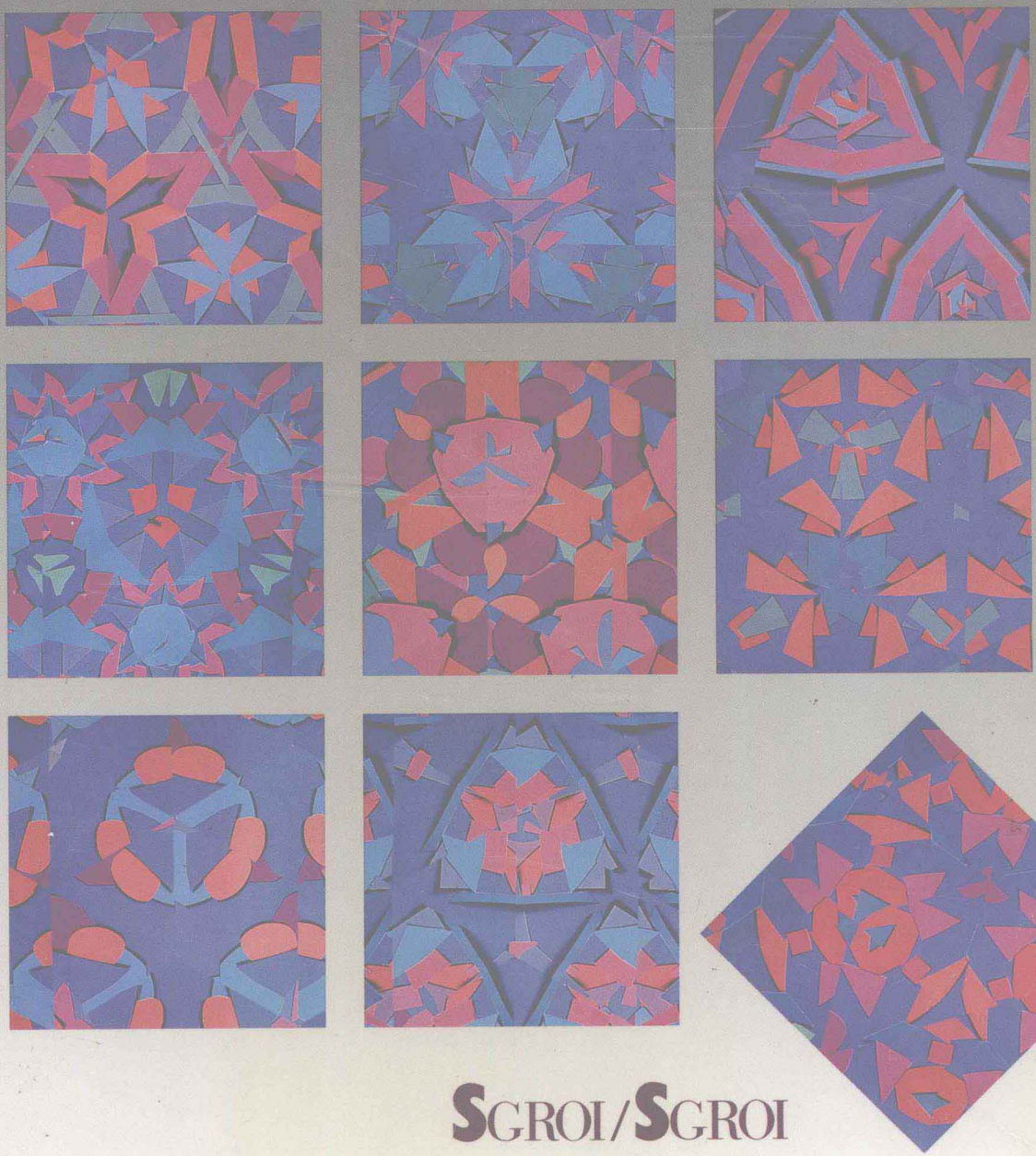
MATHEMATICS FOR ELEMENTARY SCHOOL TEACHERS

PROBLEM-SOLVING INVESTIGATIONS



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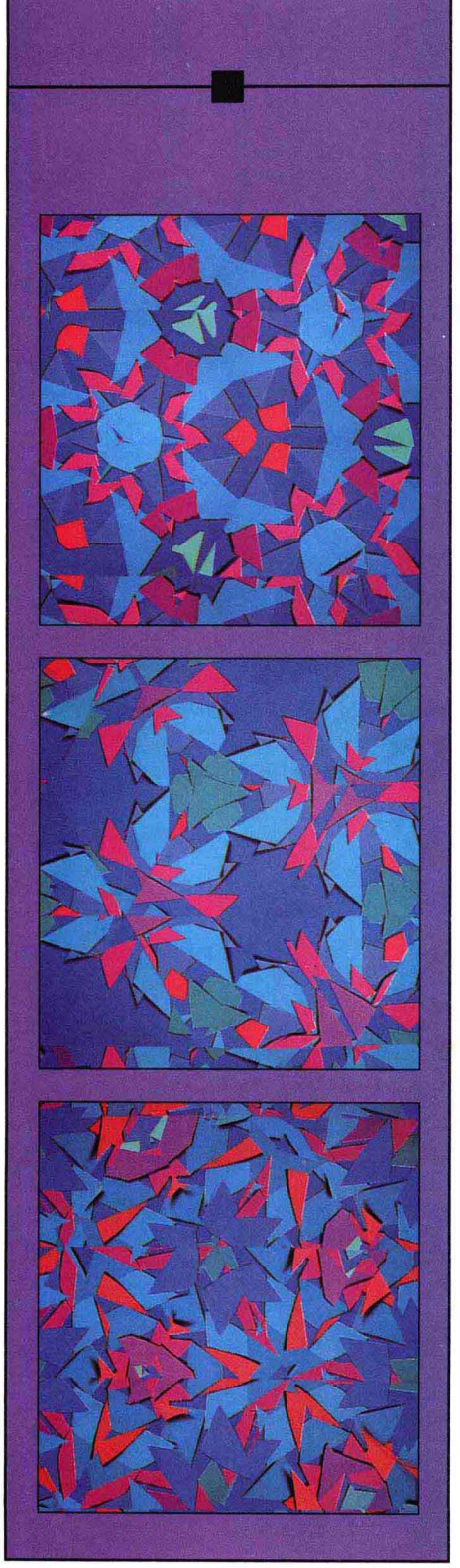

Mathematics for Elementary School Teachers

Problem-Solving Investigations

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PWS Publishing Company
Boston

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Publishing Company

20 Park Plaza
Boston, Massachusetts 02116

To the memory of Professor Marc Belth:
mentor, friend, and inspiration
and
To our children,
Allison, Elizabeth, and Kathryn

Acquisitions Editor: Tim Anderson
Production Editors: S. London and Patricia Adams
Developmental Editor: Barbara Lovenvirth
Manufacturing Coordinator: Marcia Locke
Cover and Interior Designer: S. London
Cover Photographer: Greg Bowl Studio. Lens courtesy of C. Bennett Scopes, Inc.
Interior Illustrators: Artscribe; TI-81 Graphics Calculators: Scientific Illustrators
Typesetter: TSI Graphics
Endpaper and Cover Printer: New England Book Components, Inc.
Text Printer/Binder: R. R. Donnelley & Sons Company

PWS Publishing Company is a division of Wadsworth, Inc.

Library of Congress Cataloging-in-Publication Data

Sgroi, Richard J.
Mathematics for elementary school teachers : problem-solving investigations / Richard J. Sgroi, Laura Shannon Sgroi.
p. cm.—(The Prindle, Weber & Schmidt series in mathematics)
Includes index.
ISBN 0-534-93255-X
1. Mathematics—Study and teaching (Elementary) I. Sgroi, Richard J. II. Title. III. Series.
QA135.5.S456 1993 92-37211
372.7—dc20 CIP

Photography Credits: Ch. 1 opener, © Greg Bowl Studio; Ch. 2 opener, © Nienhuis Montessori USA; Ch. 3 opener, © Steve Dunwell/The Image Bank; Ch. 4 opener, © Flip Chalfant/The Image Bank; Ch. 5 opener, © Joe Sohm/Panoramic Images, Chicago 1992; Ch. 6 opener, © Greg Bowl Studio; Ch. 7 opener, © John P. Kelly/The Image Bank; Ch. 8 opener, © Greg Bowl Studio; Ch. 9 opener, © AccuWeather, Inc.; Ch. 10, © Gary Gladstone/The Image Bank; Ch. 11 opener, © Greg Bowl Studio; Ch. 12, © Gary Gay/The Image Bank; Ch. 13 opener, © Co Rentmeester/The Image Bank.

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Printed in the United States of America.
2 3 4 5 6 7 8 9 10—97 96 95 94 93

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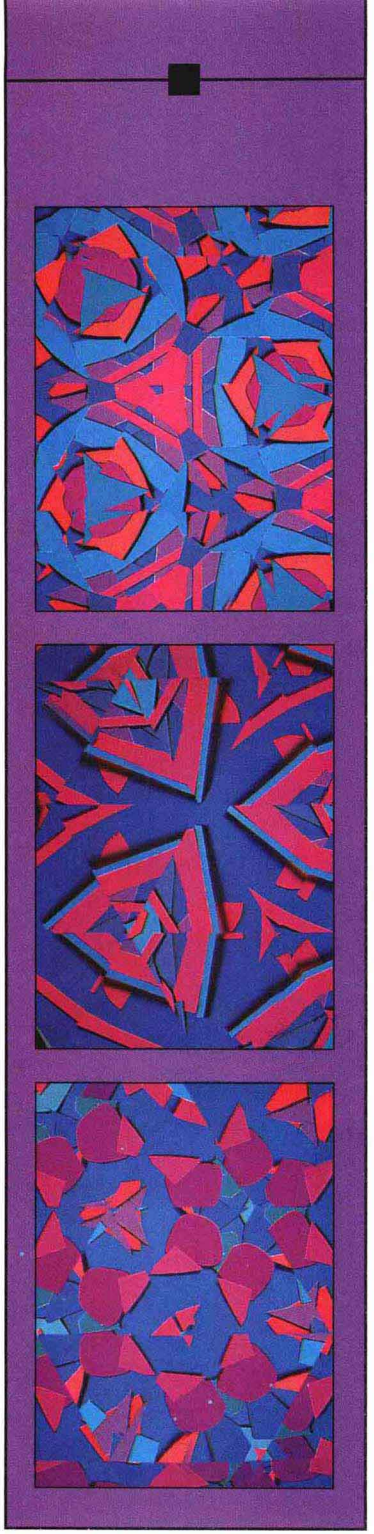


Preface

As we approach the turn of the century, many exciting changes are taking place in the field of mathematics education. A number of these changes are influenced by modifications in the world economy, which places greater and greater emphasis on mathematical skills. These changes are also necessitated by continuing technological innovations, which are constantly producing more powerful, affordable, and accessible calculators and computers.

NCTM Standards

The professional organization of mathematics educators from the United States and Canada, the National Council of Teachers of Mathematics (NCTM), has taken an active role in helping the mathematics education community achieve greater parity between mathematics as taught in schools and that which is likely to be useful to working professionals in the early part of the twenty-first century. As a step toward achieving this goal, the NCTM has issued a document, *The Curriculum and Evaluation Standards for School Mathematics* (1989), or as it is commonly referred to, *The Standards*. In this document, the educators identify certain curricular strands that merit increased attention, such as problem solving, or decreased attention, such as memorization of rules. *The Standards* also recommend employing specific pedagogical strategies, such as active learning, the use of manipulatives, writing to learn, and group activities. (Excerpts from *The Standards* are included in Appendix B.)

Because we as authors and educators recognize the importance of *The Standards* and agree with their goals, many of NCTM's suggestions are incorporated throughout this text. The recommendations contained within *The Standards* have influenced our selection of topics and the manner in which they are presented. Each chapter emphasizes problem solving, the meaning behind various algorithms or procedures, careful concept development, and the use of physical models.

Pedagogy and Features

Because of the importance of problem-solving skills to today's students, problem solving has not been relegated to a single chapter; we do not view it as a discrete topic. Rather, we view problem solving as the focus of this text and the focus of elementary school mathematics. Our combined teaching experience (which spans all educational levels, from kindergarten through graduate students) has led us to conclude that the ability to solve problems is acquired slowly. We present problems in a variety of contexts and for a variety of purposes, enabling the student to begin to understand that mathematics is a way of thinking about and understanding the world.

Problem-solving **Investigations**, found in each section of each chapter, are key to the presentations of concepts. Many of these investigations employ purposeful problem-solving strategies and use the four-step plan for solving problems introduced by George Polya in his book *How to Solve It* (1985). Beginning with the first section in Chapter 1, Polya's four-step plan is continually referred to and expanded upon throughout the text. Numerous problems and exercises can be found in the **Assessments** at the end of each section and chapter, providing the student with opportunities to practice and reinforce the use of Polya's four steps.

Other features in this text are also the result of recommendations contained in *The Standards*. There is evidence to suggest that students who study and work through math assignments in a group format are more successful than students who work in isolation. At the end of each section and chapter, we include the **Cooperative Activity**. These contain two parts: *Individual Accountability* and *Group Goal*. The student is first asked to make some contribution to the group effort. In some of the activities, the tasks are the same for all members of the group; in others, each group member works on a different aspect of the activity. Each student then brings to the group his or her findings and results, sharing strategies and refining plans based on comparing the methods used. The group is then given a problem to solve collaboratively, using the skills developed in the individual component of the activity.

Following each section and chapter, **In Other Words** offer students the opportunity to refine their thinking and understanding by writing explanations of selected mathematical concepts. Reflecting on a concept and then writing an explanation will help engage students in the mathematical communication process while structuring and strengthening their reasoning abilities.

At the end of each chapter, a **Vocabulary** of key terms reinforces and reminds the student of important issues discussed in the chapter. A **Review** is also provided at the end of each chapter. These reviews contain additional exercises and problems for the entire chapter.

Each chapter opener serves as an informal visual introduction to the concepts presented in the chapter. Students are asked to examine the photograph shown and answer questions about what they observe, using problem-solving and estimation

skills. The photographs selected for the openers are repeated in color in the Contents, enabling students to obtain another perspective if they so desire.

Endpapers in color depict the Fibonacci Sequence of Numbers (front) and the Golden Proportion (back). Each of these charts provides information helpful to students and is related to text discussion. The Fibonacci sequence can be found in the *Appendix A—Spreadsheets*, **Investigation 2**, and the Golden Ratio is discussed in Chapter 11.

Calculator Use and Estimation Skills

One of the assumptions made in *The Standards* is that all students will have a calculator. Therefore, throughout this text, access to a calculator is assumed, and specific applications and instructions for efficient use of the calculator are included where appropriate. We do not indicate when to use a calculator, leaving this determination to the student. Knowing when to utilize a calculator and when to perform the work mentally or with paper and pencil is a judgment the student will learn to make while working through this text. Acquiring this ability will necessitate and reinforce the development and strengthening of students' estimation skills. Calculator keystroke sequences are introduced and used wherever appropriate.

Alternative Models and Visuals

This text recognizes that students learn in a variety of ways. For this reason, we offer many alternative models and algorithms that can be used to view and interpret a particular problem situation. In addition, this text contains well over 2000 illustrations that serve to pictorially represent arithmetic, algebraic, and geometric concepts and problems.

Laminated Card and Problem Solving

In writing this book, we struggled with the traditional example–solution techinique in which authors present a problem for the student and then immediately solve it. We believe an important part of the problem-solving process is to allow the student to reflect before he or she acts. When teaching problem solving in a classroom setting, there are many opportunities to give your students time to ponder the problem, to offer a hint to steer them in the right direction, and to further extend the problem. This subtle interplay is not possible within the framework of a textbook. In order to give students the opportunity to first attempt to solve the **Investigations** on their own, we designed the **Laminated Card**, which is included with this text. We suggest that the students use this card to cover the solutions to each investigation and make a serious effort to solve the problems before checking the solution. A symbol  appears in the margin at the start of each investigation as a reminder to the student to first use the card. Both the simulated geoboard and graph paper that appear on this card will be useful in the solutions of some of the problems,

especially in Chapters 10–13, which deal with geometry. This card is laminated to allow the student to reuse it as often as needed, with a felt-tipped pen provided for this purpose.

Number Theory

We have purposely chosen to introduce and use number theory topics in contexts where relevant. We made this decision because we strongly feel that, taken out of context, some number theory rules and properties tend to be memorized rather than understood. When we place number theory in the contexts where it is most frequently used, students can develop a better understanding of its meaning and purpose.

Manipulatives

Also packaged with this text as an aid to both students and instructors are **Manipulatives transparencies** for hands-on use with the exercises found throughout the text. These **Manipulatives** will also be extremely useful with the *Student Activity Book* designed to be used with this text, and for any classroom activities chosen by instructors.

Philosophy of Standards and Authors

Just as important as the specific recommendations contained within *The Standards* is the basic philosophy that the document promotes. This philosophy, embraced by the text, adheres to the belief that learning mathematics is a task that can be accomplished by almost anyone who is willing to apply effort, perseverance, and diligence and who brings a questioning mind to the study of mathematics. If students, as they work through this text, find themselves asking why something is so, or where a particular concept came from and is leading to, or if there is an alternative explanation or method, then they are on their way to becoming learners of mathematics. It is necessary to become a learner of mathematics before one can become a teacher of mathematics.

Supplements

For Instructors

Instructor's Manual—contains complete solutions to all exercises found in the text and for all chapter reviews. Each chapter opens with an introduction to the material being covered, as well as a discussion of the chapter opening photograph in the text. Each section begins with the section objectives, followed by teaching suggestions for classroom activities, and a discussion of the cooperative group activities found in the Assessments. Related readings for each of the chapters are recommended. Transparency Masters of selected illustrations from the text are also included in the manual.

EXPTest—a computerized test bank for the IBM PC and compatibles, consisting of open-ended, multiple-choice, and true/false questions. Test items can be edited, rearranged, and expanded on according to need. Users may also add test items to the disk. A demo disk is available.

ExamBuilder—a computerized test bank for the Macintosh. Features and questions are identical to EXPTest. A demo disk is available.

For Students

Student Activity Book by Victoria Garrison—contains activities designed to accompany each chapter of the text. Each section will open with an "In Our World" activity motivating the student to see the applications of each chapter topic to the real world. The activities require selected materials, including cut-out manipulatives found at the back of the Activity Book, and actual manipulatives provided with the text. Each section will also contain pages entitled "Check Your Understanding," which the instructor can collect to monitor the students' progress. This manual enables students to use and reinforce skills acquired in each text chapter, and to build their expertise and confidence. Answers to the activities are provided. (Answers to "*Check Your Understanding*" are found in the Instructor's Manual.)

Problem-Solving Strategies by Robert Matulis—this workbook provides students with strategies for using the problem-solving skills they are learning. Chapters 1–13 follow the chapter sequence of the text, and each section contains problems utilizing Polya's Four Steps. Students are asked to try each problem before using the hints and suggestions provided by the author. Students are encouraged to use the laminated card from the text as they work on these problems. A final chapter entitled *True Problems* offers miscellaneous problems from all the earlier chapters without indicating which of the chapters are the source, allowing the students to make this determination.

Acknowledgments

This book has grown out of a firm desire to improve mathematics education for both students and teachers. We believe that students must come through the courses taught for preservice elementary school teachers with a feeling of accomplishment, understanding, and self-confidence. Their mathematical reasoning and problem-solving abilities must be strong, and their understanding of mathematics must go beyond rules and formulas. Only then will pedagogy make sense and methods of teaching mathematics have a firm foundation.

We must acknowledge many people who have assisted us in this endeavor. We greatly appreciate all of the efforts on the part of those at PWS-KENT. Many people worked on our project in the editorial and production departments. In particular, we recognize (in alphabetical order), Patty Adams, Tim Anderson, Susan London, and Barbara Lovenvirth. We would also like to thank Sally Stickney for her many long hours of impeccable copy editing. Her comments and suggestions were insightful and always well-taken.

Many reviewers helped to mold and shape our manuscript into a viable textbook. We greatly appreciate all of the efforts of the following reviewers.

Mary K. Alter
University of Maryland

Tom Bassarear
Keene State College

Janet J. Brougher
University of Oregon

Max Coleman
Sam Houston State University

Philip Downum
SUNY College at Oswego

Cynthia Fleck
Wright State University

Charlotte K. Lewis
University of New Orleans

Robert S. Matulis
Millersville University

Myron Morford
Edinboro University of Pennsylvania

Barbara J. Shabell
California State Polytechnic University-Pomona

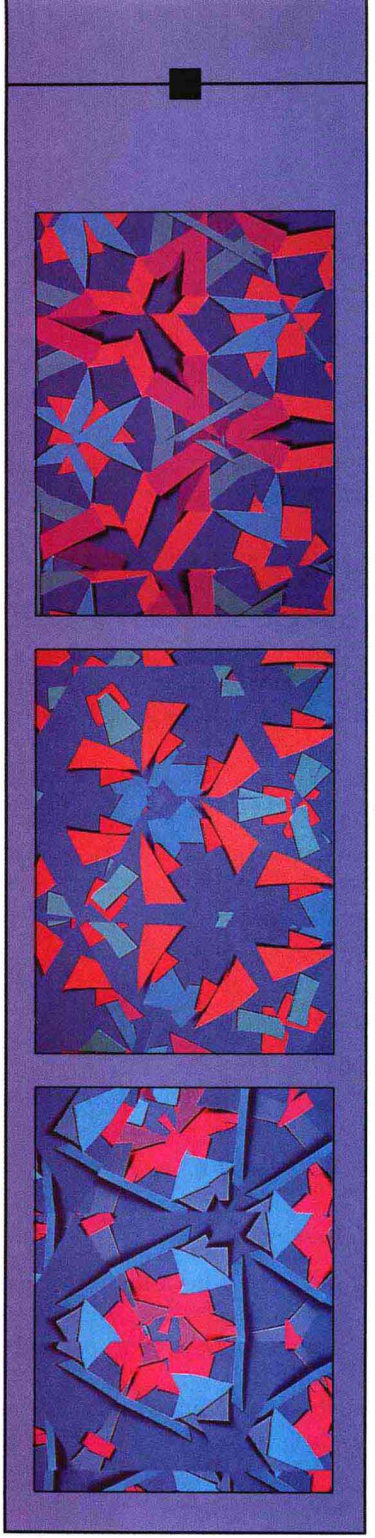
Joe K. Smith
Northern Kentucky University

Judith Wells
Illinois State University

We also wish to thank Linda and Robert Gerver for reviewing the assessments and the solutions, Victoria Garrison for all of her hard work in developing a student resource book that has vision and purpose, and Robert Matulis for developing the problem-solving supplement to this textbook.

Finally, we wish to recognize our mentor, the late Professor Marc Belth, to whom this book is dedicated. His influence and guidance have helped shape our philosophy of education.

L. S. S.
R. J. S.



To the Student: Addressing Math Anxiety

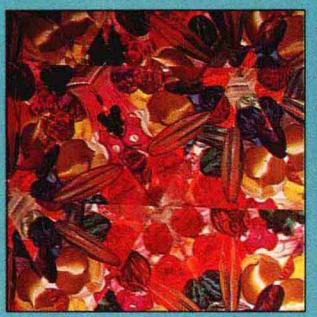
One impediment to learning mathematics is the phenomenon commonly referred to as "math anxiety." This phrase (which first appeared in *Ms. Magazine* in 1976) describes the feelings of distress, lack of confidence, dread, and even panic that some students encounter when confronted with learning or using mathematics. These feelings most typically occur when a student has a poor background in mathematics or associates a particular trauma with mathematics or learning mathematics.

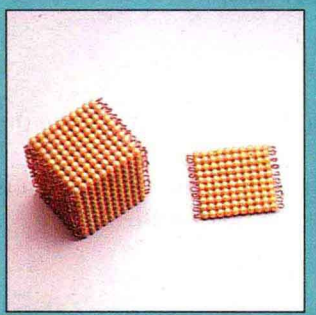
Researchers have found that levels of math anxiety often correspond inversely with levels of math achievement, that is, the lower the achievement level, the higher the anxiety level. Because mathematics has historically been viewed as a "male-oriented" subject, it is not surprising that math anxiety is found disproportionately in female students. This finding increases the need to address this phenomenon, since the majority of students entering the field of elementary education are women. Researchers have learned that math anxiety is eased in a positive and supportive classroom environment and that students who experience math anxiety prefer to study and work on math assignments in a group. In a supportive setting, students feel more comfortable asking questions for clarification because their peer group both understands and accepts the feelings math anxiety can produce.

Others who experience math anxiety have learned to use certain relaxation techniques, such as concentrating on a peaceful scene, breathing deeply for a few minutes, or taking a walk prior to math class or before sitting down to do math assignments. Students also report that it is helpful to leave the assignment for a little while and do something else if they are feeling anxious. On returning to the assignment, they feel more relaxed and able to focus on the work at hand. You may wish to try some of these techniques. You may also want to consider keeping two notebooks: one in class and one at home. Each evening, transcribe your class notes to your notebook at home. Rewriting the class notes in your own language personalizes the mathematics and gives you an opportunity to explain a concept in your own words.

For those of you who have experienced math anxiety, it may be of some comfort to know that as future teachers you have the opportunity to break the cycle that leads to avoidance of mathematics and therefore poor performance. Knowing from your own experience how debilitating math anxiety can be will help you better understand the importance of increasing your students' confidence and sense of competence.

We hope that this text, with its emphasis on math as a sense-making endeavor, its attention to understanding concepts behind the procedures, its step-by-step investigations of problems throughout, and its concern with illustrating and creating models of various abstract concepts will help you to think of yourself as capable of learning, and eventually teaching, mathematics.





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