$q\mathbf{v} \times \mathbf{B}$	$q\frac{\mathbf{v} \times \mathbf{B}}{c}$
Magnetic force in fluid plasma per unit volume	$\mathbf{J} \times \mathbf{B}$	$\frac{\mathbf{J} \times \mathbf{B}}{c}$
Magnetic energy density (or magnetic pressure)	$\frac{B^2}{2\mu_0}$	$\frac{B^2}{8\pi}$
Electric energy density (or electric pressure)	$\frac{\varepsilon_0 E^2}{2}$	$\frac{E^2}{8\pi}$
Alfvén speed	$\frac{B}{\sqrt{\mu_0 \rho}}$	$\frac{B}{\sqrt{4\pi\rho}}$
Plasma frequency of the αth species	$\sqrt{\frac{ne^2}{\varepsilon_0 m_\alpha}}$	$\sqrt{\frac{4\pi ne^2}{m_\alpha}}$
gyro frequency of the αth species	$\frac{eB}{m_\alpha}$	$\frac{eB}{m_\alpha c}$