CALCULUS WITH ANALYTIC GEOMETRY

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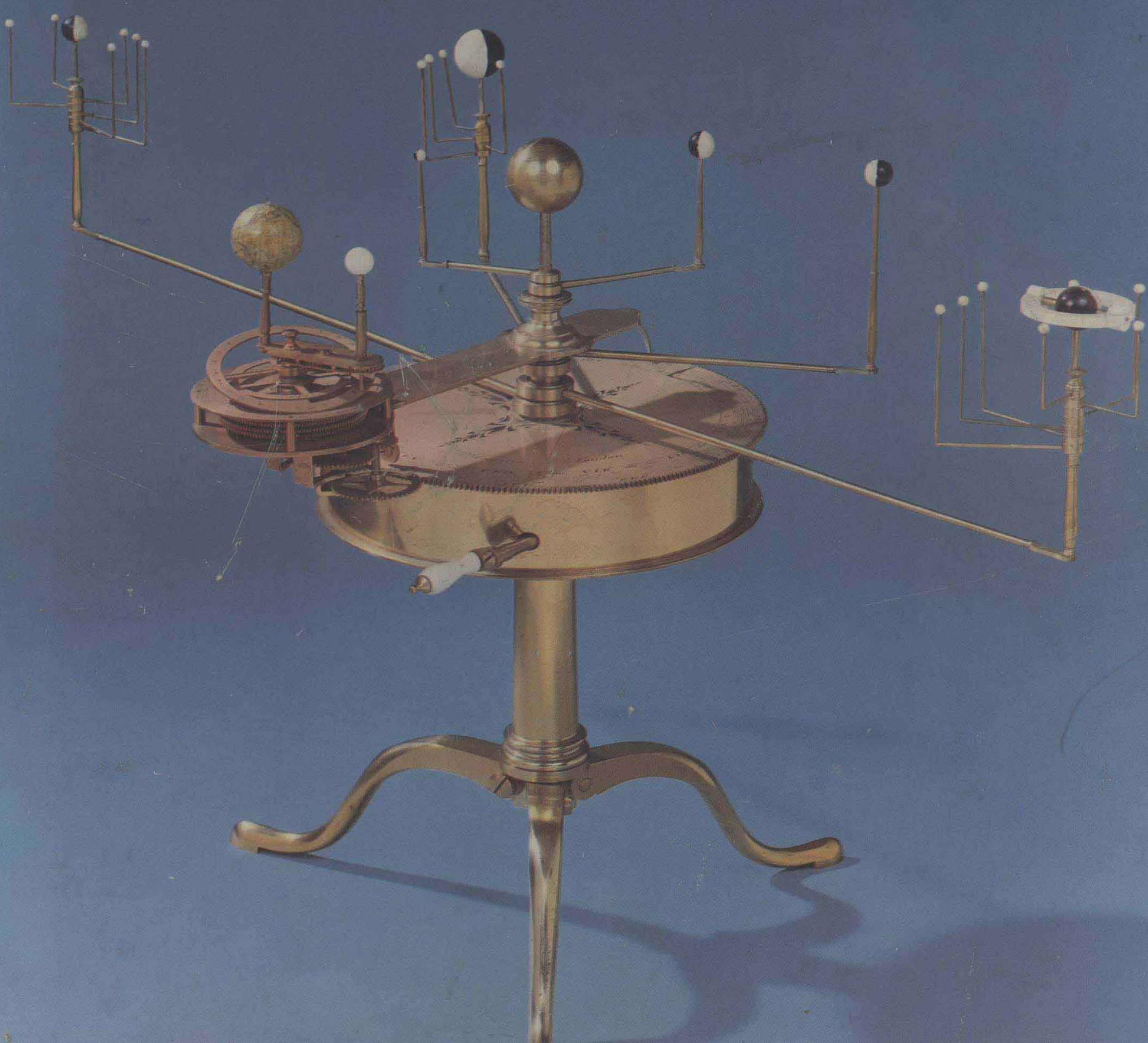

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George F. Simmons

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CALCULUS

WITH ANALYTIC GEOMETRY

For Gertrude Clark,
the great teacher in my life.

Tradition cannot be inherited, and if you want it you must obtain it by great labour. — *T. S. Eliot*

Science and philosophy cast a net of words into the sea of being, happy in the end if they draw anything out besides the net itself, with some holes in it. — *Santayana*

La vraie définition de la science, c'est qu'elle est l'étude de la beauté du monde. (The true definition of science is that it is the study of the beauty of the world.) — *Simone Weil*

To me, logic and learning and all mental activity have always been incomprehensible as a complete and closed picture and have been understandable only as a process by which man puts himself *en rapport* with his environment. It is the battle for learning which is significant, and not the victory. Every victory that is absolute is followed at once by the Twilight of the Gods, in which the very concept of victory is dissolved in the moment of its attainment.

We are swimming upstream against a great torrent of disorganization, which tends to reduce everything to the heat-death of equilibrium and sameness described in the second law of thermodynamics. What Maxwell, Boltzmann, and Gibbs meant by this heat-death in physics has a counterpart in the ethics of Kierkegaard, who pointed out that we live in a chaotic moral universe. In this our main obligation is to establish arbitrary enclaves of order and system. These enclaves will not remain there indefinitely by any momentum of their own after we have once established them. Like the Red Queen, we cannot stay where we are without running as fast as we can.

We are not fighting for a definitive victory in the indefinite future. It is the greatest possible victory to be, to continue to be, and to have been. No defeat can deprive us of the success of having existed for some moment of time in a universe that seems indifferent to us. — *Norbert Wiener*

PREFACE

It is a curious fact that people who write thousand-page textbooks still seem to find it necessary to write prefaces to explain their purposes. Enough is enough, one would think. However, every textbook—and this one is no exception—is both an expression of dissatisfaction with existing books and a statement by the author of what he thinks such a book ought to contain, and a preface offers one last chance to be heard and understood. Furthermore, anyone who adds to the glut of introductory calculus books should be called upon to justify his action (or perhaps apologize for it) to his colleagues in the mathematics community.

This book is intended to be a mainstream calculus text that is suitable for every kind of course at every level. It is designed particularly for the standard course of three semesters for students of science, engineering, or mathematics. Students are expected to have a background of high school algebra and geometry.

On the other hand, no specialized knowledge of science is assumed, and students of philosophy, history, or economics should be able to read and understand the applications just as easily as anyone else. There is no law of human nature which decrees that people with a strong interest in the humanities or social sciences are automatically barred from understanding and enjoying mathematics. Indeed, mathematics is the stage for many of the highest achievements of the human imagination, and it should attract humanists as irresistibly as a field of wildflowers attracts bees. It has been truly said that mathematics can illuminate the world or delight the mind, and often both. It is therefore clear that a student of philosophy (for example) is just as crippled without a fairly detailed knowledge of this great subject as a student of history would be without a broad understanding of economics and religion. As for students of history, how can they afford to neglect the fact (and it is a fact!) that the rise of mathematics and science in the seventeenth century was the crucial event in the development of the modern world, much more profound in its historical significance than the American, French, and Russian Revolutions combined? We teachers of mathematics have an obligation to help such students with this part of their education, and calculus is an excellent place to start.

The text itself—that is, the 21 chapters without considering the appendixes—is traditional in subject matter and organization. I have placed great emphasis on *motivation* and *intuitive understanding,* and the refinements of theory are downplayed. Most students are impatient with the theory of the subject, and justifiably so, because the essence of calculus does not lie in theorems and how to prove them, but rather in tools and how to use them. My overriding purpose has been to present calculus as a problem-solving art of immense power which is indispensable in all the quantitative sciences. Naturally, I wish to convince the student that the standard tools of calculus are reasonable and legitimate, but not at the expense of turning the subject into a stuffy logical discipline dominated by extra-careful definitions, formal statements of theorems, and meticulous proofs. It is my hope that every mathematical explanation in these chapters will seem to the thoughtful student to be as natural and inevitable as water flowing downhill along a canyon floor. The main theme of our work is what calculus is good for—what it enables us to do and understand—and not what its logical nature is as seen from the specialized (and limited) point of view of the modern pure mathematician.

There are several features of the text itself that it might be useful for me to comment on.

Precalculus Material Because of the great amount of calculus that must be covered, it is desirable to get off to a fast start and introduce the derivative quickly, and to spend as little time as possible reviewing precalculus material. However, college freshmen constitute a very diverse group, with widely different levels of mathematical preparation. For this reason I have included a first chapter on precalculus material which I urge teachers either to omit altogether or else to skim over as lightly as they think advisable for their particular students. This chapter is written in enough detail so that individual students who need to spend more time on the preliminaries should be able to absorb most of it on their own with a little extra effort.*

Trigonometry The problem of what to do about trigonometry in calculus courses has no satisfactory solution. Some writers introduce the subject early, partly in order to be able to use trigonometric functions in teaching the chain rule. This approach has the disadvantage of clogging the early chapters of calculus with technical material that is not really essential for the students' primary aims at this stage, which are to grasp the meanings and some of the uses of derivatives and integrals. Another disadvantage of this early introduction of the subject is that many students take only a single semester of calculus, and for these students trigonometry is an unnecessary complication that perhaps they should be spared. The fact is that trigonometry becomes really indispensable only when formal methods of integration must finally be confronted.

* A more complete exposition of high school mathematics that is still respectably concise can be found in my little book, *Precalculus Mathematics In a Nutshell* (William Kaufmann, Inc., Los Altos, Calif., 1981), 119 pages.

For these reasons I introduce the calculus of the trigonometric functions in Chapter 9, so that all the ideas will be fresh in the mind when students begin Chapter 10 on methods of integration. A full exposition of trigonometry from scratch is given in Section 9.1. For most students this will be a needed review of material that was learned (and mostly forgotten) in high school. For those who have studied no trigonometry at all, the explanations are complete enough so that they should be able to learn what they need to know from this single section alone.

For teachers who prefer to take up trigonometry early—and there are good reasons for this—I point out that Sections 9.1 and 9.2 can easily be introduced directly after Section 4.5, and Sections 9.3 and 9.4 at any time after Chapter 6. The only necessary adjustments are to warn students away from parts (b), (c), and (d) of Example 2 in Section 9.2, and also to make sure that the following problems are not assigned as homework: in Section 9.2, 15-18; in Section 9.3, 12, 16, 17, 29; and in Section 9.4, 11, 12, and 24.

Problems For students, the most important parts of their calculus book may well be the problem sets, because this is where they spend most of their time and energy. There are more than 5800 problems in this book, including many old standbys familiar to all calculus teachers and dating back to the time of Euler and even earlier. I have tried to repay my debt to the past by inventing new problems whenever possible. The problem sets are carefully constructed, beginning with routine drill exercises and building up to more complex problems requiring higher levels of thought and skill. The most challenging problems are marked with an asterisk (*). In general, each set contains approximately twice as many problems as most teachers will want to assign for homework, so that a large number will be left over for students to use as review material.

Most of the chapters conclude with long lists of additional problems. Many of these are intended only to provide further scope and variety to the problems sets at the ends of the sections. However, teachers and students alike should treat these additional problems with special care, because some are quite subtle and difficult and should only be attacked by students with ample reserves of drive and tenacity.

I should also mention that there are several sections scattered throughout the book with no corresponding problems at all. Sometimes these sections occur in small groups and are merely convenient subdivisions of what I consider a single topic and intend as a single assignment, as with Sections 6.1, 6.2, 6.3 and 6.4, 6.5. In other cases (Sections 9.7, 14.12, 15.5, 19.4, and 20.9) the absence of problems is a tacit suggestion that the subject matter of these sections should be touched upon only lightly and briefly.

There are a great many so-called "story problems" spread through the entire book. All teachers know that students shudder at these problems, because they usually require nonroutine thinking. However, the usefulness of mathematics in the various sciences demands that we try to teach our students how to penetrate into the meaning of a story problem, how to judge what is relevant to it, and how to translate it from words into sketches and equations. Without these skills—which are equally valuable for students who will become doctors, lawyers, financial analysts, or thinkers of any kind—there is no mathematics education worthy of the name.*

Infinite Series Any mathematician who glances at Chapter 14 will see at once that infinite series is one of my favorite subjects. In the flush of my enthusiasm, I have developed this topic in greater depth and detail than is usual in calculus books. However, some teachers may not wish to devote this much time and attention to the subject, and for their convenience I have given a shorter treatment in Chapter 13 that may be sufficient for the needs of most students who are not planning to go on to more advanced mathematics courses. Those teachers who consider this subject to be as important as I do will probably use both chapters, the first to give students an overview, and the second to establish a solid foundation and nail down the basic concepts. The spirit of these chapters is quite different, and there is surprisingly little repetition.

Differential Equations and Vector Analysis Each of these subjects is an important branch of mathematics in its own right. They should be taught in separate courses, after calculus, with ample time to explore their distinctive methods and applications. One of the main responsibilities of a calculus course is to prepare the way for these more advanced subjects and take a few preliminary steps in their direction, but just how far one should go is a debatable question. Some writers on calculus try to include mini-courses on these subjects in large chapters at the ends of their books. I disagree with this practice and believe that few teachers make much use of these chapters. Instead, in the case of differential equations I prefer to introduce the subject as early as possible (Section 5.4) and return to it in a low-key way whenever the opportunity arises (Sections 5.5, 7.8, 8.5, 8.6, 9.6, 17.7, 19.9); and in vector analysis I believe that Green's Theorem is just the right place to stop, with Stokes' Theorem—which after all is one of the most profound and far-reaching theorems in all of mathematics—being left for a later course. For those teachers who wish to include more vector analysis in their calculus course, I give a brief treatment of the divergence theorem and Stokes' Theorem—with problems—in Appendixes A.22 and A.23.

One of the major ways in which this book is unique and quite different from all its competitors can be understood by examining the appendixes, which I will now comment on very briefly. Before doing so, I emphasize that this material is entirely separate from the main text and can be carefully studied, dipped into occasionally, or completely ignored, as each individual student or instructor desires.

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* I cannot let the opportunity pass without quoting a classic story problem that appeared in the *New Yorker* magazine many years ago. "You know those terrible arithmetic problems about how many peaches some people buy, and so forth? Well, here's one we *like*, made up by a third-grader who was asked to think up a problem similar to the ones in his book: 'My father is forty-four years old. My dog is eight. If my dog was a human being, he would be fifty-six years old. How old would my father plus my dog be if they were both human beings?' "

Appendix A In teaching calculus over a period of many years, I have collected a considerable number of miscellaneous topics from number theory, geometry, science, etc., which I have used for the purpose of opening doors and forging links with other subjects . . . and also for breaking the routine and lifting the spirits. Many of my students have found these "nuggets" interesting and eye-opening. I have collected most of these topics in this appendix in the hope of making a few more converts to the view that mathematics, while sometimes rather dull and routine, can often be supremely interesting.

Appendix B This material amounts to a brief biographical history of mathematics, from the earliest times to the mid-nineteenth century. It has two main purposes.

First, I hope in this way to "humanize" the subject, to make it transparently clear that great men created it by great efforts of genius, and thereby to increase the students' interest in what they are studying. The minds of most people turn away from problems—veer off, draw back, avoid contact, change the subject, think of something else at all costs. These people—the great majority of the human race—find solace and comfort in the known and the familiar, and avoid the unknown and unfamiliar as they would deserts and jungles. It is as hard for them to think steadily about a difficult problem as it is to hold together the north poles of two strong magnets. In contrast to this, a tiny minority of men and women are drawn irresistibly to problems: their minds embrace them lovingly and wrestle with them tirelessly until they yield their secrets. It is these who have taught the rest of us most of what we know and can do, from the wheel and the lever to metallurgy and the theory of relativity. I have written about some of these people from our past in the hope of encouraging a few in the next generation.

My second purpose is connected with the fact that many students from the humanities and social sciences are compelled against their will to study calculus as a means of satisfying academic requirements. The profound connections that join mathematics to the history of philosophy, and also to the broader intellectual and social history of Western civilization, are often capable of arousing the passionate interest of these otherwise indifferent students.

Appendix C In the main text, the level of mathematical rigor rises and falls in accordance with the nature of the subject under discussion. It is rather low in the geometrical chapters, where for the most part I rely on common sense together with intuition aided by illustrations; and it is rather high in the chapters on infinite series, where the substance of the subject cannot really be understood without careful thought. I have constantly kept in mind the fact that most students have very little interest in purely mathematical reasoning for its own sake, and I have tried to prevent this type of material from intruding any more than is absolutely necessary. Some students, however, have a natural taste for theory, and some instructors feel as a matter of principle that all students should be exposed to a certain amount of theory for the good of their souls. This appendix contains virtually all of the theoretical material that by any stretch of the imagination might be considered appropriate for the study of calculus. From the purely mathematical point of view, it is possible for instructors to teach courses at many different levels of sophistication by using — or not using — material selected from this appendix.

In summary, therefore, the main body of this book is straightforward and traditional, while the appendixes make it convenient for teachers with many different interests and opinions to offer a wide variety of courses tailored to the needs of their own classes. I have aimed at the utmost flexibility of use.

Every project of this magnitude obviously depends on the cooperative efforts of many people. On the publisher's staff, I am especially grateful to Peter Devine, who as editor knew very well when to provide gentle guidance and when to let me go my own way; to Jo Satloff, the editorial supervisor, whose sympathy, tact, and highly skilled professionalism mean a great deal to me; and to Joan O'Connor, the designer, whose willingness to listen to an amateur's suggestions is very much appreciated.

Also, I offer my sincere thanks to the publisher's reviewers: Joe Browne, Onondaga Community College; Carol Crawford, United States Naval Academy; Bruce Edwards, University of Florida; Susan L. Friedman, Baruch College; Melvin Hausner, New York University; Louis Hoelzle, Bucks County Community College; Stanley M. Lukawecki, Clemson University; Peter Maserick, Pennsylvania State University; and David Zitarelli, Temple University. These people shared their knowledge and judgment with me in many important ways.

For the flaws and errors that undoubtedly remain, there is no one to blame but myself. I will consider it a kindness if colleagues and student users will take the trouble to inform me of any blemishes they detect, for correction in future editions.

George F. Simmons

TO THE STUDENT

Appearances to the contrary, no writer deliberately sets out to produce an unreadable book; we all do what we can and hope for the best. Naturally, I hope that my language will be clear and helpful to students, and in the end only they are qualified to judge. However, it would be a great advantage to all of us—teachers and students alike—if student users of mathematics textbooks could somehow be given a few hints on the art of reading mathematics, which is a very different thing from reading novels or magazines or newspapers.

In high school mathematics courses most students are accustomed to tackling their homework problems first, out of impatience to have the whole burdensome task over and done with as soon as possible. These students read the explanations in the text only as a last resort, if at all. This is a grotesque reversal of reasonable procedure, and makes about as much sense as trying to put on one's shoes before one's socks. I suggest that students should read the text first, and when this has been thoroughly assimilated, *then and only then* turn to the homework problems. After all, the purpose of these problems is to nail down the ideas and methods described and illustrated in the text.

How should a student read the text in a book like this? Slowly and carefully, and in full awareness that a great many details have been deliberately omitted. If this book contained every detail of every dicussion, it would be five times as long, which God forbid! There is an old French proverb: "He who tries to explain everything soon finds himself talking to an empty room." Every writer of a book of this kind tries to walk a narrow path between saying too much and saying too little.

The words "clearly," "it is easy to see," and similar expressions are not intended to be taken literally, and should never be interpreted by any student as a putdown on his or her abilities. These are code-phrases that have been used in mathematical writing for hundreds of years. Their purpose is to give a signal to the careful reader that in this particular place the exposition is somewhat condensed, and perhaps a few details of calculations have been omitted. Any phrase like this amounts to a friendly hint to the student that it might be a good idea to read even more carefully and thoughtfully in order to fill in omissions in the exposition, or perhaps get out a piece of scratch paper to verify omitted details of calculations. Or better yet, make full use of the margins of this book to emphasize points, raise questions, perform little computations, and correct misprints.

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PART III