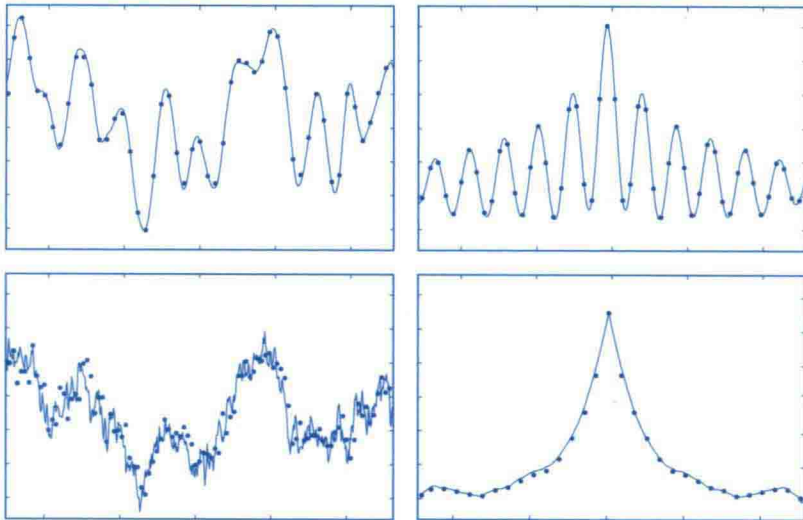


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TECHNISCHE UNIVERSITÄT MÜNCHEN
Lehrstuhl für Baumechanik

**Output-only measurement-based parameter
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load processes**

Katrin S. Runtemund

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Abstract

In the present work a new output-only measurement based method is proposed which allows identifying the modal parameters of structures subjected to natural loads such as wind, ocean waves, traffic or human walk. The focus lies on the dynamic excitation of structures by wind turbulences and wind-induced ocean waves modeled as stationary Gaussian random process. In contrast to the existing output-only identification techniques which model the unmeasured load as white noise process, statistical information about the dynamic excitation, e.g. obtained by measurements of the wind fluctuations in the vicinity of the structure, are taken into account which improve the identification results as well as allow identifying the unmeasured load process exciting the structure.

The identification problem is solved on basis of a recently developed method called *H-fractional spectral moment (H-FSM) decomposition* of the transfer function $H(\omega)$ which allows representing Gaussian random processes with known power spectral density (PSD) function as output of a linear fractional differential equation with white noise input.

In the present work the efficiency and accuracy of this method is improved by the use of an alternative fractional operator and a modification is proposed which makes it applicable to short as well as long memory processes. The most widely used wind and ocean wave model spectra are compared and discussed, and the corresponding H-FSMs are provided in closed form allowing to simulate realization of the processes in a straight forward manner. Based on the FSM decomposition a state space representation of arbitrarily correlated Gaussian processes is developed in closed form which neither requires the factorization of the PSD function nor any optimization procedure. Combined with the state space model of the structure, it leads to an overall model with white noise input, which can be efficiently combined with any state-space model-based parameter identification algorithms such as the well known (weighted) extended Kalman filter algorithm used here. The method is successfully applied for the stiffness and damping estimation of single and multi-degree of freedom systems subjected to wind and wind-wave turbulences as well as for the estimation of the unmeasured load process. Finally, a sensitivity analysis of the filter accuracy is conducted in order to improve the accuracy and efficiency of the method.

Zusammenfassung

In der vorliegenden Arbeit wird eine neue Methode zur Identifikation modaler Parameter dynamischer Systeme entwickelt, die auf (Output-only) Messungen der Systemantwort infolge der natürlichen Anregung durch Lasten wie z.B. Wind, Wellen, Verkehr oder Personen basiert. Der Fokus der Arbeit liegt hierbei auf der stochastischen Anregung durch Windturbulenzen und windinduzierten Wellen, welche als Realisation stationärer Gaußscher Prozesse modelliert werden. Im Gegensatz zu bestehenden Output-only Identifikationsverfahren, die die unbekannten Lasten vereinfacht als weiße Rauschprozesse beschreiben, werden hier zusätzliche statistische Informationen, die beispielsweise durch Windmessung in der Nähe der Struktur gewonnen werden, berücksichtigt. Dies führt nicht nur zu einer Verbesserung der Parameterschätzung, sondern ermöglicht auch die gleichzeitige Lastidentifikation.

Das entwickelte Identifikationsverfahren basiert auf einer kürzlich entwickelten Methode, der sogenannten „*H-fractional spectral moment (H-FSM) decomposition*“, d.h. der Zerlegung der Übertragungsfunktion $H(\omega)$ mit Hilfe von spektralen Momenten fraktionaler Ordnung. Die Methode erlaubt einen Gaußschen Prozess mit gegebener Leistungsspektraldichte (PSD) als Output einer linearen fraktionalen Differentialgleichung mit weißem Rauschen als Input zu simulieren.

Im Rahmen dieser Arbeit wird die Effizienz und die Genauigkeit dieser Methode durch die Verwendung eines alternativen fraktionalen Integraloperators verbessert und die Definition der H-FSMs derart modifiziert, dass die Methode nicht nur für sogenannte „Short Memory“ Prozesse mit exponentiell abklingender Autokorrelation, sondern auch für langkorrelierte („Long Memory“) Prozesse anwendbar ist. Die gebräuchlichsten Wind und Windwellen charakterisierenden Modellspektren werden diskutiert und die zugehörigen H-FSMs in analytischer Form zur Verfügung gestellt, mit Hilfe derer, Realisationen der Prozesse in einfacher Weise generiert werden können. Auf der H-FSM Zerlegung aufbauend, wird ein für beliebig korrelierte Lastprozesse gültiges lineares Zustandsraummodell in analytischer Form hergeleitet, das im Gegensatz zu gebräuchlichen Methoden weder die spektrale Faktorisierung der Leistungsspektraldichte noch die Anwendung eines Optimierungsverfahrens erfordert. Es erlaubt die Lasten in die Systemgleichungen zu integrieren, so dass das System mit korrelierten Lasten auf ein Gesamtsystem höherer Ordnung mit weißem Rauschen als Input

zurückgeführt werden kann, dessen Parameter dann mit einem beliebigen zustandsraum-basierten Verfahren, wie z.B. das hier verwendete Erweiterte Kalman Filter, identifiziert werden können. Die Methode wird für die Schätzung der Steifigkeits- und Dämpfungsp-arameter von Ein- und Mehrfreiheitsgradsystemen unter wind- und welleninduzierten Lasten sowie für die Schätzung des unbekannten Lastprozesses verwendet. Schließlich wird eine Sensitivitätsanalyse durchgeführt, mit dem Ziel, die Genauigkeit und die Effizienz des Algo-rithmus weiter zu verbessern.

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List of Symbols and Acronyms

Typical units are given in square brackets.

Subscripts

a	[-]	augmented
c	[-]	continuous-time
d	[-]	discrete-time
k	[-]	time step k at time $t = k\tau$, where τ is the sampling interval
Exp	[-]	Exponential
Kar	[-]	von Kármán
PM	[-]	Pierson Moskowitz

Superscripts

A^{-1}	[-]	inverse of A (matrix)
A^T	[-]	transpose of A (matrix)
X^*	[-]	complex conjugate of X
$\hat{X}$	[-]	estimate of X
$\bar{X}$	[-]	mean value of X
$\tilde{X}$	[-]	true (undisturbed) value of X

Greek letters

α	[-]	Power exponent of the wind profile
$\alpha_k(\gamma), \alpha_{c,k}(\gamma)$	[-]	Coefficients of the (centered) GL discretization
$\eta = Im\gamma$	[-]	Imaginary part of the complex number $\gamma \in \mathbb{C}$
$\Delta\eta$	[-]	Discretization step width along the imaginary axis
$\eta(t, x)$	[m]	Sea surface profile at time t and location x
γ_i	$[m^2, (m/s)^2]$	Mean square of the i th element of the posterior error $d_{i,k k}$; s. Eq. (6.42a)
γ_i^N	[-]	Mean square of the i th element of the posterior error $d_{i,k k}$ normalized with respect to the undisturbed system response; s. Eq. (6.43a)
$\hat{\gamma}_i^N$	[-]	Normalized mean square of the i th element of the posterior error $d_{i,k k}$ normalized with respect to the noisy system response; s. Eq. (6.42b)
κ	[-]	Surface drag coefficient
λ	[m]	Wavelength
$\lambda_X(\gamma)$	$[N^2/s\gamma]$	Spectral moments of the PSD function of the load process $\{X(t)\}$ of order $\gamma \in \mathbb{N}_0$
$\Lambda_X(\gamma)$	$[N^2/s\gamma]$	Fractional spectral moments of the PSD function of the process $\{X(t)\}$ of order $\gamma \in \mathbb{C}$
$\mu(t)$	[m, N]	Mean value at time t , e.g. of displacement or force
$\phi(t)$	[rad]	Phase angle at time t
$\Pi_H(\gamma)$	$[s^{-(1/2+\gamma)}]$	H-fractional spectral moments of the transfer function $H(\omega)$ of order $\gamma \in \mathbb{C}$
ω	[rad/s]	Angular frequency
ω_i	[-]	Sample point
Ω	[-]	Sample space
$\psi_a(z; f)$	[Ns/m]	Aerodynamic admittance function relating wind force and wind velocity
$\rho = Re\gamma$	[-]	Real part of the complex number $\gamma \in \mathbb{C}$
ρ_a, ρ_w	$[kg/m^3]$	Air, water density
$\rho_u(x, y, z; \tau)$	[-]	Normalized AC function of the longitudinal wind velocity fluctuations
σ	[m, N]	Standard deviation, e.g. displacement, force
σ^2	$[m^2, N^2]$	Variance, e.g. displacement, force
σ_η	[m]	Standard deviation of the surface evaluation

σ_F	[N]	Standard deviation of the dynamic drag force $F_D'(z;t)$
$\sigma_u, \sigma_v, \sigma_w$	[m/s]	Standard deviation of the longitudinal, lateral and vertical wind fluctuations
σ_y	[m]	Standard deviation of the floor displacements
$\sigma_{\dot{y}}$	[m/s]	Standard deviation of the floor velocities
σ_k	[N/m, Ns/m]	Standard deviation of estimated stiffness and damping parameters
$\Sigma_{yy,k}$	[m ²]	Covariance matrix of the floor displacements
$\Sigma_{\dot{y}\dot{y},k}$	[(m/s) ²]	Covariance matrix of the floor velocities
$\Sigma_{pp,k}$	[(N/m) ² , (Ns/m) ²]	Covariance matrix of the unknown stiffness and damping parameters
$\Sigma_{xx,k}$	[m ²]	Covariance matrix of the state estimates
θ^j	[m ² , (m/s) ²]	Objective function, i.e. the average of all measurement square errors at the end of the j th iteration 6.30a
θ_{min}^j	[-]	Minimum of the objective function
θ_{min}^N	[-]	Minimum of the objective function normalized with respect to the undisturbed system response, s. Eq. (6.42c)
$\hat{\theta}_{min}^N$	[-]	Minimum of the objective function normalized with respect to the noisy system response, s. Eq. (6.43b)
τ	[s]	Time shift/lag, sampling interval

Latin letters

$a(x, z; t)$	[m/s ²]	Horizontal water particle acceleration
$a_0, \dots, a_p$	[-]	Coefficients of the autoregressive model $AR(p)$ of order p
$d\beta(t) = W(t)dt$		Increment of the Brownian motion process
A	[m ²]	Surface area of the structure in the wind flow
$\mathbf{A}$	[-]	State transition coefficient matrix
$AR(p)$	[-]	Autoregressive model of order p
$ARV(n, p)$	[-]	n -dimensional vector AR model of order p
$ARMA(p, q)$	[-]	Autoregressive moving average model of order p, q
$ARMA(n, p, q)$	[-]	n -dimensional ARMA model of order p, q
$b_0, \dots, b_q$	[-]	Coefficients of the moving average model
B	[m]	Characteristic length of an object in the wind flow
$B(t)$	[N]	Brownian motion process
ΔB	[N]	Increments of the Brownian motion process

B	$[-]$	Force transition matrix
c	$[m/s]$	Propagation velocity of the sea wave
c	$[Ns/m]$	Viscous damping coefficient
c_1, c_2, c_3	$[Ns/m]$	Viscous damping coefficients of the three story shear building (s. Fig. 6.6a)
$c_0, c_1, \dots$	$[-]$	Coefficients of the infinite moving average model $MA(\infty)$
C_d	$[-]$	Dimensionless drag coefficient
C_F	$[-]$	Dimensionless force coefficient
C_m	$[-]$	Dimensionless inertia coefficient
$C_X(t_1, t_2)$	$[N^2]$	Autocovariance function of the process $\{X(t)\}$
C	$[-]$	Observation transition matrix
$d_0, d_1, \dots$	$[-]$	Coefficients of the infinite autoregressive model $AR(\infty)$
d	$[m]$	Height above the ground where the mean wind velocity is zero
$\mathbf{d}_k$	$[m, m/s]$	Innovation: Discrepancy between actual measurement and prediction
$\mathbf{d}_{k k}$	$[m, m/s]$	Posterior innovation: Discrepancy between actual measurement and optimal estimate
D	$[m]$	Diameter of a cylinder
D	$[-]$	Ratio of critical damping
D	$[-]$	Measurement noise transition matrix
$e_{c,0}$	$[-]$	Initial relative error of the damping estimates
$e_{k,0}$	$[-]$	Initial relative error of the stiffness estimates
$\mathbf{e}_{x,k}$	$[m, m/s]$	Posterior error at time k
$\mathbf{e}_{x,k+1 k}$	$[m, m/s]$	Prior prediction error at time $k+1$ including all information up to time k
f	$[Hz]$	Frequency
F, F_{min}	$[m]$	Storm fetch, minimum fetch
$F_d(x, z; t)$	$[N]$	Horizontal drag force of the sea wave
$F_D(z; t)$	$[N]$	Aerodynamic drag (or along wind) force
$\bar{F}_D(z; t)$	$[N]$	Mean value of the aerodynamic drag force
$F'_D(z; t)$	$[N]$	Dynamic time-dependent part of the aerodynamic drag force
$F_m(x, z; t)$	$[N]$	Horizontal inertia force of the sea wave
$g = 9.81$	$[m/s^2]$	Gravitational acceleration
$G_X(\omega), G_X(f)$	$[N^2s]$	One-sided PSD function of the load process $\{X(t)\}$

$\mathbf{G}$	$[-]$	Process noise transition matrix
$h(t)$	$[\text{s/kg}]$	Impulse response function
H	$[\text{m}]$	Wave height (measured from wave crest to trough)
H_0	$[\text{m}]$	Most probable wave height
$\bar{H}$	$[\text{m}]$	Mean probable wave height
H_{rms}	$[\text{m}]$	Root-mean-square wave height
H_s	$[\text{m}]$	Significant wave height
$H(\omega, z)$	$[\text{rad/s}]$	Transfer function relating sea surface elevation, velocity and acceleration
$H(\omega), \mathbf{H}(\omega)$	$[\text{s}^{1/2}]$	Transfer function and transfer matrix relating white noise input and colored load process
$H(\gamma)$	$[\text{s/kg}]$	Matrix transfer function obtained by FSM decomposition, s. Eq. (5.31)
I_u, I_v, I_w	$[-]$	Turbulence intensities of the fluctuation components u_D , v_D and w_D
I_F	$[-]$	Intensity of the dynamic drag force fluctuations $F_D'(z; t)$
$\mathbf{I}_{n \times n}$	$[-]$	Identity matrix of order $n \times n$
KC	$[-]$	Keulegan-Carpenter number
$k = \frac{2\pi}{\lambda}$	$[-]$	Wavenumber
k	$[\text{N/m}]$	Stiffness coefficient
k_1, k_2, k_3	$[\text{N/m}]$	Stiffness coefficients of the three story shear building (s. Fig. 6.6a)
L	$[-]$	Backshift operator
$L_{C,u}(z)$	$[\text{m}]$	Longitudinal integral length scale of wind eddies proposed by Couninhan
$L_u(z)$	$[\text{m}]$	Longitudinal integral length scale of wind eddies
$L_i^j(z)$	$[\text{m}]$	Longitudinal, lateral and vertical integral length scale of eddies at height z in $j = x, y, z$ direction
m	$[\text{kg}]$	Mass
m	$[-]$	Number of considered fractional spectral moments
m_1, m_2, m_3	$[\text{kg}]$	Mass coefficients of the three story shear building (s. Fig. 6.6a)
$M = p\tau$	$[\text{s}]$	Considered process memory (s. Eq. 5.5.1)
$\text{MA}(q)$	$[-]$	Moving average model of order p
$\text{MA}(n, q)$	$[-]$	n -dimensional MA model of order p
$n = \frac{f L_u}{U_z}$	$[-]$	Dimensionless (reduced) frequency

$n_z = \frac{fz}{U_z}$	[-]	Monin similarity coordinate (dimensionless frequency)
$\{N(t)\}$	[N]	Autocorrelated random process
p	[-]	Number of load coefficients
$p_v(z; t)$	[N/m ²]	Velocity pressure
$p(X)$	[-]	Probability density function of the random number X
$\mathbf{p}_k$	[-]	parameter vector
q_W	[N ²]	Intensity of the white noise process
$Q_k, \mathbf{Q}_k$	[N ²]	Process noise covariance (matrix) at time k
$R_k, \mathbf{R}_k$	[m ² , (m/s) ²]	Measurement noise covariance (matrix) at time k
$R_X(t)$	[N ²]	AC function of the stationary process $\{X(t)\}$
$R_X(t_1, t_2)$	[N ²]	AC function of the non-stationary process $\{X(t)\}$
$R_{XY}(t)$	[N ²]	Cross correlation function of the stationary processes $\{X(t)\}$ and $\{Y(t)\}$
$R_{XY}(t_1, t_2)$	[N ²]	Cross correlation function of the non-stationary processes $\{X(t)\}$ and $\{Y(t)\}$
s	[-]	Complex frequency of the Laplace domain
$S_X(\omega), S_X(f)$	[N ² s]	two-sided PSD function of the load process $\{X(t)\}$
t	[s]	Time
t_s	[s]	Sampling interval
T	[s]	(Averaging) time period
T_p	[s]	Wave period
$T_u(z)$	[s]	Heights dependent integral time scale of the longitudinal wind fluctuations
u_D, v_D, w_D	[m/s]	Longitudinal (along wind), lateral and vertical component of the dynamic wind velocity fluctuations
U_*	[m/s]	Friction velocity
U_g	[m/s]	Gradient wind velocity
$\bar{U}(z), \bar{U}_z$	[m/s]	Longitudinal mean wind speed
$U(\omega)$	[-]	Unit step function
$\mathbf{U}_D(x, y, z; t)$	[m/s]	Random vector process of the dynamic wind velocity fluctuation at location x, y, z in space and at time t
$v(x, z; t)$	[m/s]	Horizontal water particle velocity
$W(t), W_k$	[N]	Elements of the white noise process $\{W(t)\}, \{W_t\}$
$\{W(t)\}, \{\mathbf{W}(t)\}$	[N]	White noise (vector) process
x, y, z	[-]	Coordinates of the Cartesian coordinate system
$\mathbf{x}_{k k}$	[m, m/s]	Posterior state estimate at time k including all measurements up to time k

$\mathbf{x}_{k+1 k}$	[m, m/s]	Prior state estimate at time $k + 1$ including all measurements up to time k
X	[-]	Random variable
X_{ref}	[m, m/s]	Reference sensor
$X(t), X_k$	[N]	Elements of the random process $\{X(t)\}, \{X_t\}$
$\{X(t)\}, \{\mathbf{X}(t)\}$	[N]	Continuous-time random (vector) process
$\{X_t\}, \{\mathbf{X}_t\}$	[N]	Discrete-time (vector) random process
$\mathbf{x}_k$	[-]	State vector
V_k	[m, m/s]	Measurement noise
z	[m]	Height over ground
$z = e^s$	[-]	Complex parameter in the z -domain
z_0	[m]	Roughness length
z_g	[m]	Gradient height
z_{ref}	[m]	Reference height
$\mathbf{z}_k$	[-]	Measurement vector

Mathematical symbols

$\{\cdot\}$	[-]	Random Process
$\binom{n}{k} = \frac{n!}{k!(n-k)!}$	[-]	Binomial coefficient
$\mathbb{C}$	[-]	Set of complex numbers
$\mathcal{C}_q$	[-]	Controllability Matrix of order q
$\delta(t - t_0)$	[-]	Dirac delta function at point t_0
δ_{kj}	[-]	Kronecker delta function
$(D^\gamma f)(t)$	[-]	(Fractional) derivative of order $\gamma \in \mathbb{C}$
$(D_c^\gamma f)(t)$	[-]	Centered Grünwald-Letnikov fractional derivative of order $\gamma \in \mathbb{C}$
$(D_+^\gamma f)(t),$	[-]	Right- and left-sided fractional derivative operator of order $\gamma \in \mathbb{C}$
$(D_-^\gamma f)(t)$	[-]	
$E[\cdot]$	[-]	Expectation operator
$F_n(x_1, \dots, x_n; t_1, \dots, t_n)$	[-]	n -th dimensional probability distribution function of a random process
${}_pF_q[a_1, \dots, a_p; b_1, \dots, b_q; z]$	[-]	Generalized hypergeometric function of order p, q
$\mathcal{F}\{\omega; t\}$	[-]	Fourier transform
$\mathcal{F}^{-1}\{t; \omega\}$	[-]	Inverse Fourier transform
$\Gamma(\cdot)$	[-]	Gamma function

$\mathcal{H}_{p,q}$	$[-]$	Hankel matrix of order p,q
$(I^\gamma f)(t)$	$[-]$	(Fractional) derivative of order $\gamma \in \mathbb{C}$
$(I_c^\gamma f)(t)$	$[-]$	Centered Grünwald-Letnikov fractional integral of order $\gamma \in \mathbb{C}$
$(I_+^\gamma f)(t),$ $(I_-^\gamma f)(t)$	$[-]$	Right- and left-sided fractional integral operator of order $\gamma \in \mathbb{C}$
$\{k; k = 0,1,2,\dots\}$	$[-]$	Discrete index set
$\ln(\cdot)$	$[-]$	Natural logarithm
$\mathcal{L}(\cdot)$	$[-]$	Linear differential operator
$\circ \longrightarrow \bullet$	$[-]$	Transform from time to Laplace domain
$\mathcal{M}\{t; \gamma\}$	$[-]$	Mellin transform
$\mathcal{M}^{-1}\{\gamma, t\}$	$[-]$	Inverse Mellin transform
$p(x_1,\dots,x_n; t_1,\dots,t_n)$	$[-]$	n -th dimensional probability density function of a random process
$\mathbb{N} = \{1,2,3,\dots\}$	$[-]$	Set of positive integer number
$\mathbb{N}_0 = \{0,1,2,\dots\}$	$[-]$	Set of non-negative integer numbers
$\mathcal{O}_p$	$[-]$	Observability Matrix of order p
$\mathbb{R}^{n \times n}$	$[-]$	$n \times n$ -dimensional set of real numbers
$\{X(t_i; \cdot)\}$	$[-]$	Family of random variables characterizing the random process
$\{X(\cdot; \omega_i)\}$	$[-]$	Realization of a random process

Acronyms

AC	Autocorrelation function
AIC	Akaike's Information Theoretic Criterion
AvTs	Ambient vibrations Tests
AR	Autoregressive model
ARMA	Autoregressive Moving Average
BR	Balanced Realization Method
CVA	Canonical Variant Analysis
CF	Characteristic Function
EMA	Experimental Modal Analysis
FPE	Akaike's Final Prediction Error Criterion
FRF	Frequency Response Functions
GL	Grünwald-Letnikov