

Polychronis Kondaxakis

# A Modular Framework for Multi-robot Localization Scenarios

Developing and Distributing Localization Algorithms  
Utilizing Kalman Filter Estimators, for Mobile  
Multi-robot Systems



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# Contents

<b>Chapter 1</b> .....	<b>9</b>
<b>Introduction and Problem Statement</b> .....	<b>9</b>
1.1 Assumptions Made in this Thesis .....	10
1.2 Problem Statement .....	10
1.3 Research Contribution.....	12
1.4 Thesis Outline .....	13
<b>Chapter 2</b> .....	<b>15</b>
<b>Literature Review</b> .....	<b>15</b>
2.1 Mobile Robot Positioning .....	15
2.1.1 Robot Position Tracking and Kalman Filters.....	16
2.2 Absolute Robot Localization .....	17
2.2.1 Landmark Based Localization .....	17
2.2.2 Dense Sensor Maps Localization .....	20
2.3 Multi-Robot Localization.....	22
2.4 Proposed Extensions to Multi-Robot Localization .....	24
<b>Chapter 3</b> .....	<b>26</b>
<b>Modeling an Ultrasonic Beacon System for Cooperative Localization</b> .....	<b>26</b>
3.1 Exteroceptive Range Sensors.....	26
3.1.1 Ultrasonic TOF Systems .....	28
3.1.2 Beam Geometry .....	29
3.2 Ultrasound Sensor Arrangement for Relative Distance and Bearing Measurements .....	33
3.2.1 The Ultrasound Transducer using the <i>muRata</i> MA40S5 Sensor .....	35
3.2.2 Design of the Reflective Cone Devices for 360° Ultrasound Tranception .....	38
3.2.3 Alternative Design of the Reflective Cone Devices for 360° Ultrasound Tranception ..	42
3.2.4 R.P.S (Relative Positioning System) Configuration .....	44
3.3 Experimental Noise Modeling of Ultrasonic Tri-Transducer system.....	48
3.3.1 Experimental Analysis of a Single-Sensor Cone Transducer.....	49
3.3.2 Experimental Analysis of a Tri-Sensor Cone Transducer .....	52
<b>Chapter 4</b> .....	<b>66</b>
<b>2-D Robot Localization and Kalman Filtering</b> .....	<b>66</b>
4.1 Discrete Kalman Filter .....	66
4.1.1 State Space Model of a Random Process.....	68
4.1.2 Computational Origins of the Kalman Filter .....	69

4.1.3 The Probabilistic Origins of the Kalman Filter.....	71
4.1.4 The Discrete Kalman Filter Algorithm .....	72
4.2 Non-linear Applications and Kalman Filtering.....	74
4.2.1 The Discrete Extended Kalman Filter.....	77
4.2.2 Computational Origins of the Extended Kalman Filter .....	78
4.2.3 The Discrete Extended Kalman Filter Algorithm.....	81
4.3 2-D Representation of a Mobile Robot Kinematics.....	83
4.3.1 The Plant Model Representation.....	83
4.3.2 The Plant Model Linearization .....	87
4.3.3 The Measurement Model Representation .....	88
4.3.4 The Measurement Model Linearization.....	90
4.4 Simulated Results of Single Robot Localization .....	92
<b>Chapter 5.....</b>	<b>99</b>
<b>Collective Localization for a Group of Mobile Robots.....</b>	<b>99</b>
5.1 Introduction to Collective Localization .....	99
5.2 Problem Statement of Multi-Robot Localization.....	100
5.3 Localization for a Group of Three Mobile Robots .....	103
5.3.1 Prediction Cycle.....	104
5.3.2 Update Cycle.....	107
5.4.1 Robots with Only Relative Measurement Capabilities.....	111
5.4.2 Robots with Relative and Absolute Measurement Capabilities.....	113
5.5 Decentralized Computational Algorithms for EKF .....	113
5.5.2 Sequential Extended Kalman filter .....	115
5.5.3 Parallel Extended Kalman filter without Feedback .....	117
5.5.4 Parallel Extended Kalman filter with Feedback .....	121
5.6 Simulated Results of Multi-Robot Localization .....	123
<b>Chapter 6.....</b>	<b>143</b>
<b>Distributed Processing of Kalman Filter for Collective Robot Localization....</b>	<b>143</b>
6.1 Introduction to Parallel Kalman Filters for Multi-Robot Localization.....	143
6.2 Real-Time Implementation Issues .....	144
6.2.1 Processor Loading Issues.....	146
6.3 Localization Interdependencies for a Group of Mobile Robots.....	147
6.4 EKF Distribution among a Team of Mobile Robots.....	151
6.4.1 Prediction Cycle before First Update.....	152
6.4.2 First Update Cycle .....	154
6.4.3 Prediction Cycle after First Update.....	163
6.4.4 Second Update Cycle.....	166
6.5 Simulated Results of the Distributed Localization Algorithm.....	171
<b>Chapter 7.....</b>	<b>180</b>
<b>Conclusion and Future Work.....</b>	<b>180</b>
7.1 Thesis Summary.....	180
7.2 Evaluating the Research.....	182

7.2.1 Developing the Extended Kalman Filter Localization Solution .....	183
7.2.2 Complexity of Distributed EKF Arbitration and Communication.....	184
7.2.3 Modularity Evaluation and Observability Performance for Different Localization Scenarios .....	185
7.2.4 Further Validation of the Proposed Localization Framework .....	186
7.3 Future Research.....	187
<b>Appendix A.....</b>	<b>190</b>
<b>Optimum Recursive Estimator (Scalar Kalman Filter) .....</b>	<b>190</b>
<b>Appendix B.....</b>	<b>198</b>
<b>Alternative Representation of Discrete Kalman Filter.....</b>	<b>198</b>
<b>Appendix C.....</b>	<b>203</b>
<b>Circuit Diagrams of the Ultrasonic Tri-Transducer.....</b>	<b>203</b>
<b>References .....</b>	<b>207</b>

# List of Figures

Figure 3. 1 Beam dispersion angle $\theta$ and spot diameter $D$ at distance $R$ .	30
Figure 3. 2 Ultrasonic ranging error due to beam divergence.	31
Figure 3. 3 Beam splitting techniques.	33
Figure 3. 4 Ultrasound transducer operation diagram.	37
Figure 3. 5 The muRata MA40S5 ultrasonic transceiver [61].	38
Figure 3. 6 Directivity in overall sensitivity of the muRata MA40S5 ultrasonic transceiver [61].	38
Figure 3. 7 Transceiver with metallic cone reflector.	39
Figure 3. 8 Angles involved in the reflective cone design.	40
Figure 3. 9 Reflection cone design.	41
Figure 3. 10 Prototype reflection cone.	42
Figure 3. 11 Reflection patterns for incident-complementary angles of less or more than $30^\circ$ .	43
Figure 3. 12 Alternative cone configuration designed to keep the sensors main lobe incident-complementary angles at $30^\circ$ .	43
Figure 3. 13 Determination of the surface curvature slope for the alternative cone configuration.	44
Figure 3. 14 Trilateration methods.	45
Figure 3. 15 Tri-transducer System.	46
Figure 3. 16 Relative position measurement using trilateration.	48
Figure 3. 17 Plot of real and measured distances vs. TOF.	51
Figure 3. 18 1-D range measurement performance.	51
Figure 3. 19 Tri-transducer sensor dimensions.	53
Figure 3. 20 Actual and measured beacon positions for 600mm distance between receiver and transmitter.	54
Figure 3. 21 Actual and measured X- positions of beacon for 600mm distance between receiver and transmitter.	55
Figure 3. 22 Actual and measured Y- positions of beacon for 600mm distance between receiver and transmitter.	55
Figure 3. 23 X-component absolute measurement error for 600mm distance.	56
Figure 3. 24 Y-component absolute measurement error for 600mm distance.	56
Figure 3. 25 Actual and measured beacon positions for 1200mm distance between receiver and transmitter.	58
Figure 3. 26 Actual and measured X- positions of beacon for 1200mm distance between receiver and transmitter.	58
Figure 3. 27 Actual and measured Y- positions of beacon for 1200mm distance between receiver and transmitter.	59
Figure 3. 28 X-component absolute measurement error for 1200mm distance.	59
Figure 3. 29 Y-component absolute measurement error for 1200mm distance.	60
Figure 3. 30 Actual and measured beacon positions for 1800mm distance between receiver and transmitter.	61
Figure 3. 31 Actual and measured X- positions of beacon for 1800mm distance between receiver and transmitter.	61
Figure 3. 32 Actual and measured Y- positions of beacon for 1800mm distance between receiver and transmitter.	62
Figure 3. 33 X-component absolute measurement error for 1800mm distance.	62
Figure 3. 34 Y-component absolute measurement error for 1800mm distance.	63
Figure 3. 35 Dependencies between sensor separation distances and resolution margins.	65

<i>Figure 4. 1 The ongoing discrete Kalman filter update cycle. The time prediction projects the current state estimate ahead in time. The measurement update adjusts the projected estimate by an actual measurement at that time.</i>	73
<i>Figure 4. 2 Illustration for two-stage Kalman computational cycle.</i>	73
<i>Figure 4. 3 The linearized Kalman filter's feedforward configuration diagram.</i>	76
<i>Figure 4. 4 The extended Kalman filter's feedback configuration diagram.</i>	76
<i>Figure 4. 5 A complete picture of the operation of the extended Kalman filter.</i>	82
<i>Figure 4. 6 The differential drive configuration.</i>	84
<i>Figure 4. 7 Estimated and actual robot trajectory with one available landmark in space.</i>	94
<i>Figure 4. 8 x coordinates of the robot as time passes, detecting one landmark.</i>	95
<i>Figure 4. 9 y coordinates of the robot as time passes, detecting one landmark.</i>	95
<i>Figure 4. 10 Picture (a) indicates the uncertainty (variance) related to the x state of the robot, picture (b) indicates the uncertainty (variance) related to the y state of the robot, and picture (c) indicates the covariance between the x and y components.</i>	96
<i>Figure 4. 11 Estimated and actual robot trajectory with three available landmarks in space.</i>	97
<i>Figure 4. 12 x coordinates of the robot as time passes, detecting three landmarks.</i>	97
<i>Figure 4. 13 y coordinates of the robot as time passes, detecting three landmarks.</i>	98
<i>Figure 4. 14 Picture (a) indicates the uncertainty (variance) related to the x state of the robot, picture (b) indicates the uncertainty (variance) related to the y state of the robot, and picture (c) indicates the covariance between the x and y components.</i>	98
<i>Figure 5. 1 Distance and bearing measurements between R1 and R2, 3 at time k.</i>	102
<i>Figure 5. 2 Distance and bearing measurements between R2 and R1, 3 at time k+1.</i>	103
<i>Figure 5. 3 Sequential Kalman Filter.</i>	117
<i>Figure 5. 4 Decentralized parallel filter with no feedback.</i>	118
<i>Figure 5. 5 Decentralized parallel filter with feedback.</i>	122
<i>Figure 5. 6 Estimated and actual trajectory of the three robots with no landmarks in space.</i>	124
<i>Figure 5. 7 x coordinates of robot 1 as time passes, with only relative measurements.</i>	127
<i>Figure 5. 8 y coordinates of the robot 1 as time passes, with only relative measurements.</i>	128
<i>Figure 5. 9 x coordinates of robot 2 as time passes, with only relative measurements.</i>	128
<i>Figure 5. 10 y coordinates of the robot 2 as time passes, with only relative measurements.</i>	129
<i>Figure 5. 11 x coordinates of robot 3 as time passes, with only relative measurements.</i>	129
<i>Figure 5. 12 y coordinates of the robot 3 as time passes, with only relative measurements.</i>	130
<i>Figure 5. 13 Picture (a) indicates the uncertainty (variance) related to the x state of robot 1, picture (b) indicates the uncertainty (variance) related to the y state of robot 1, and picture (c) indicates the covariance between the x and y components.</i>	130
<i>Figure 5. 14 Picture (a) indicates the uncertainty (variance) related to the x state of robot 2, picture (b) indicates the uncertainty (variance) related to the y state of robot 2, and picture (c) indicates the covariance between the x and y components.</i>	131
<i>Figure 5. 15 Picture (a) indicates the uncertainty (variance) related to the x state of robot 3, picture (b) indicates the uncertainty (variance) related to the y state of robot 3, and picture (c) indicates the covariance between the x and y components.</i>	131
<i>Figure 5. 16 X-axis measurement innovation between robot 1 and robot 3.</i>	132
<i>Figure 5. 17 Estimated and actual trajectory of the three robots with one available landmark in space.</i>	133
<i>Figure 5. 18 x coordinates of robot 1 as time passes, with absolute and relative measurements.</i>	135
<i>Figure 5. 19 y coordinates of robot 1 as time passes, with absolute and relative measurements.</i>	136



<i>Figure 5. 20 x coordinates of robot 2 as time passes, with only relative measurements.</i> .....	136
<i>Figure 5. 21 y coordinates of the robot 2 as time passes, with only relative measurements.</i> .....	137
<i>Figure 5. 22 x coordinates of robot 3 as time passes, with only relative measurements.</i> .....	137
<i>Figure 5. 23 y coordinates of the robot 3 as time passes, with only relative measurements.</i> .....	138
<i>Figure 5. 24 Picture (a) indicates the uncertainty (variance) related to the x state of robot 1, picture (b) indicates the uncertainty (variance) related to the y state of robot 1, and picture (c) indicates the cross-correlation between the x and y components.</i> .....	138
<i>Figure 5. 25 Picture (a) indicates the uncertainty (variance) related to the x state of robot 2, picture (b) indicates the uncertainty (variance) related to the y state of robot 2, and picture (c) indicates the cross-correlation between the x and y components.</i> .....	139
<i>Figure 5. 26 Picture (a) indicates the uncertainty (variance) related to the x state of robot 3, picture (b) indicates the uncertainty (variance) related to the y state of robot 3, and picture (c) indicates the cross-correlation between the x and y components.</i> .....	139
<i>Figure 5. 27 Inversely proportional decrease in the error covariance values of robot's 1 x- component to the number of robots in the team.</i> .....	141
<i>Figure 5. 28 Inversely proportional decrease in the error covariance values of robot's 1 y- component to the number of robots in the team.</i> .....	141
<i>Figure 5. 29 The cross-correlation error covariance values of robot's 1 x and y component as the number of robots in the team increases.</i> .....	142
<i>Figure 6. 1 Timeline for a real implementation of Kalman filter's update cycle.</i> .....	145
<i>Figure 6. 2 Timeline for a real implementation of Kalman filter's update cycle with additional intended software delay.</i> .....	146
<i>Figure 6. 3 x coordinates of robot 1 as time passes, obtaining only relative measurements.</i> .....	173
<i>Figure 6. 4 y coordinates of the robot 1 as time passes, obtaining only relative measurements.</i> .....	174
<i>Figure 6. 5 Evolution of the x coordinates of robot 2, obtaining only relative measurements.</i> .....	174
<i>Figure 6. 6 Evolution of the y coordinates of the robot 2, obtaining only relative measurements.</i> .....	175
<i>Figure 6. 7 Evolution of the x coordinates of robot 3, obtaining only relative measurements.</i> .....	175
<i>Figure 6. 8 Evolution of the y coordinates of the robot 3, obtaining only relative measurements.</i> .....	176
<i>Figure 6. 9 Picture (a) indicates the uncertainty (variance) related to the x state of robot 1, picture (b) indicates the uncertainty (variance) related to the y state of robot 1, and picture (c) indicates the covariance between the x and y components.</i> .....	176
<i>Figure 6. 10 Picture (a) indicates the uncertainty (variance) related to the x state of robot 2, picture (b) indicates the uncertainty (variance) related to the y state of robot 2, picture (c) indicates the covariance between the x and y components.</i> .....	177
<i>Figure 6. 11 Picture (a) indicates the uncertainty (variance) related to the x state of robot 3, picture (b) indicates the uncertainty (variance) related to the y state of robot 3, picture (c) indicates the covariance between the x and y components.</i> .....	177
<i>Figure 6. 12 Time required by the c-EKF to execute 100 iterations as the number of robots in the team increases gradually.</i> .....	179
<i>Figure 6. 13 Time required by the distributed extended Kalman filter to execute 100 iterations as the number of robots in the team increases gradually.</i> .....	179
<i>Figure A. 1 Model of random signal and Measurement process.</i> .....	192
<i>Figure A. 2 Optimum recursive estimator.</i> .....	194

*Figure B. 1 Alternative Kalman filter recursive loop. .... 200*

*Figure C. 1 Receiver circuit of the muRata MA40S5 ultrasound transceiver. .... 204*

*Figure C. 2 Activation control circuit of the three muRata MA40S5 receiver circuits. .... 205*

*Figure C. 3 Transmitter circuit of muRata MA40S5 ultrasound transceiver. .... 206*



# Chapter 1

## Introduction and Problem Statement

Mobile robots are fast becoming one of the most prominent applications of robotics. They have moved from the factory floor and are now being sent on missions to other planets [35], remote areas [87], or dangerous radioactive sites [16]. The potential applications for mobile robots do not include only special missions. They are also being used as guides in museums [13], [81] and for entertainment purposes [36]. In order for a mobile robot to travel from one location to another, it has to know its position at any given time. In most cases today, this is achieved by either having a human in the navigation loop [15] who directs the vehicle remotely or by constraining the robot to operate in a certain area, precisely mapped [19], [81] or suitably engineered; i.e. marked with beacons [47] or other artificial landmarks [62]<sup>1</sup>. The level of autonomy depends predominantly on the ability of the robot to know its exact location using minimal *a priori* information about the environment, represented in a simple way.

In order for a robot to navigate effectively, it must be able to quickly and accurately determine its location. Fairly accurate position estimates can be obtained by integrating kinetic information from the robot's *proprioceptive* sensors (dead-reckoning). The error accumulation in these estimates when traveling over long distances can lead to unacceptable performance. An effective way of observing the surroundings when the robot is uncertain of its position estimates is by focusing on distinguishing characteristics of the environment such as landmarks. A landmark is defined as a feature (or a combination of features) of the environment that the robot's *exteroceptive* sensors are capable of detecting. When a landmark is sensed, the robot can estimate its own pose<sup>2</sup> by invoking information regarding the feature's pose<sup>3</sup> with respect to some global frame. Two poor assumptions usually invoked by existing localization schemes are that: (i) the world is populated with distinct landmarks, and (ii) these can be sensed at all times. Finally, the increasing need for robots working as teams has created the demand for algorithms supporting cooperative localization

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<sup>1</sup> From now on the words landmark and feature will be used interchangeably.

<sup>2</sup> As *pose* we define the position and orientation of the robot. For example, in case of a robot moving on a 2-D plane its position is determined by the pair  $(x, y)$  where  $x$  is the displacement along the  $x$ -axis and  $y$  is the displacement along the  $y$ -axis with respect to a global frame of reference. The orientation is usually denoted as  $\theta$  and it describes the cumulative rotation of the robot with respect to the global frame of reference. The pose vector combines the position and orientation information in a single *pose* vector.

<sup>3</sup> A known feature does not always provide information regarding all three degrees of freedom  $x, y, \theta$ . For example, stellar objects (e.g. the sun or stars), the magnetic pole of the earth, the gravitational center of the earth, the horizon, or objects in the horizon (e.g. mountain peaks) can be used as landmarks that provide only attitude information while the rest of the positional degrees of freedom remain undetermined.

of groups of mobile robots. Failure to address the information inter-dependencies issue has led to schemes with unacceptable limitations: (i) At least one member of the group has to remain stationary at all times, and (ii) visual contact between the stationary robot and the rest of the group has to be sustained at all times.

## 1.1 Assumptions Made in this Thesis

In this thesis we study the case of mobile robots that theoretically carry a variety of *proprioceptive* sensors that monitor the motion of the vehicle. These sensors can be accelerometers, gyroscopes, wheel-shaft encoders, optical flow odometric systems, Doppler radars, or a combination of the above in an *inertial navigation system* (INS). These devices are commonly found as parts of dead-reckoning systems where the position of the robot is tracked by integrating the kinetic information. The signals from these sensors contain components of noise. In our case, the assumption made is that the noise is Gaussian with zero-mean (white process).

We also consider a robot equipped with *exteroceptive* sensors that monitor the environment for features. These sensors are usually sonars, laser scanners, cameras, sun sensors or star trackers. There are two kinds of *exteroceptive* measurements that we exploit for localization purposes and each of them provides a different level of localization information: (i) Landmarks for absolute position estimation and (ii) for the case of groups of robots, each member of the group carries relative *exteroceptive* sensors for detecting other robots of the same team and measures the relative distance and bearing between each other. A sensor with such capability is described in Chapter 3 more analytically. These robots are also assumed to be capable of communicating with each other when required.

One last but crucial assumption made in this thesis is that all robots' initial position and state is known. This assumption lifts the critical burden to initialize the robots *pose* with respect to their surrounding environment and gives the developed localization algorithm the capability to demonstrate its full potential.

## 1.2 Problem Statement

This thesis addresses the following 3 problems:

- **Problem 1 – Single robot localization problem:** Given a number of known and identified landmarks in space, we need to determine the optimal, in the minimum mean-square error sense, sensor fusion scheme that combines all the collected information, previous and current, to accurately track the robot's sequential *poses* when it is moving freely in a rectangular room with known dimensions.
- **Problem 2 – Multi-robot localization problem:** The optimal cooperative localization scheme, in the minimum mean-square error sense, needs to be determined. It must combine previously related (non-independent) position information from different robots with current relative and absolute pose measurements and compensates for the existing data inter-dependencies without requiring that (i) at least one of the robots is stationary at any time, (ii) all robots collect absolute measurements at all times, and (iii) all the robots sustain visual contact with each other.
- **Problem 3 – Localization algorithm parallelization problem:** In case of groups of mobile robots, we need to take advantage of the local processing power provided by each of the available robots in a team and parallelize the multi-robot localization algorithm developed for Problem 2 by equally distributing it among the robots. Furthermore, the modularity of the algorithm is addressed by examining the problem in which a robot loses contact (no measurements and no communication) with the other robots from the group ("*lost robot*" problem) and after a time period reinserts itself in the team by reestablishing contact with the other robots.

This thesis presents the following approaches for solving each of the previously stated problems:

- **Approach to Problem 1:** In the single robot localization scheme, we need to efficiently estimate the robots' sequential positions by fusing the *proprioceptive* and *exteroceptive* sensor information into a single algorithm to obtain a *pose* estimate update at every time instant. In order for a single robot to localize itself, it needs to keep constant track of its rotational speed signals provided by the Inertial Navigation System (INS). We have assumed that these signals are corrupted by white Gaussian noise, thus the resulting *pose* estimates degrade with time. Every time at least one absolute measurement is collected from a known and identified landmark point in space by *exteroceptive* sensors capable of extracting distance and bearing information from the landmark. These two kinds of information are then processed and fused into an Extended Kalman Filter (EKF) configuration, which provides the minimum mean-square error estimates of the sequential robot *poses*.

- **Approach to Problem 2:** This problem is addressed by performing collective localization in a group of robots, which is viewed as one entity – “group organism” – with multiple “limbs” (the individual robots in the group) and multiple virtual “joints” visualized as connecting each robot with every other member of the team. This leads us to adopt a *centralized* extended Kalman filter estimator, which accommodates the dynamic states of all available robots in the group and combines the independent robot measurements and cross-correlation interdependencies into a single *centralized* form. Thus, a single *centralized* filter is responsible for estimating the robots *poses* at every filter update.
- **Approach to Problem 3:** When groups of two or more robots are sensing each other and exchanging information in order to improve their localization accuracy, the information exchanged during a previous meeting can seriously affect the validity of their estimates. It can become unrealistically over-optimistic. The information interdependencies could be compensated by the *centralized* Kalman estimator, developed for Problem 2, which would combine all the information collected by all the robots at all times. Since such an estimator has very large communication and computational requirements, it is desired to formulate this *centralized* estimator so that it can be distributed amongst the robots. The resulting scheme requires communication only when the robots require updating their previous pose estimates. Furthermore, this distributed scheme is an ideal solution for the “*lost robot*” problem since each of the robots handles the Kalman equations associated only with it at all time.

## 1.3 Research Contribution

The proposed research describes a new approach to multi-robot localization using Kalman filtering techniques. This work has scientific contributions to both extended Kalman filtering algorithms and multi-robot localization areas. In the field of extended Kalman filtering, the authors provide a new approach to filter parallelism by introducing a method to distribute a *centralized* system, among different agents described by different groups of states. A main assumption in this thesis is that the agents can communicate with each other in order to exchange vital information that relates their dynamic states. The proposed fully decoupled algorithm allows the semi-asynchronous operation of each agent, which requires synchronization only when a full system update occurs. This method overcomes problems of previous *decentralization* approaches where a remote *centralized* master filter is needed to optimally combine the state vector estimations of every

individual agent [34], [59], or where the *centralized* full state vector is required to be maintained by all agents [69], [12]. The theoretical implementation of this method in a localization scenario for a group of three mobile robots is analytically explained in Chapter 6.

In the area of mobile robot-group localization, it provides important contributions to both hardware and software. Chapter 3 describes a novel ultrasound sensor arrangement, which can provide a team of robots with 2D relative observation capabilities. This arrangement uses reflective metallic cones to disperse and receive ultrasound pulses in 360°. Such an inexpensive sensor can outperform previous methods that are capable of collecting only 1D relative measurements [63], or exhibit non-linear characteristics in short and long distances [45]. Furthermore, an inexpensive and simple sensor, similar to the proposed ultrasound arrangement, can successfully replace their much more complex and expensive camera counterparts [78], with only reduced loss in measurement accuracy.

Finally, Chapter 5 validates a collective multi-robot localization scenario where a centralized extended Kalman filter is implemented in a group of three mobile robots to simultaneously estimate their position and orientation in a global frame of reference. These estimations are updated using relative and absolute 2D measurements collected from moving robots in the group and stationary landmarks in space. Moreover, it presents an observability case study as well as simulation results on the reduction of the estimation error when more robots are added in the *centralized* localization algorithm. The work of this chapter has been published in [48], which supports the novelty and significance of the obtained results.

## 1.4 Thesis Outline

This thesis is composed of seven chapters. Chapter 2 presents a literature review of the current state-of-the-art in single and multi-robot localization research. Chapter 3 examines a range of hardware sensors that can be used to equip a real-robot implementation of the proposed localization architecture. In addition, we present a solution utilizing an ultrasound sensor for the purposes of measuring relative distance and bearing between robots. The sensor is described sufficiently with a mathematical noise model. Chapter 4 presents the linear and non-linear Kalman filter algorithms and examines the implementation of an Extended Kalman Filter (EKF) algorithm in a single robot scenario using stationary landmark measurements to update the robot's position and orientation estimate. In Chapter 5 we extend the single robot localization problem to a multi-robot scenario using a centralized Extended Kalman Filter (c-EKF) algorithm. The robots can only rely on relative distance and bearing measurements collected between them to estimate their



position and orientation states. The effects of external stationary landmarks in the c-EKF are explored, as well as the reduction of the estimated error as the number of robots in the group increases linearly. Chapter 6 examines the distribution of the c-EKF algorithm amongst the robots in the team in order for the system to exhibit parallelism. We also investigate how a robot can be isolated from the rest of the team and after a period of time to reinsert itself into the group without causing any system degradation. Finally, Chapter 7 presents a conclusion to this thesis as well as a discussion, which sets the ground for future work possibilities.