

Springer Series in

# Optical Sciences

D.L. MacAdam

# Color Measurement

Theme and Variations

Second Revised Edition



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Theme and Variations

Second Revised Edition

With 92 Figures and 4 Colorplates

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Berlin Heidelberg New York Tokyo

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*This book is dedicated to the memory of*  
Arthur Cobb Hardy



Arthur Cobb Hardy (1895 – 1977)

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Inventor of recording spectrophotometer  
Inspiration and teacher of pioneers in applied colorimetry  
Professor of Optics and Photography at Massachusetts Institute of Technology  
Teacher of leaders of renaissance of optics  
Co-author of *Principles of Optics* (McGraw-Hill, New York, London 1932)  
Founder of MIT Color Measurement Laboratory  
Coauthor of *Handbook of Colorimetry*  
Chief of Camouflage Section of NDRC – Research on visibility and influence of color on perception of distant objects  
President of Optical Society of America and Secretary for 17 years  
Recipient of Ives Medal, Longstreth Medal of Franklin Institute, Modern Pioneer Award of National Association of Manufacturers, and Progress Medal of Society of Motion Picture and Television Engineers  
Honorary D.Sc. (St. Lawrence University)  
Honorary LL.D. (University of California)  
In gratitude for inspiration and guidance, and  
In tribute to his unique contributions to applied optics and colorimetry

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## Preface to the Second Edition

Reviews and use of the first edition as the textbook for a senior-division university course indicated the need for a number of corrections and clarifications. Although no new topics have been introduced, the new edition should be more clear and useful. A novelty in the Notes and Sources Appendix should facilitate reference from the notes back to the text. For that purpose, the page number of the text to which each note refers is indicated in square brackets following the serial number of the note.

The FMC1 color-difference formula has been substituted everywhere for the Friele-MacAdam formula, including the reference to the sources in Note 52. The FMC1 formula was actually used in the investigations reviewed in Sects. 8.3 and 8.4. The Friele-MacAdam formula given on page 151 of the first edition, which I thought was equivalent to the FMC1, was erroneous and should not be used.

The formulas for the geodesic chromaticity diagram, on p. 153 of the first edition, were based on observations by 14 normal observers (last reference in Note 51). They have been replaced by the formulas based on the observations of PGN, for consistency with all other formulas and discussions in the book. Figure 8.29 in the first edition was based on the PGN data and on the formulas printed below it in the new edition. Therefore, Fig. 8.29 is unchanged.

No other corrections merit mention. Very few of them are likely to be noticed without line-by-line or character-by-character comparison of the second with the first edition.

The constants given for Planck's formula, on pp. 26 and 94 (of the first edition), to which one reviewer took exception, are correct and consistent with the units of wavelength, power, and area specified on those pages. The dimension specification given for  $c_1$  on p. 94 was incomplete and has been completed in the second edition.

I thank the reviewers and my students (who tripled in number, from 19 to 57, in the four years since we began to use the first edition) for pointing out errors, for suggesting clarifications, and for patience and forbearance.

Rochester NY, April 1985

*David L. MacAdam*



## Preface to the First Edition

Color is attractive and interesting to everyone. Consequently, control of color is important to all producers, buyers, sellers, and users of colored materials. In various ways, color is an indication of freshness, quality, or other desirable (or undesirable) characteristics of goods. To assure acceptability, saleability, and favorable price — especially in contracts and monitoring of conformance to specifications — numerical expression of color is greatly superior to verbal descriptions. Disagreements concerning words or visual comparisons with samples are all too likely and frequent. Such disagreements underlie much unpleasantness and loss in commerce in consumer goods. Such loss of money and good will must amount to billions of dollars per year, world wide.

Persistent efforts to substitute measurements of color for visual judgment have marked the twentieth century. Because visual perception of small color differences is so acute, the requirements for accuracy and world-wide reproducibility of color measurements have been severe. Only during the last half century have practical spectrophotometers with adequate accuracy been available.

The characteristics of color vision differ even among persons with normal color vision. If different laboratories or countries used different visual data for interpretation of the significance of measurements obtained from spectrophotometers, the need for interchangeably reproducible color specifications would be subverted. To forestall that danger, the International Commission on Illumination (CIE) recommended data that characterize a standard observer for colorimetry. Related to those data, a coordinate system for maplike representation of the results of color measurements was recommended at the same time (1931) by the CIE. Methods for using the CIE data and coordinate system constitute the subject of colorimetry.

The earliest and still most successful exposition of colorimetry was the MIT *Handbook of Colorimetry* (1936). It was written for newcomers because there were then very few cognoscenti. It effectively introduced colorimetry and the CIE standards to industry.

The first three chapters and the first three sections of each of Chapters 4 and 5 of this book consist of revisions and abridgments of the *Handbook of Colorimetry*. The revisions conform to current agreements on terminology by the CIE and other international organizations.

The later sections of Chapters 4 and 5 begin the variations on themes introduced by the *Handbook of Colorimetry* that are promised by the subtitle of

this book. They deal with some alternate methods and with some implications of colorimetry.

The remainder of the book presents further variations on the theme of colorimetry. Chapters 6 and 7 discuss applications to two important kinds of problems, colors of light and objects, respectively. Daylight and light from man-made sources are subjects of Chapter 6. Colorant mixtures are discussed in Chapter 7.

Chapter 8 is concerned with the abilities of normal observers to see small color differences and with representation and use of such information to assess color differences evaluated by colorimetry.

Chapter 9 describes color-order systems, particularly their relations to the uniform color scales of the Optical Society of America. Those color scales are illustrated by four pages printed in color.

Chapter 10 discusses the dependence of color-matching data on the chromaticities of primaries, in terms of the CIE observer data and coordinate system. The alychne and apeiron are defined; orthogonal color-matching functions and self-conjugate primaries are discussed.

Chapter 11 is devoted to chromatic adaptation and color constancy. The important effect of color constancy is to make objects seem to retain their colors unaltered despite significant changes of spectral distributions of the light with which they are illuminated.

Readers are not expected to be acquainted with mathematics beyond high-school algebra. The only symbols that may be unfamiliar to some are  $\Sigma$  for summation and  $\Delta$  for the difference between two quantities. Each is explained in a note, at its first occurrence. Calculus and vector and matrix algebra, which are sometimes encountered in expositions of colorimetry, are unnecessary embellishments and are omitted, without loss.

In general, useful information and formulas are given without formal derivations or proof. The major exception is the proof of Ostwald's theorem concerning the spectrophotometric curves of optimal colors. The proof is given as an example of the power and method of use of Newton's center-of-gravity principle. Ostwald offered the theorem unproved; Schrödinger's proof was very long and difficult.

To maintain continuity without interruptions or digressions, all accessory information is placed in an Appendix entitled "Notes and Sources". Those notes are numbered in sequence. A superscript number is placed at the place in the text to which each note is related. Readers to whom the material is new may ignore those numbers during their first reading. Subsequently, like readers already familiar with the subject, they may find the notes helpful. Some of the notes are merely references.

There is no pretense that the references are complete. They are given only as suggestions for sources of further information or data, or to give credit for pioneering contributions to the subject, especially to authors named in the text.

This does not purport to be a reference book. The subject matter was determined by my own experience and interests. Therefore there are no discussions of the anatomy, physiology, or psychology of color vision, or of color-vision theories, or of anomalies of color vision or color blindness. Topics in colorimetry that are obsolete or, in my opinion, obsolescent have been omitted to save space for some new topics that I think may become significant.

The newest exciting application of colorimetry that is not included is discussed in *Color Theory and Its Application in Art and Design*, by G. A. Agoston (Springer, Berlin, Heidelberg, New York 1979), Springer Series in Optical Sciences, Vol. 19, the same series to which this book belongs.

The portrait of Professor Hardy, used as a frontispiece, was furnished by the MIT Museum and Historical Collection.

Permission has been granted by the MIT Press for inclusion of revisions of text, figures, and selected tables and charts from the *Handbook of Colorimetry*. I was a co-author of that book.

Generous career-long support of my work in colorimetry by the Eastman Kodak Company is gratefully acknowledged. The most recent instance of that support is entry of the typescript of this book into the Document Processing system by Debe Jayne of the Research Laboratories, whom I sincerely thank. The figures were prepared over a period of four decades by the Photographic Service Department, with a recent burst to unify style and nomenclature.

This is to thank also my beloved wife, Muriel, who converted my atrocious handwriting into typescript during the summer of 1980 at Cape Cod.

Rochester NY, January 1981

David L. MacAdam

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## Multiresolution Image Processing and Analysis

Editor: **A. Rosenfeld**

1984. 198 figures. VIII, 385 pages. (Springer Series in Information Sciences, Volume 12). ISBN 3-540-13006-3

**Contents:** Image Pyramids and Their Uses. – Architectures and Systems. – Modelling, Processing, and Segmentation. – Features and Shape Analysis. – Region Representation and Surface Interpolation. – Time-Varying Analysis. – Applications. – Index of Contributors. – Subject Index.

## Physical and Biological Processing of Images

Proceedings of an International Symposium Organised by the Rank Prize Funds, London, England, September 27–29, 1982

Editors: **O. J. Braddick, A. C. Sleigh**

1983. 227 figures. XI, 403 pages. (Springer Series in Information Sciences, Volume 11). ISBN 3-540-12108-0

**Contents:** Overviews. – Local Spatial Operations on the Image. – Early Stages of Image Interpretation. – Pattern Recognition. – Spatially Analogue Processes. – Higher Level Representations in Image Processing. – Postscript. – Index of Contributors.

## Picture Engineering

Editors: **K. S. Fu, T. L. Kunii**

1982. 166 figures. VIII, 303 pages. (Springer Series in Information Sciences, Volume 6). ISBN 3-540-11822-5

**Contents:** Pictorial Database Management. – Picture Representation. – Picture Computer Architecture. – Office Automation. – Computer-Aided Design. – Computer Art. – Index of Contributors.

**H. Niemann**

## Pattern Analysis

1981. 112 figures. XIII, 302 pages. (Springer Series in Information Sciences, Volume 4). ISBN 3-540-10792-4

**Contents:** Introduction. – Preprocessing. – Simple Constituents. – Classification. – Data. – Control. – Knowledge Representation, Utilization, and Acquisition. – Systems for Pattern Analysis. – Things to Come. – References. – Subject Index.

**G. A. Agoston**

## Color Theory and Its Application in Art and Design

1979. 55 figures, 6 color plates, 12 tables. XI, 137 pages (Springer Series in Optical Sciences, Volume 19)  
ISBN 3-540-09654-X



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# Frontiers in Computer Graphics

Proceedings of Computer Graphics  
Tokyo '84

Editor: **T. L. Kunii**

1985. 266 figures, 82 of them in color.  
XI, 443 pages. ISBN 3-540-70004-8

**Contents:** Geometry Modelling. – Graphic Languages. – Visualization Techniques. – Human Factors. – Interactive Graphics Design. – CAD/CAM. – Graphic Displays and Peripherals. – Graphics Standardization. – Author Index. – Subject Index.



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Computer graphics, taken as a whole, is an area in which progress is very rapid. It is not easy for anyone, including the experts, to keep abreast of the various basic and applied fields. This book has been compiled to present you with the substance of progress in computer graphics. It also serves as the final version of the Proceedings of Computer Graphics Tokyo '84 held in Tokyo, Japan, in April. Eight major frontiers of computer graphics are covered: geometry modelling, graphic languages, visualization techniques, human factors, interactive graphic design, CAD/CAM, graphic displays and peripherals, and graphics standardization.

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