

OPTICAL WAVEGUIDE MODES

Polarization, Coupling, and Symmetry

Richard J. Black
Langis Gagnon

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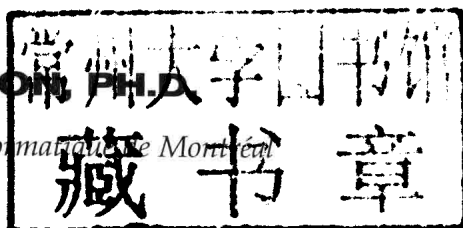
Polarization, Coupling,
and Symmetry

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This book is about the modes of single- and few-mode optical waveguides with an emphasis on single-core and multicore optical fibers and couplers including a large range of geometries and anisotropies, both standard and exotic. It provides both an “atlas” of modal field forms and an understanding of the physical properties resulting from waveguide symmetries. In addition to optical waveguide and fiber-optic designers, researchers, and students, this book may appeal to quantum and solid-state chemists and physicists interested in the application by analogy of techniques they know well in the continually expanding field of photonics.

To aid in rapid understanding, we emphasize a building-block approach with approximate modes and simplified structures forming a basis for more exact analyses and more complex structures. Accordingly we commence with single-core fibers and the symmetry consequences arising from specific forms of the azimuthal and radial dependence of the index profile.

The mathematical tools involve (1) the weak-guidance perturbation formalism facilitating the incorporation of polarization effects following a scalar analysis together with (2) a group theoretic approach for systematic exploitation of symmetry.

Scalar modes provide a basis for vector modes. Field constructions for transverse and hybrid polarized modes in terms of both linearly and circularly polarized modes are given. Degeneracy splittings and vector mode field transformations are considered depending on the relative strengths of the refractive index profile height and deformations from a circular cross section (e.g., elliptical, triangular, square) or birefringence (linear, radial or azimuthal, circular). Both microscopic and macroscopic anisotropies are considered: The polarization effects arising from a single interface may be regarded as a macroscopic manifestation of form birefringence. Single-core results are then used as a building block in the analysis arrays of few-mode lightguides: multicore fibers and multifiber couplers.

The organization of material is as follows:

- Chapter 1 provides an introduction including a motivation for the study of waveguide mode forms.
- Chapter 2 starts from the fundamental Maxwell equations for electrically anisotropic and isotropic media to provide a

comprehensive treatment of the resulting wave equations. For longitudinally invariant optical waveguides, it emphasizes the weak-guidance formalism which in general leads to perturbation expansion in terms of the typically small fractional refractive index difference between the waveguide core and cladding.

- Chapter 3 considers the scalar and vector modes of circular optical fibers. It includes a tutorial introduction to the consequences of symmetry, using a group theoretic approach in degeneracy determination and field construction of different modes of circularly symmetric fibers.
- Chapter 4 examines elliptical, triangular, and square deformations of circular waveguide cross sections as illustrations of the modal degeneracy splitting and field transformation resulting when the *azimuthal* circular symmetry is lowered to n -fold rotation-reflection symmetry.
- Chapter 5 considers linearly, radially, and circularly birefringent (gyrotropic) fibers.
- Chapter 6 is devoted to the construction of modes of multi-core fibers and multifiber coupler arrays.
- Chapter 7 provides a summary of the results and discusses extensions of the concept of modes for longitudinally invariant structures to modes for structures with longitudinal variations, such as periodic structures and Kerr-type nonlinear waveguides where intensity-dependence induces longitudinal variation in the presence of a propagating wave.
- The appendix provides the essential results of elementary applied group representation theory used for the analysis of many physical and chemical systems involving symmetry. Together with the symmetry tutorial included in Sec. 3.2, this provides an alternative introduction to and/or illustration of concepts which students might apply by analogy in many other fields such as quantum, solid-state, and molecular chemistry and physics.

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Langis Gagnon

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Following some inspiring discussions and correspondence with Prof. Geoff Stedman in 1986, this book had its origins in two manuscripts [1, 2] prepared by us in the mid to late 1980s while Richard J. Black (RJB) was at the École Polytechnique de Montréal and Langis Gagnon (LG) was at the Université de Montréal. Following the encouragement of Prof. Carlo Someda, the first full version of this book was prepared in 1991–1992, with relevant references up until that time, while RJB was at the École Polytechnique de Montréal. It was later revised for part of a course presented by RJB at the Swiss Federal Institute of Technology [École Polytechnique de Lausanne (EPFL)] in January–February 1995. The present 2010 revised version followed from discussions between RJB and Taisuke Soda of McGraw-Hill, who we thank, together with all the McGraw-Hill and Glyph International team, particularly Shruti Vasishta, for expert preparation of the book. We also thank colleagues at Photon Design, Technix by CBS, IFOS, and CRIM, and many other colleagues, family, and friends too numerous to mention, for their contributions and support.

The present new version includes (a) a simple intuitive introduction to waveguide modes (Sec. 1.5) aimed at those encountering them for the first time, (b) recent developments (Secs. 7.4 through 7.8), and (c) 78 additional references. (With regard to references, the first 138 appeared in the 1992 manuscript, and Refs. 139 and 140 were added for the 1995 manuscript.) While the fundamental theory of optical waveguide modes presented herein remains the same, since 1992 we have witnessed considerable growth in photonics in the commercial sector (particularly rapid in telecom in the late 1990s with steady progress in photonic sensors to the present), with technical and scientific developments in many areas, for example, periodic lightguides [fiber Bragg gratings (FBGs), photonic crystal fibers, and photonic crystals] and waveguide modeling packages. We touch on these areas in added Refs. 141 and above together with new Secs. 7.4 through 7.8.

*Richard J. Black
Langis Gagnon
2010*

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Richard J. Black
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1992

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Introduction

In this chapter, Secs. 1.1 through 1.4 introduce the major themes of the book. Section 1.5 provides an intuitive introduction to optical waveguide modes and Sec. 1.6 provides a chapter-by-chapter outline of the remainder of the book highlighting major results.

1.1 MODES

This is a book about *lightguide mode forms*. In particular,

1. We emphasize the basic structure of **modal field patterns** in optical fiber cross sections **transverse** to the direction of propagation.
2. We consider the relative **longitudinal** dependencies of modal fields in terms of their *propagation constant degeneracies or splittings*. Our major objective is to provide an understanding of **how transverse optical waveguide geometry influences modal polarization properties** with refractive index variations ranging from macroscopic “global anisotropies” down to scales much smaller than a wavelength where the local refractive indices of the constituent waveguide media can be treated as anisotropic. As in Refs. 1 and 2, we undertake the analysis using extensions of the weak-guidance perturbation formalism [3] together with elementary group representation theory [4–6]; see also Refs. 7 through 10.

As well as providing the basic general electromagnetic formalism and structural description appropriate for analysis of the lowest-order

or fundamental modes (i.e., the two polarization states of the modes referred to as HE_{11} and LP_{01} or CP_{01}), we go beyond that mainstay of present-day long-distance telecommunications and include a detailed introduction and classification of diverse forms of higher-order modes and various polarization manifestations thereof, e.g., modes of polarization that are transverse electric (TE), transverse magnetic (TM), hybrid (HE or EH), linear (LP), circular (CP), and “triangular” (TP). We give particular attention to the *second-order modes*; e.g., for circular fibers, these are the TE_{01} , TM_{01} , and HE_{21} modes, each of which may be constructed in terms of two linearly polarized (LP_{11}) “pseudo-modes” or alternatively in terms of circularly polarized (CP_{11}) modes.

Apart from the applications, since the original circular fiber modal classification scheme due to Snitzer [11], few-mode lightguide problems have attained a particular physical interest in their own right, e.g., Ref. 12. Indeed, our major aim is simply **to provide an understanding of the fundamental physics of mode structure**. It is our belief that a valuable basis for future novel waveguide designs and exploitations will be provided by a thorough knowledge of how waveguide structure—ranging from standard to exotic—can be used to create and manipulate modes with the desired properties.

While *we mostly restrict ourselves to the concept of monochromatic independently propagating modes of idealized lightguides with longitudinally invariant linear refractive indices*, **these ideal “linear” modes** may form the **basis for adaptations to perturbed and other less idealized situations** including **longitudinal variations** and “**nonlinear**” effects using coupled-mode, local-mode, coupled-local-mode, and other approaches [e.g., 3, 13]. In the context of few-mode fibers, we mention but some of the adaptations of topical interest for which a full understanding of ideal linear guide modes as a fundamental building block can provide useful added insight:

1. Nonlinear (Kerr-type) intensity-dependent modal interferometry [14–17]
2. Nonlinear (second harmonic) frequency conversion via phase matchings of different-order modes [18]
3. (Permanent-) Grating induced frequency conversion and filtering [19, 20]

In general, few-mode lightguides have received attention ranging from visual photoreceptor studies [21] to a particular recent interest in **modal interferometry** [22, 23] and applications thereof, such as