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symposia on theoretical physics and mathematics

Lectures presented at the
1969 Seventh Anniversary
Symposium of the Institute
of Mathematical Sciences
Madras, India

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Edited by
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Introduction

This volume, the tenth in the series, comprises the proceedings of the Seventh Anniversary Symposium, as well as some important seminars held during 1968–1969 at the Institute of Mathematical Sciences, in Madras. The volume opens with an article by Professor Shreeram Abhyankar on the life and work of the famous Indian mathematician, Srinivasa Ramanujan, whose memory is being cherished by the Institute through a visiting professorship in his name. By a fortunate circumstance, we were able to include as a sequel an article by Professor Robert A. Rankin, in which is given a survey of problems, solved and unsolved, concerning Ramanujan's function $\tau(n)$.

The first half of the volume contains the mathematical contributions, mainly in analysis, and we are grateful to our visiting professors, J. G. Krzyż and Tord Ganelius, for participating in the symposium. The contribution of Krzyż deals with close-to-convex functions, while Ganelius considers the application of Fourier methods, in combination with a theorem of Poincaré on solutions of difference equations, to the characterization of regularly varying functions. V. I. Gavrilov discusses some uniqueness theorems on meromorphic functions on the unit disc, and K. Srinivasacharyulu proves a fixed point theorem for completely continuous perturbations on nonexpansive mappings. There is a lecture by B. Sz.-Nagy discussing the problem of investigating the behaviour of completely nonunitary operators on Hilbert space.

In the theoretical physics section, F. Riahi from Teheran, using the axiomatic approach, discusses nonrelativistic many-body scattering theory and the analytic properties of many-particle scattering amplitudes, which are far more complicated than the usual, well worked out, two-particle amplitudes. This is probably the direction in which theories on scattering, both relativistic and non-relativistic, will concentrate in future. José R. Fulco and Gordon L. Shaw make a critical analysis of the utilization of the peripheral model in relation to $\pi\pi$ phase shifts. A group theoretical study of spaces characterized by symmetries and their relations to various branches of physics is made by A. Sankaranarayanan, while V. Devanathan studies distortion effects in nuclear reactions.

In a series of three articles, Alladi Ramakrishnan and his collaborators summarize the results of the recent work at the Institute on L-matrices, dealing with the role of Pauli matrices as a primary tool of mathematical physics. Their investigations have almost led them to believe that "modern physics is Pauli matrices recollected in tranquillity"! This may sound like a hyperbole, but it does emphasize that there is much more to Pauli matrices than meets the eye.

The next contribution is that of E. C. G. Sudarshan, who presents ideas currently agitating the minds of physicists on the possible existence of particles travelling faster than light. It brings out adequately the speculative spirit of a one-day symposium held at the Institute with Professor Sudarshan as the principal invited speaker.

Finally, V. Devanathan offers an article on the effects of short-range correlations in nuclear reactions.

Alladi Ramakrishnan

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The Contributions of Ramanujan to Mathematics

SHREERAM SHANKAR ABHYANKAR

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Recently on behalf of the Sir C. P. Ramasway Aiyar Foundation, Professor Alladi Ramakrishnan asked me to give a lecture to be entitled "The Contributions of Ramanujan to Mathematics."

Ever since I was a schoolboy I had heard wonderful stories about the world-famous Indian mathematical genius Srinivasa Ramanujan—naturally I felt very honored to be asked to speak about him. However, I was being asked to speak not just about Ramanujan, but specifically about his contributions to mathematics. And so, to prepare myself for this assignment, I started reading the collected mathematical works of Ramanujan, and also Hardy's lectures on Ramanujan's work. I soon realized that his work is very beautiful and profound, and at the same time quite mysterious, and written in a terse and cryptic style. I therefore came to the conclusion that I really could not measure up to the task—at any rate not on such short notice. It would take me at least five years of concentrated study to gain a reasonable degree of understanding of his work. So what I really ought to have done is to postpone the assigned lecture for such a period. However, I could not resist the temptation of sharing with you some interesting stories about Ramanujan. I hope that this occasion will inspire some of you to undertake a study of his mathematical work. I, for one, have now already been so inspired. Thus, with a promise to do some justice to the title of today's lecture at

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some time in the future, let me present some interesting stories about Ramanujan.

One of Ramanujan's distinguishing characteristics was his love and understanding of numbers, i.e., positive integers. Every number was, so to speak, his personal friend. One amusing anecdote—but a true one—regarding this is what happened once when Ramanujan was lying ill in a hospital in England and the famous English mathematician Hardy went to visit him. Hardy had arrived in a taxi and had happened to notice that the taxi number was 1729. Upon entering Ramanujan's bedroom Hardy remarked that his taxi number was 1729 which seemed to him to be a rather dull number, and he hoped it was not an unfavorable omen. "No," replied Ramanujan immediately, "it is a very interesting number; it is the smallest number which can be expressed as a sum of two cubes in two different ways; namely, $1729 = 12^3 + 1^3 = 10^3 + 9^3$." Hardy then asked Ramanujan whether he could also tell the solution of the corresponding problem for fourth powers. After a moment's thought Ramanujan replied that he knew no obvious example but supposed that the first such number must be very large. Indeed, the simplest known example of this is very large, namely, the following one found by Euler:

$$158^4 + 59^4 = 134^4 + 133^4 = 635{,}318{,}657$$

Another of Ramanujan's interests was in algebraical formulas, especially those concerned with infinite series, infinite products, infinite continued fractions, and so on. With his insight for and mastery over algebraic formulas, he has never been surpassed by any other mathematician, and can be compared only with the two famous European mathematicians Euler and Jacobi. Many of the algebraic formulas which Ramanujan discovered are of such an astonishing nature that most mathematicians could not even dream of them. Indeed, according to Ramanujan, it was the goddess of Namakkal who used to visit him in his dreams and tell him the formulas. In any case, it is a remarkable fact that frequently, upon rising from bed, he would note results and rapidly verify them, although often he was not able to supply a rigorous proof.

By the time Ramanujan was in his early twenties, he had two notebooks filled with theorems and formulas that he had discovered.

Owing to his passionate interest in mathematics Ramanujan had completely neglected all other subjects, and as a result twice failed the first-year college examination. His formal education stopped at that level. Thus, Ramanujan was an entirely self-taught mathematician.

Some time after leaving college, Ramanujan obtained employment as a clerk in the accounts department of the Madras Port Trust. However, his pursuit of mathematics never slackened.

The fact that Ramanujan was very much a self-taught mathematician showed up in his mathematical work in two ways. First, not having had the advantage of knowing the work of past mathematicians, a substantial part of his work turned out to be rediscovery. Second, he never developed a clear-cut idea of what is meant by a proof; if a significant piece of reasoning occurred somewhere and the total mixture of evidence and intuition gave him certainty, he looked no further. As a consequence, same of his theorems and formulas turned out to be incorrect. In spite of these two aspects of his work, Ramanujan discovered an amazing number of beautiful and profound formulas and theorems.

As I have just said, Ramanujan was often not able to supply rigorous proofs of his results. Indeed, after his untimely death, a number of mathematicians expended considerable effort in deciphering his notebooks and giving rigorous mathematical proofs of many of the results conjectured by him. Nevertheless, a few of Ramanujan's conjectures remain unproved to this day.

Ramanujan continued his pursuit of mathematics completely on his own until he reached the age of twenty-six. Then, taking the advice of some of his friends, he wrote a letter to G. H. Hardy, who was a professor at Cambridge University. The major part of the letter consisted of the statements of about 120 results, mostly formulas, extracted from Ramanujan's notebooks. Of course, Hardy was most surprised to receive such a letter from an unknown clerk. All the distinguishing features of Ramanujan's work which I have mentioned before, stood out quite clearly in that letter. Namely, Hardy found that out of the formulas stated in the letter some were known, some were easy to prove, some were quite difficult to prove, some he could not prove, and some were false. However, as Hardy has remarked, each one of Ramanujan's formulas, whether true or false, is extremely fascinating.

Soon after this, it was arranged that Ramanujan go to Cambridge. The next six years were a time of very fruitful collaboration between Hardy and Ramanujan. Hardy has said that his discovery of Ramanujan and their happy collaboration was the one most romantic incidents in Hardy's life.

To give a very small sample of Ramanujan's work, I shall now state some of his results concerning partitions. These show both his deep knowledge of numbers and his mastery of algebraical formulas.

Let $p(n)$ denote the number of partitions of a positive integer n, i.e.,