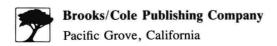
TRIGONOMETRY FOR COLLEGE STUDENTS

KARL J. SMITH FOURTH EDITION

TRIGONOMETRY FOR COLLEGE STUDENTS

KARL J. SMITH Santa Rosa Junior College

This book is a unit-circle approach to trigonometry. For a right-triangle treatment of the subject, see the author's other book, *Essentials of Trigonometry*, *Second Edition*, also published by Brooks/Cole Publishing Company.



To my mother,
Rosamond,
with love and thanks.

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Preface

There is perhaps nothing which so occupies, as it were, the middle position of mathematics, as trigonometry.

J. F. Herbart, 1890

Trigonometry, The Course

Trigonometry is the first mathematics course that provides a transition from elementary mathematics to the more advanced mathematics courses that focus on concepts and ideas, rather than manipulative skills. Six functions, called the *trigonometric functions*, will be defined; the remainder of the course is then devoted to investigating and understanding these functions, their graphs, their relationships to one another, and ways in which they can be used in a variety of different disciplines. Trigonometry is a precalculus mathematics course that follows intermediate algebra, so students using this book should have completed a course equivalent to second-year high school algebra. Even though a course in geometry is also a desirable prerequisite, I have found that more and more students are taking trigonometry without having taken high school geometry. This text is written to present all the necessary ideas from geometry as they are needed. In many schools, this course is the last precalculus course, and in such cases it is desirable to introduce the conic sections prior to beginning calculus. Optional Chapter 9 is included for these schools.

Trigonometry, The Book

Trigonometry for College Students, Fourth Edition, introduces the fundamental ideas of trigonometry in a text designed for a one-semester or one-quarter course. There are three commonly accepted methods for presenting circular functions: (1) as right-triangle ratios, (2) as functions defined on a unit circle, and (3) as wrapping functions. This book defines the circular functions on a unit circle first, then in terms of a point on the terminal side of the angle, and finally in terms of functions of real numbers. Chapter 6, "Solving Triangles," gives the right-triangle definition. The emphasis in this book is in working with radians, thereby providing a smooth transition to trigonometry as it is used in more advanced mathematics courses.

Why Use This Trigonometry Book?

It seems as if every new book reports innovation, state-of-the-art production, supplementary materials, readability, abundant problems, and relevant applications. How, then, is one book chosen over another? How does this book differ from other trigonometry books, and what factors were taken into consideration as this textbook was being written?

Content First the book must cover the appropriate topics, and in the right order. In this book I begin with angles and radian measure of angles early in Chapter 1. We then discuss the nature of the trigonometric functions and learn how to evaluate trigonometric functions by calculator, exact value, and table. The emphasis in this chapter is on understanding that we are indeed dealing with functions of an angle, which can be measured in degrees, radians, or real numbers. Graphing is introduced early (in Chapter 2) and is given special consideration. I feel that graphing the generalized trigonometric functions is one of the most important concepts to be taught to the students. In this book, you will find an innovative and unique method presented—I call it framing. With this framing technique, I am able to quickly and efficiently communicate the concepts of amplitude, period, and phase displacement without the usual difficulties of plotting "ugly coordinates." Two complete chapters are devoted to trigonometric identities. Over the years I have found it helpful to separate the fundamental identities from all the other identities in the students' minds. For this reason, in Chapter 3 the focus is on the fundamental identities, and I have included the other identities separately in Chapter 4. Chapter 5 discusses inverse relations and solving trigonometric equations, while Chapter 6 solves both right and oblique triangles. The immediate introduction of the law of cosines eliminates the confusion that sometimes arises with the law of sines and its ambiguous cases. You will find an unusually complete treatment of the ambiguous case. The remaining chapters are optional and can be covered as time permits. These include complex numbers, polar curves, and the conic sections.

Style The author's writing style is another factor that distinguishes one textbook from another. My writing style is informal, and I write with the student always in mind. I offer study hints along the way and use all kinds of pedagogical aids to let the student know what is important. There is a danger in a writer doing this because individual professors will, no doubt, have their own ideas about what should or should not be memorized or what is particularly important. Nevertheless, I feel that I must take a personal interest in each and every reader of this book. For example, if you ask most professors to list the trigonometric ratios they will say, "sine, cosine, tangent, . . ." However, in this book I present them in the order "cosine, sine, tangent, . . ." Why? Professors will say sine-cosine simply because that is the way other books have done it; so why change? Because, if the student always remembers that the cosine is first, the material will be easier to remember. Cosine comes first alphabetically and is associated with x (the first component), with the first component on a unit circle, and with the *first* two quadrants when dealing with inverse functions. Throughout the book I have kept in mind how a student goes about learning and how to make this process as smooth and natural as possible for the student seeing this material for the first time. Trigonometry requires more memory work than most other mathematics courses, and so I feel it is important to help the students with their memory work by providing mnemonic or memory hints whenever appropriate.

Problems Another determining factor in the selection of a textbook is the problems. In this book I provide abundant examples and problems for both drill and theory. The problems are divided into A, B, and C categories by order of difficulty. The student is generally required to work at the A and B level, whereas C problems are designed for an honors class or to present a challenge to most students. An innovation in this book is the possibility of a standard mathematics assignment. This means that I have used a, b, and c parts for particular problems so that all of the problem sets in the book have uniform length, with the different types of problems distributed within each problem set in basically the same way. This means, for example, that a typical assignment might be Problems 3-30, multiples of 3, and this assignment would apply throughout the text. Spiral assignments are therefore particularly easy to assign (since they can be the same day after day). For example "Problems 5-30, multiples of 5, and Problems 7, 14, 21, and 28 of the previous problem set" might routinely be assigned. The problems are presented in pairs of similar problems so that the answers can be used as a meaningful learning aid. Numerous applied problems are presented in almost every section to enhance the material and to make the subject more interesting for the students. In addition, extended Applications for Further Study provide supplementary material in more depth than is possible in the problem sections. These supplementary sections at the ends of the chapters illustrate some of the utility of trigonometry. Answers to the odd-numbered problems are provided in Appendix D.

Contemporary Nearly every trigonometry book published today advertises calculator usage, but this book *develops* calculator skills. Look, for example, at the treatment of inverse trigonometric functions and the way various books treat this topic (see page 139, for example). How about topics like linear interpolation? Do they have a place in trigonometry? I believe that interpolation *is* an important skill, but *not* in the context of trigonometric tables (which are better done by calculator). I therefore develop interpolation in terms of a sunrise/sunset table in the Application for Further Study on page 106. It is presented so that it can be covered or omitted depending on the goals of the course. Computers can and should be used when the facilities are available, and therefore a computer supplement for this book is also available. (See the Application for Further Study at the end of Chapter 2.)

Supplementary Materials for the Student

The availability of inexpensive calculators is one of the most exciting recent developments in the teaching and learning of trigonometry. Students should be encouraged to own calculators; however, it is only desirable and not mandatory that each student have a calculator. Use of a calculator enhances the reader's understanding of the material presented, allows for more interesting problems, and leaves more time for trigonometry because less time is needed for lengthy calculations.

Extensive reviews are presented throughout the text. Each chapter concludes with a list of new and important terms, a listing of objectives for that chapter, and a sample test with questions keyed to the corresponding objectives. I have made every effort to make it clear to the student exactly what is expected and that a mastery of these objectives should insure success. In addition to these chapter reviews, there are three cumulative reviews, each with two different forms of sample tests in the text to provide even more drill and practice for the student. The answers to all the problems in these cumulative reviews are provided in the back of the book.

A computer supplement is also available. Programs in graphing the trigonometric functions and in solving triangles provide an important learning supplement for those who have access to a computer.

Supplementary Material for the Instructor

The material is designed to provide for easy lesson planning. Each section should take about one 50-minute class session, and the problem sets at the end of each section are of uniform length. The material can easily be adapted to a three-times-per-week quarter or semester class or a twice-per-week semester class. Sections that can be omitted easily are indicated by an asterisk in the Contents. Sections 3.2 and 3.3, 4.1 and 4.2, 5.1 and 5.2, or 7.1 and 7.2 can be combined if time is limited.

There is an *Instructor's Manual* that contains complete solutions for all the exercises in the text, as well as over 400 sample test questions.

Computer supplements are also available. The *Computer Supplement* provides for the graphing of trigonometric functions as well as solving both right and oblique triangles. A testing program is also available on a computer disk that allows you to create an almost unlimited number of examinations.

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Karl J. Smith Sebastopol, California

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^{*} Optional section

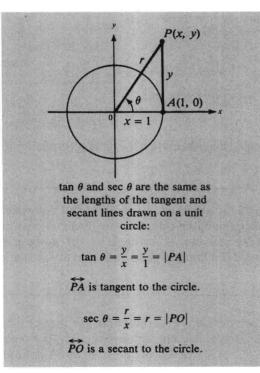
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Circular Functions

The origins of trigonometry are obscure, but we do know that it began more than 2000 years ago with the Mesopotamian, Babylonian, and Egyptian civilizations. Much of the knowledge from these civilizations was passed on to the Greeks, who formally developed many of the ideas in this book. The word trigonometry comes from the words "triangle" (trigon) and "measurement" (metry), and the ancients used trigonometry

in a very practical way to measure triangles. However, today it is used in a much more theoretical way, not only in mathematics but also in electronics, engineering, and computer science. Today we can characterize trigonometry as the branch of mathematics that deals with the properties and applications of six functions: the cosine, sine, tangent, secant, cosecant, and cotangent. These functions are defined in this chapter.

Historical Note

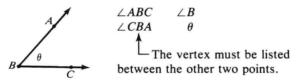


The words tangent and secant come from the relationship of the ratios to the lengths of the tangent and secant lines drawn on a circle. However, the word sine has a more interesting origin. According to Howard Eves in his book In Mathematical Circles, Arvabhata called it jya-adga (chord half) and abbreviated it jya, which the Arabs first wrote as jiba but later shortened to ib. Later writers saw ib, and, since jiba was meaningless to them. they substituted jaib (cove or bay). The Latin equivalent for jaib is sinus, from which our present word sine is derived. Finally, Edmund Gunter (1581-1626) first used the prefix co to invent the words cosine and cotangent. In 1620, Gunter published a sevenplace table of the common logarithms of the sines and tangents of angles for intervals of a minute of an arc. Gunter originally entered the ministry but later decided on astronomy as a career. He was such a poor preacher that Eves states that Gunter left the ministry in 1619 to the "benefit of both occupations."

1.1 Angles and Degree Measure

Essential to the definition of the circular functions is the idea of an angle. Recall from geometry that an **angle** is defined as two rays or line segments with a common endpoint. The rays or line segments are called the **sides** of the angle, and the common endpoint is called the **vertex**. In geometry you named angles by naming the vertex or the three points as shown in Figure 1.1. In trigonometry it is also common to use lowercase Greek letters to denote angles. A list of the common Greek letters is included as part of Figure 1.1. (Note that π (pi) is a lowercase Greek letter that will not be used to represent an angle, because it denotes an irrational number approximately equal to 3.141592654.)

The symbol \angle denotes angle. The given angle can be denoted in the following ways:



Symbol	Name	
α	alpha	
β	beta	
γ	gamma	
δ	delta	
θ	theta	
λ	lambda	
ϕ or φ	phi	
ω	omega	

Figure 1.1 Ways to denote an angle

In trigonometry and more advanced mathematics this geometric definition of an angle is generalized.

Generalized Definition of an Angle An **angle** is formed by rotating a ray about its endpoint (called the **vertex**) from some initial position (called the **initial side**) to some terminal position (called the **terminal side**). The measure of an angle is the amount of rotation. An angle is also formed if a line segment is rotated about one of its endpoints.

If the rotation of the ray is in a counterclockwise direction, the measure of the angle is called **positive.** If the rotation is in a clockwise direction, the measure of the angle is called **negative.** A small curved arrow is used to denote the direction and amount of rotation.

There are several units of measurement used for measuring angles. Historically, the most common scheme uses degrees. Draw a circle with any nonzero radius r and center at O. An **arc** is part of a circle, so, if P and Q are any two

distinct points on the circle, then \widehat{PQ} is an arc of the circle. Angle POQ is called a **central angle** that is **subtended** by \widehat{PQ} as shown in Figure 1.2.

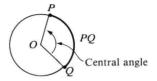


Figure 1.2 Central angle and an arc

The distance around a circle is called the **circumference** of the circle. A **degree** is an angle measurement that is defined as the size of a central angle subtended by an arc equal to $\frac{1}{360}$ of the length of the circumference. This definition of degree measure is independent of the size of the radius of the circle.

To measure angles you can use a protractor showing degree measure (see Figure 1.3).

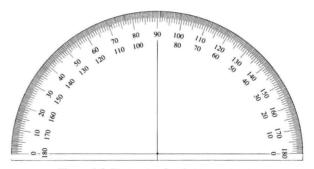
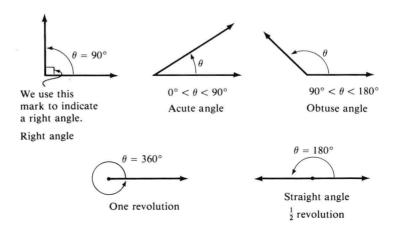


Figure 1.3 Protractor for degree measure

You will not need to use a protractor in this class, but you will need to know the approximate size of various angles measured in degrees. The degree symbol is ° and is used as shown:



Other common angles are shown in Example 1. When we write $\angle B = 45^{\circ}$ we mean the measure of angle B is 45° , and when we write $\theta = 30^{\circ}$ we mean the measure of angle θ is 30° .

Example 1 Draw angles whose measures are given.

a.
$$\alpha = 30^{\circ}$$

b.
$$/G = 45^{\circ}$$

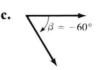
$$\beta = -60^{\circ}$$

d.
$$v = 270^{\circ}$$

Solution









You should remember the approximate size of the angles 0°, 30°, 45°, 60°, 90°, 180°, 270°, and 360° because they are frequently used in mathematics.

A degree can be divided into 60 equal parts, each called a **minute**, denoted by $'(1^{\circ} = 60')$. Furthermore, a minute can be divided into 60 equal parts, each called a **second**, denoted by $''(1' = 60''; 1^{\circ} = 60' = 3600'')$. With the widespread use of calculators, it is becoming more and more common to represent minutes and seconds as decimal degrees. In this book you should use decimal degrees rather than minutes and seconds unless you are directed to do otherwise.

Example 2 Convert the degree measures to decimal degrees.

Solution

a. Since 1° = 60′, the number of minutes should be divided by 60 in order to convert it to decimal degrees.

$$30' = \frac{30^{\circ}}{60} = \frac{1^{\circ}}{2} = .5^{\circ}$$

Thus, $25^{\circ}30' = 25.5^{\circ}$.

b.
$$14' = \frac{14^{\circ}}{60} = .2\overline{3}^{\circ} \approx .23^{\circ}$$

If an angle is measured in minutes, then round the converted decimal to the nearest hundredth of a degree. Thus, $128^{\circ}14' \approx 128.23^{\circ}$.

Calculator conversion:

