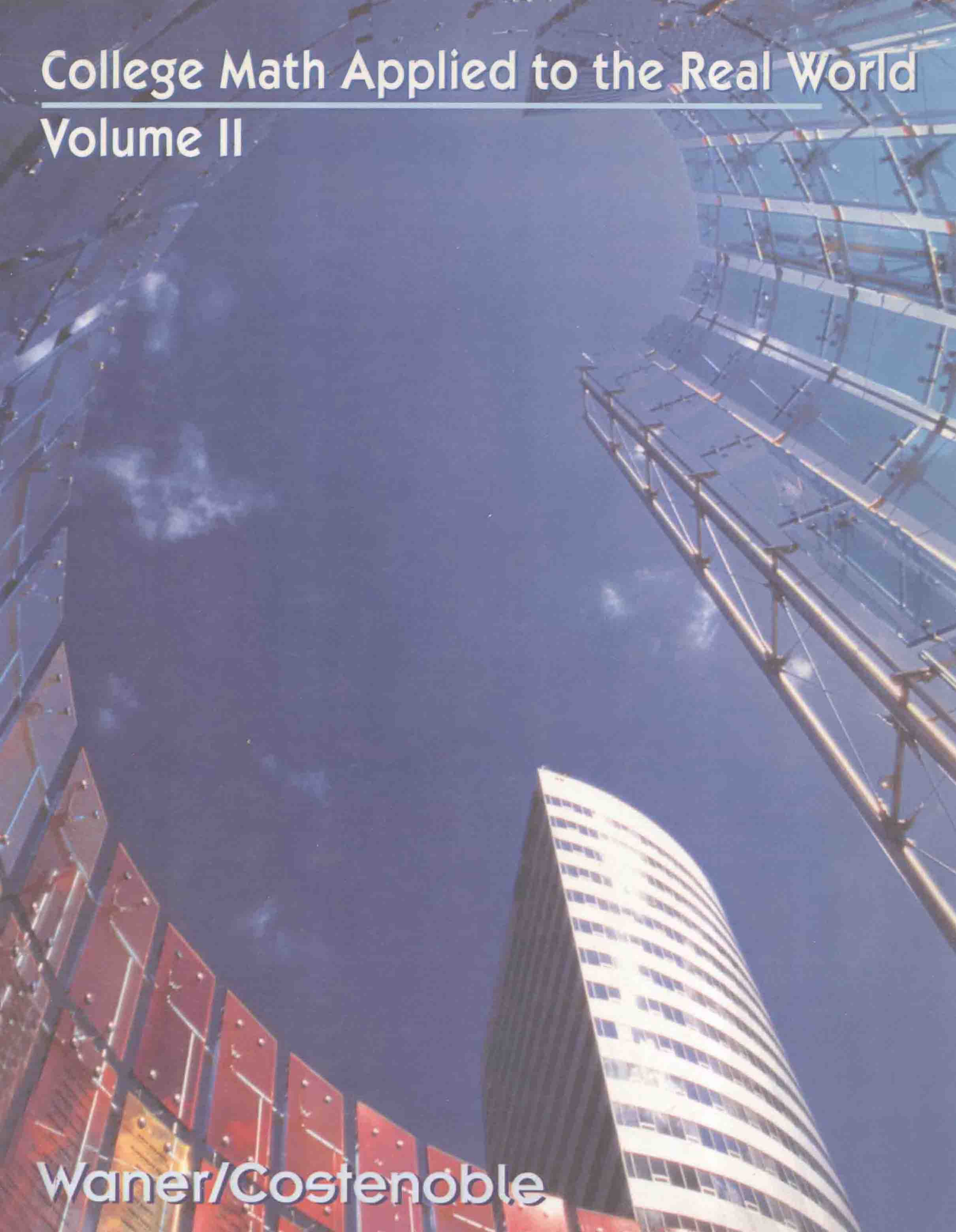
College Math Applied to the Real World

Volume II



Waner/Costenoble

College Mathematics Applied to the Real World Vol. II

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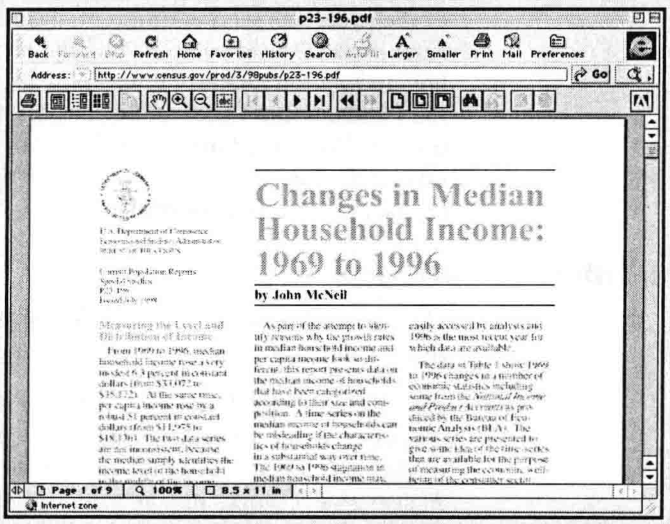
Custom Contents

1

You're the Expert

Wage Inflation

As assistant personnel manager for a large corporation, you have been asked to estimate the average annual wage earned by a worker in your company, from the time the worker is hired to the time the worker retires. You have data about wage increases. How can you estimate this average?



Internet Resources for This Chapter

At the web site, follow the path

Web site → Everything for Calculus → Chapter 6

where you will find links to step-by-step tutorials for the main topics in this chapter, a detailed chapter summary you can print out, a true–false quiz, and a collection of sample test questions. You will also find downloadable Excel tutorials for each section, an on-line numerical integration utility, and other resources. Complete text and interactive exercises have been placed on the web site covering the following optional topic:

- Numerical Integration

The Integral

Introduction

Roughly speaking, calculus is divided into two parts: **differential calculus** (the calculus of derivatives) and **integral calculus,** which is the subject of this chapter and the next. Integral calculus is concerned with problems that are in some sense the reverse of the problems seen in differential calculus. For example, whereas differential calculus shows how to compute the rate of change of a quantity, integral calculus shows how to find the quantity if we know its rate of change. This idea is made precise in the **Fundamental Theorem of Calculus.** Integral calculus and the Fundamental Theorem of Calculus allow us to solve many problems in economics, physics, and geometry, including one of the oldest problems in mathematics—computing areas of regions with curved boundaries.

6.1 The Indefinite Integral

Having studied differentiation in the preceding chapters, we now discuss how to *reverse* the process.

Question If the derivative of $F(x)$ is $4x^3$, what was $F(x)$?
Answer After a moment's thought, we recognize $4x^3$ as the derivative of x^4. So, we might have $F(x) = x^4$. However, on thinking further, we realize that, for example $F(x) = x^4 + 7$ works just as well. In fact, $F(x) = x^4 + C$ works for any number C. Thus, there are *infinitely many* possible answers to this question.

In fact, we will see shortly that the formula $F(x) = x^4 + C$ covers *all* possible answers to the question. Let us give a name to what we are doing.

Antiderivative

An **antiderivative** of a function f is a function F such that $F' = f$.

Quick Examples

1. An antiderivative of $4x^3$ is x^4. Because the derivative of x^4 is $4x^3$
2. Another antiderivative of $4x^3$ is $x^4 + 7$. Because the derivative of $x^4 + 7$ is $4x^3$
3. An antiderivative of $2x$ is $x^2 + 12$. Because the derivative of $x^2 + 12$ is $2x$

Thus,

If the derivative of $A(x)$ is $B(x)$, then an antiderivative of $B(x)$ is $A(x)$.

We call the set of *all* antiderivatives of a function the **indefinite integral** of the function.

Indefinite Integral

The expression

$$\int f(x)\,dx$$

is read "the **indefinite integral** of $f(x)$ with respect to x" and stands for the set of all antiderivatives of f. Thus, $\int f(x)\,dx$ is a *collection of functions;* it is not a single function nor a number. The function f that is being **integrated** is called the **integrand,** and the variable x is called the **variable of integration.**

Quick Examples

1. $\int 4x^3\,dx = x^4 + C$ Every possible antiderivative of $4x^3$ has the form $x^4 + C$.
2. $\int 2x\,dx = x^2 + C$ Every possible antiderivative of $2x$ has the form $x^2 + C$.

The **constant of integration,** C, reminds us that we can add any constant and get a different antiderivative.

example 1

Indefinite Integral

Check that $\int x\,dx = \frac{x^2}{2} + C$.

Solution

We check by taking the derivative of the right-hand side:

$$\frac{d}{dx}\left(\frac{x^2}{2} + C\right) = \frac{2x}{2} + 0 = x \checkmark$$

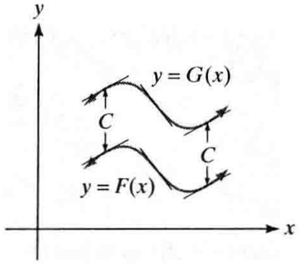


Figure 1

Question If $F(x)$ is one antiderivative of $f(x)$, then why must all other antiderivatives have the form $F(x) + C$?

Answer Suppose $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$, so that $F'(x) = G'(x)$. Consider what this means by looking at Figure 1. If $F'(x) = G'(x)$ for all x, then F and G have the *same slope* at each value of x. This means that their graphs must be *parallel* and hence remain exactly the same vertical distance apart. But that is the same as saying that the functions differ by a constant—that is, that $G(x) = F(x) + C$ for some constant C.[1]

[1]This argument can be turned into a more rigorous proof—that is, a proof that does not rely on geometric concepts such as "parallel graphs." We should also say that the result requires that F and G have the same derivative *on an interval* $[a, b]$.

Now, we would like to make the process of finding indefinite integrals (antiderivatives) more mechanical. For example, it would be nice to have a power rule for indefinite integrals similar to the one we already have for derivatives. Two cases suggested by the examples above are

$$\int x\,dx = \frac{x^2}{2} + C \qquad \text{and} \qquad \int x^3\,dx = \frac{x^4}{4} + C$$

We can check the last equation by taking the derivative of its right-hand side. These cases suggest the following general statement.

Power Rule for the Indefinite Integral, Part I

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \text{if } n \neq -1$$

In Words

To find the integral of x^n, add 1 to the exponent and then divide by the new exponent. This rule works provided n is not -1.

Quick Examples

1. $\int x^{55}\,dx = \frac{x^{56}}{56} + C$

2. $\int \frac{1}{x^{55}}\,dx = \int x^{-55}\,dx$ — Exponent form

$= \frac{x^{-54}}{-54} + C$ — When we add 1 to -55, we get -54, *not* -56.

$= -\frac{1}{54x^{54}} + C$

3. $\int 1\,dx = x + C$ — Since $1 = x^0$. This is an important special case.

Notes

The integral $\int 1\,dx$ is commonly written as $\int dx$.

Similarly, the integral $\int \frac{1}{x^{55}}\,dx$ may be written as $\int \frac{dx}{x^{55}}$.

We can easily check the power rule formula by taking the derivative of the right-hand side:

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1+C}\right) = \frac{(n+1)x^n}{n+1} = x^n \checkmark$$

Question What is the reason for the restriction $n \neq -1$?

Answer Let us answer the question with a question: Does the right-hand side of the power rule formula make sense if $n = -1$?

Question Well, no, so what is

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx$$

Answer Think before reading on: Have you ever seen a function whose derivative is $1/x$? Prodding our memories a little, we recall that $\ln x$ has derivative $1/x$. In fact, as we pointed out when we first discussed it, $\ln|x|$ also has derivative $1/x$, but it has the advantage that its domain is the same as that of $1/x$. Thus, we can fill in the missing case as follows.

Power Rule for the Indefinite Integral, Part II

$$\int x^{-1}\,dx = \ln|x| + C \qquad \text{Equivalently, } \int \frac{1}{x}\,dx = \ln|x| + C.$$

Note Consider the function

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if } x > 0 \\ \ln|x| + C_2 & \text{if } x < 0 \end{cases}$$

where C_1 and C_2 are possibly different constants. This function also has derivative $1/x$, and so we should really write

$$\int x^{-1}\,dx = \begin{cases} \ln|x| + C_1 & \text{if } x > 0 \\ \ln|x| + C_2 & \text{if } x < 0 \end{cases}$$

However, most books ignore this subtle point, as we shall also, and implicitly assume that $C_1 = C_2$. Thus, we will continue to write

$$\int x^{-1}\,dx = \ln|x| + C$$

Here are some other indefinite integrals that come from the corresponding formulas for differentiation.

Indefinite Integrals of Some Exponential and Trig Functions

$$\int e^x\,dx = e^x + C \qquad \text{Because } \frac{d}{dx}(e^x) = e^x$$

$$\int \cos x\,dx = \sin x + C \qquad \text{Because } \frac{d}{dx}(\sin x) = \cos x$$

$$\int \sin x\,dx = -\cos x + C \qquad \text{Because } \frac{d}{dx}(-\cos x) = \sin x$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \text{Because } \frac{d}{dx}(\tan x) = \sec^2 x$$

Question What about more complicated functions, such as $2x^3 + 6x^5 - 1$?

Answer We need the following rules for integrating sums, differences, and constant multiples.

Rules for the Indefinite Integral

Sum and Difference Rules

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

In Words

The integral of a sum is the sum of the integrals, and the integral of a difference is the difference of the integrals.

Constant Multiple Rule

$$\int kf(x)\,dx = k\int f(x)\,dx \qquad (k \text{ constant})$$

In Words

The integral of a constant times a function is the constant times the integral of the function. (In other words, *the constant "goes along for the ride."*)

Quick Examples

1. Sum rule: $\int (x^3 + 1)\,dx = \int x^3\,dx + \int 1\,dx = \frac{x^4}{4} + x + C$ $\quad f(x) = x^3;\ g(x) = 1$
2. Constant multiple rule: $\int 5x^3\,dx = 5\int x^3\,dx = 5\frac{x^4}{4} + C$ $\quad k = 5;\ f(x) = x^3$
3. Constant multiple rule: $\int 4\,dx = 4\int 1\,dx = 4x + C$ $\quad k = 4;\ f(x) = 1$

Why are these rules true? Because the derivative of a sum is the sum of the derivatives, and similarly for differences and constant multiples.

example 2

Using the Sum and Difference Rules

Find the integrals.

a. $\int (x^3 + x^5 - 1)\,dx$ **b.** $\int \left(x^{2.1} + \frac{1}{x^{1.1}} + \frac{1}{x}\right) dx$

c. $\int (e^x - \sin x + \cos x)\,dx$

Solution

a. $\int (x^3 + x^5 - 1)\,dx = \int x^3\,dx + \int x^5\,dx - \int 1\,dx$ Sum/difference rule

$= \frac{x^4}{4} + \frac{x^6}{6} - x + C$ Power rule

b. $\int \left(x^{2.1} + \frac{1}{x^{1.1}} + \frac{1}{x}\right) dx = \int (x^{2.1} + x^{-1.1} + x^{-1})\,dx$ Exponent form

$= \int x^{2.1}\,dx + \int x^{-1.1}\,dx + \int x^{-1}\,dx$ Sum rule

$= \frac{x^{3.1}}{3.1} + \frac{x^{-0.1}}{-0.1} + \ln|x| + C$ Power rule

$= \frac{x^{3.1}}{3.1} - \frac{10}{x^{0.1}} + \ln|x| + C$ Back to fraction form

c. $\int (e^x - \sin x + \cos x)\,dx = e^x + \cos x + \sin x + C$ Two steps in one

Before we go on . . . As usual, you should check each of the answers by differentiating.

Question Why is there only a single arbitrary constant C in each of the answers?

Answer We could have written the answer to part (a) as

$$\frac{x^4}{4} + D + \frac{x^6}{6} + E - x + F$$

where D, E, and F are all arbitrary constants. Now suppose that, for example, we set $D = 1$, $E = -2$, and $F = 6$. Then the particular antiderivative we get is $x^4/4 + x^6/6 - x + 5$, which has the form $x^4/4 + x^6/6 - x + C$. Thus, we could have chosen the single constant C to be 5 and obtained the same answer. In other words, the answer $x^4/4 + x^6/6 - x + C$ is just as general as the answer $x^4/4 + D + x^6/6 + E - x + F$, but simpler.

In practice we do not explicitly write the integral of a sum as a sum of integrals. We just "integrate term by term" [see part (c) in Example 2] much as we learned to differentiate term by term.

example 3

Combining the Rules

Find the integrals.

a. $\int (10x^4 + 2x^2 - 3e^x)\,dx$ **b.** $\int \left(\frac{2}{x^{0.1}} + \frac{x^{0.1}}{2} - 3\sin x\right) dx$

Solution

a. We need to integrate separately each of the terms $10x^4$, $2x^2$, and $3e^x$. To integrate $10x^4$ we use the rules for constant multiples and powers:

$$\int 10x^4\,dx = 10\int x^4\,dx = 10\frac{x^5}{5} + C = 2x^5 + C$$

The other two terms are similar. We get

$$\int (10x^4 + 2x^2 - 3e^x)\,dx = 10\frac{x^5}{5} + 2\frac{x^3}{3} - 3e^x + C = 2x^5 + \frac{2}{3}x^3 - 3e^x + C$$

b. We first convert to exponent form and then integrate term by term:

$$\int \left(\frac{2}{x^{0.1}} + \frac{x^{0.1}}{2} - 3\sin x\right) dx = \int \left(2x^{-0.1} + \frac{1}{2}x^{0.1} - 3\sin x\right) dx \quad \text{Exponent form}$$

$$= 2\frac{x^{0.9}}{0.9} + \frac{1}{2}\frac{x^{1.1}}{1.1} + 3\cos x + C \quad \text{Integrate term by term.}$$

$$= \frac{20x^{0.9}}{9} + \frac{x^{1.1}}{2.2} + 3\cos x + C$$

---

example 4

Different Variable Name

Find $\int \left(\frac{1}{u} + \frac{1}{u^2}\right) du$.

Solution

This integral may look a little strange because we are using the letter u instead of x, but there is really nothing special about x. We get

$$\int \left(\frac{1}{u} + \frac{1}{u^2}\right) du = \int (u^{-1} + u^{-2})\,du \quad \text{Exponent form}$$

$$= \ln|u| + \frac{u^{-1}}{-1} + C \quad \text{Integrate term by term.}$$

$$= \ln|u| - \frac{1}{u} + C \quad \text{Simplify the result.}$$

Before we go on . . . When we compute an indefinite integral, we want the independent variable in the answer to be the same as the variable of integration. Thus, if the integral had been written in terms of x rather than u, we would have written

$$\int\left(\frac{1}{x}+\frac{1}{x^2}\right)dx = \ln|x| - \frac{1}{x} + C$$

Application: Cost and Marginal Cost

example 5

Finding Cost from Marginal Cost

The marginal cost to produce baseball caps at a production level of x caps is $3.20 - 0.001x$ dollars per cap, and the cost of producing 50 caps is \$200. Find the cost function.

Solution

We are asked to find the cost function $C(x)$, given that the *marginal* cost function is $3.20 - 0.001x$. Recalling that the marginal cost function is the derivative of the cost function, we have

$$C'(x) = 3.20 - 0.001x$$

and we must find $C(x)$. Now $C(x)$ must be an antiderivative of $C'(x)$, so we write

$$C(x) = \int(3.20 - 0.001x)\,dx$$

$$= 3.20x - 0.001\frac{x^2}{2} + K \qquad K \text{ is the constant of integration.}$$

$$= 3.20x - 0.0005x^2 + K$$

(Why did we use K and not C for the constant of integration?) Now, unless we know a value for K, we don't really know what the cost function is. However, there is another piece of information we have ignored: The cost of producing 50 baseball caps is \$200. In symbols,

$$C(50) = 200$$

Substituting in our formula for $C(x)$, we have

$$C(50) = 3.20(50) - 0.0005(50)^2 + K$$

$$200 = 158.75 + K$$

$$K = 41.25$$

Now that we know what K is, we can write the cost function.

$$C(x) = 3.20x - 0.0005x^2 + 41.25$$

Before we go on . . .
Question What is the significance of the term 41.25?
Answer If we substitute $x = 0$, we get

$$C(0) = 3.20(0) - 0.0005(0)^2 + 41.25$$

or $$C(0) = 41.25.$$

Thus, \$41.25 is the cost of producing zero items; in other words, it is the **fixed cost.**

Application: Motion in a Straight Line

An important application of the indefinite integral is in the study of motion. The application of calculus to problems about motion is an example of the intertwining of mathematics and physics that is an important part of both. We begin by bringing together some facts, scattered throughout the last several chapters, that have to do with an object moving in a straight line.

Position, Velocity, and Acceleration: Derivative Form

If $s = s(t)$ is the **position** of an object at time t, then its **velocity** is given by the derivative

$$v = \frac{ds}{dt}$$

In other words, *velocity is the derivative of position.*

The **acceleration** of an object is given by the derivative

$$a = \frac{dv}{dt}$$

In other words, *acceleration is the derivative of velocity.* On the planet Earth, a freely falling body experiencing no air resistance accelerates at approximately 32 feet per second per second, or 32 ft/s^2 (or 9.8 m/s^2).

We may rewrite the derivative formulas above as integral formulas.

Position, Velocity, and Acceleration: Integral Form

$$s(t) = \int v(t)\,dt \qquad \text{because } v = \frac{ds}{dt}$$

$$v(t) = \int a(t)\,dt \qquad \text{because } a = \frac{dv}{dt}$$

example 6

Motion in a Straight Line

You toss a stone upward at a speed of 30 feet per second.

a Find the stone's velocity as a function of time. How fast and in what direction is it going after 5 seconds?

b. Find the position of the stone as a function of time. Where will it be after 5 seconds?

c. When and where will the stone reach its **zenith,** its highest point?

Solution

a. Let us measure heights above the ground as positive, so that a rising object has positive velocity and the acceleration downward due to gravity is negative. Thus, the acceleration of the stone is given by

$$a(t) = -32 \text{ ft/s}^2$$

We wish to know the velocity, which is an antiderivative of acceleration, so we compute

$$v(t) = \int(-32)\,dt = -32t + C$$

This is the velocity as a function of time t. But what is the value of C? Now, we are told that you tossed the stone upward at 30 ft/s, so when $t = 0$, $v = 30$; that is, $v(0) = 30$. Thus,

$$30 = v(0) = -32(0) + C$$

so $C = 30$ and the formula for velocity is $v(t) = -32t + 30$. In particular, after 5 seconds the velocity is

$$v(5) = -32(5) + 30 = -130 \text{ ft/s}$$

Thus, after 5 seconds the stone is *falling* with a speed of 130 feet per second.

b. We wish to know the position, but position is an antiderivative of velocity. Thus,

$$s(t) = \int v(t)\,dt = \int(-32t + 30)\,dt = -16t^2 + 30t + C$$

Now, to find C we need to know the initial position $s(0)$. We are not told this, so let us measure heights so that the initial position is zero. Then

$$0 = s(0) = C$$

and $s(t) = -16t^2 + 30t$. In particular, after 5 seconds the stone has a height of

$$s(5) = -16(5)^2 + 30(5) = -250 \text{ ft}$$

In other words, the stone is now 250 ft *below* where it was when you first threw it, as shown in Figure 2.

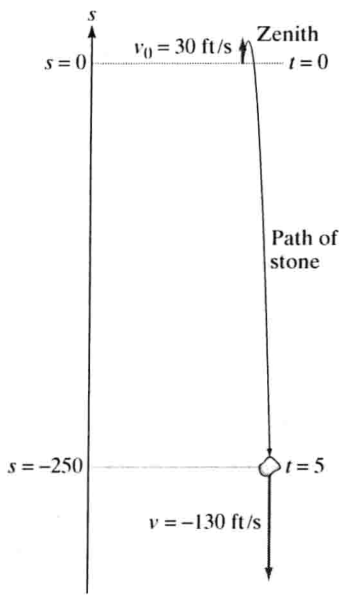


Figure 2

c. The stone reaches its zenith when its height $s(t)$ is at its maximum value, which occurs when $v(t) = s'(t)$ is zero. So we solve

$$v(t) = -32t + 30 = 0$$

to get $t = \frac{30}{32} = \frac{15}{16} = 0.9375$ s. This is the time when the stone reaches its zenith. The height of the stone at that time is

$$s\left(\frac{15}{16}\right) = -16\left(\frac{15}{16}\right)^2 + 30\left(\frac{15}{16}\right) = 14.0625 \text{ feet}$$

6.1 exercises

Evaluate the integrals in Exercises 1–10 mentally.

1. $\int x^5\,dx$

2. $\int x^7\,dx$

3. $\int 6\,dx$

4. $\int (-5)\,dx$

5. $\int x\,dx$

6. $\int (-x)\,dx$

7. $\int (x^2 - x)\,dx$

8. $\int (x + x^3)\,dx$

9. $\int (1 + x)\,dx$

10. $\int (4 - x)\,dx$

Evaluate the integrals in Exercises 11–38.

11. $\int x^{-5}\,dx$

12. $\int x^{-7}\,dx$

13. $\int (x^{2.3} + x^{-1.3})\,dx$

14. $\int (x^{-0.2} - x^{0.2})\,dx$

15. $\int (u^2 - 1/u)\,du$

16. $\int (1/v^2 + 2/v)\,dv$

17. $\int \sqrt{x}\,dx$

18. $\int \sqrt[3]{x}\,dx$

19. $\int (3x^4 - 2x^{-2} + x^{-5} + 4)\,dx$

20. $\int (4x^7 - x^{-3} + 1)\,dx$

21. $\int \left(\frac{1}{x} + \frac{2}{x^2} - \frac{1}{x^3}\right) dx$

22. $\int \left(\frac{3}{x} - \frac{1}{x^5} + \frac{1}{x^7}\right) dx$

23. $\int (3x^{0.1} - x^{4.3} - 4.1)\,dx$

24. $\int \left(\frac{x^{2.1}}{2} - 2.3\right) dx$

25. $\int \left(\frac{3}{x^{0.1}} - \frac{4}{x^{1.1}}\right) dx$

26. $\int \left(\frac{1}{x^{1.1}} - \frac{1}{x}\right) dx$

27. $\int (2e^x + 5/x + 1/4)\,dx$

28. $\int (-e^x + x^{-2} - 1/8)\,dx$

29. $\int \left(\frac{6.1}{x^{0.5}} + \frac{x^{0.5}}{6} - e^x \right) dx$

30. $\int \left(\frac{4.2}{x^{0.4}} + \frac{x^{0.4}}{3} - 2e^x \right) dx$

Δ 31. $\int (\sin x + \cos x)\, dx$

Δ 32. $\int (\cos x - \sin x)\, dx$

Δ 33. $\int (2 \cos x - 4.3 \sin x - 9.33)\, dx$

Δ 34. $\int (4.1 \sin x + \cos x - 9.33/x)\, dx$

Δ 35. $\int \left(3.4 \sec^2 x + \frac{\cos x}{1.3} - 3.2e^x \right) dx$

Δ 36. $\int \left(\frac{3 \sec^2 x}{2} + 1.3 \sin x - \frac{e^x}{3.2} \right) dx$

37. $\int \frac{x + 2}{x^3}\, dx$

38. $\int \frac{x^2 - 2}{x}\, dx$

39. Find $f(x)$ if $f(0) = 1$ and the tangent line at $(x, f(x))$ has slope x.

40. Find $f(x)$ if $f(1) = 1$ and the tangent line at $(x, f(x))$ has slope $1/x$.

41. Find $f(x)$ if $f(0) = 0$ and the tangent line at $(x, f(x))$ has slope $e^x - 1$.

42. Find $f(x)$ if $f(1) = -1$ and the tangent line at $(x, f(x))$ has slope $2e^x + 1$.

Applications

43. ***Marginal Cost*** The marginal cost of producing the xth box of lightbulbs is $5 - (x/10{,}000)$ and the fixed cost is \$20,000. Find the cost function $C(x)$.

44. ***Marginal Cost*** The marginal cost of producing the xth box of computer disks is $10 + (x^2/100{,}000)$ and the fixed cost is \$100,000. Find the cost function $C(x)$.

45. ***Marginal Cost*** The marginal cost of producing the xth roll of film is $5 + 2x + 1/x$. The total cost to produce one roll is \$1000. Find the cost function $C(x)$.

46. ***Marginal Cost*** The marginal cost of producing the xth box of videotape is $10 + x + 1/x^2$. The total cost to produce 100 boxes is \$10,000. Find the cost function $C(x)$.

47. ***Interest Rates*** Between 1990 and 1998 the discount interest rate in Japan declined at a rate of 0.7 percentage point per year.[2] Given that the discount interest rate was 6% in 1992, use an indefinite integral to find a formula for the interest rate I as a function of time t since 1990 ($t = 0$ represents 1990), and use your formula to calculate the interest rate in 1998.
Source: Bloomberg Financial Markets/Japan External Trade Organization/*New York Times*, September 20, 1998, p. WK5.

48. ***Real Estate*** Between 1990 and 1997 the price of a square foot of land in Tokyo declined at an average rate of \$25 per year. Given that the price in 1990 was \$375, use an indefinite integral to find a formula for the price p as a function of time t since 1990 ($t = 0$ represents 1990), and use your formula to calculate the price in 1997.
Source: See the source in Exercise 47.

49. ***Employment*** In 1988 statewide employment in Massachusetts was approximately 3,100,000. The following quadratic model approximates the rate of increase in employment, in thousands of people per year, in Massachusetts from 1988 through 1994:[3]

$$C(t) = 25t^2 - 137t + 68$$

where t is the number of years since 1988. Use the model and the 1988 employment figure for Massachusetts to obtain a model for the total number of people $N(t)$ employed in Massachusetts as a function of t.

50. ***Hawaiian Tourism*** The rate of visitor spending, in billions of dollars per year, in Hawaii during the years 1985 to 1993 can be approximated by[4]

$$r(t) = -0.164t^2 + 1.60t + 6.71$$

where $t = 0$ represents June 30, 1985. According to the model, how much revenue did Hawaii earn from visitor spending between June 30, 1985, and June 30, 1990? (Give your answer to the nearest billion dollars.)

51. ***Medicare Spending*** The rate of federal spending on Medicare (in constant 1994 dollars) increased more or less linearly from \$50 billion per year in 1972 to \$160 billion per year in 1994.[5] Obtain a linear model for the rate of increase of federal spending as a function of the time t in years since 1972, and use your model to obtain the total amount $C(t)$ spent on Medicare since 1972.
Source: Health Care Financing Administration, Economic Report of the President/*New York Times*, July 23, 1995, p. 1.

52. ***Certified Financial Planners*** The number of people who passed the exam to become a certified financial plan-

[2] This was the average rate of change over the given period.

[3] The model is a quadratic regression based on actual data from the Massachusetts Department of Employment, DRI/McGraw Hill/*New York Times*, January 27, 1995, p. D4.

[4] The model is based on a best-fit quadratic for (approximate) tourism data, as measured in constant 1993 dollars. *Source for data:* Hawaii Visitors Bureau/*New York Times*, September 5, 1995, p. A12.

[5] Figures are rounded to the nearest \$10 billion.