Mark Burgin

Editor

Topics in Integration Research

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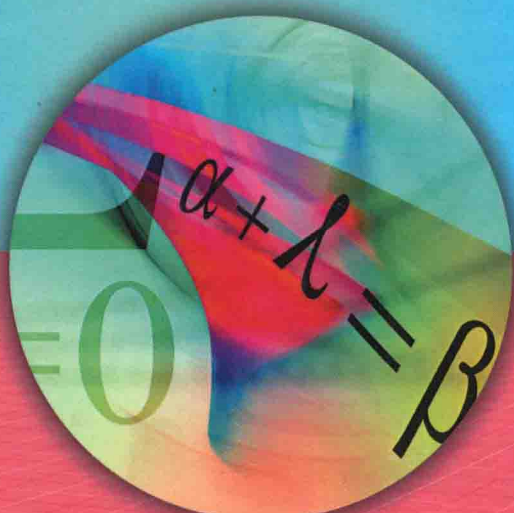

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PREFACE

In calculus, we integrate functions using two types of integration – definite integration and indefinite integration. In functional analysis, we integrate operators. To find a solution of a differential equation, we integrate this equation. Going beyond mathematics, we see that in databases, we integrate data, as well as database schemas. In electronics, integrated circuits have become central components of computers, calculators, cellular phones, and other digital appliances, which are now inextricable parts of the structure of modern societies. In economics, we have integration of the economy of one country into the economy of a union of other countries, e.g., integration of economy of Hungary into the European Union economy. There is political integration and there is social integration. Thus, we can see many types and kinds of integration. Design of complex database schemas is based on a gradual integration of external schemas.

In the most general sense, *integration means making the whole of some entities.*

For instance, definite integration in mathematics is an operation in which values of a function, e.g., numbers for a numerical function, are converted to or represented by one value, e.g., by one number.

Research presented in this book studies integration in mathematics and its applications. However, it is not only classical integration of functions but also fuzzy integration, integration of structures, probability as integration of random characteristics and integral operators in bundles with a hyperspace base. In each part of the book, chapters are ordered according to the alphabetical order of the authors' names.

In the first part of the book, theoretical problems of integration are studied.

Numerical integration is an important area, which has numerous applications in science, engineering and economics. In Chapter 1, B.S. Bhadauria and Ashok Singh study numerical integration based on a cubic spline interpolation formula. In their approach, the integration domain is divided into sub-intervals of unequal widths. This situation is typical when values of the function are obtained in experiment or observation. The suggested schema of numerical integration allows achieving higher accuracy and efficiency in comparison with other similar techniques.

Another area of active research is fuzzy integration. In Chapter 2, Slavka Bodjanova and Martin Kalina study fuzzy integration of real functions based on the concepts of a T-evaluator of a given t-norm and an S-evaluator of a given t-conorm, which were introduced in their previous publications. This allows the authors to treat fuzzy integrals as evaluators of Borel-measurable functions. This type of function is often used in soft computing for modeling vague concepts by fuzzy sets. A generalization of the Choquet integral from (Greco, et al.,

2011) is extended to the Sugeno integral and several types of level-dependent Sugeno integrals are introduced. In addition, the authors build four generalizations of the Choquet integral: the lower Sugeno integral, the upper Sugeno integral and the proper Sugeno integrals of type I and of type II. Conditions under which the Sugeno integral and its level-dependent modifications satisfy properties of T-evaluators and S-evaluators are described. The concepts of a strong T-evaluators and S-evaluators are introduced, studied and linked to domination of aggregation functions. It is demonstrated that strong T-evaluators are T-evaluators and strong S-evaluators are S-evaluators. However, there are T-evaluators and S-evaluators that are not strong.

Chapter 3 by Slavka Bodjanova and Martin Kalina is devoted to level-dependent modifications of Sugeno integral studied in the previous chapter. The level-dependent Choquet integral was proposed in (Greco, et al., 2011). Here this idea is extended to the Shilkert integral using the concept of a generalized measure. The authors introduce three types of Shilkret integrals: two of them are based on a generalized measure and one is based on a generalized capacity. It is demonstrated that the level-dependent Sugeno integral proposed in (Mesiar, et al., 2008; 2009) is the upper Sugeno integral studied in the previous chapter. In addition, the properties of generalized measures and generalized capacities, under which a particular level-dependent Sugeno integral is a T- evaluator or/and S-evaluator, are obtained.

Structure integration is an operation when a system of structures is converted (integrated) into one structure. In Chapter 4, by Mark Burgin, structure integration is defined and studied for named sets, as well as for their sequences, arrays and chains. It is important to study operations with named sets because it is proved that a *named set* (a *fundamental triad*) is the most fundamental structure in mathematics, physics and other sciences (Burgin, 2011). Named sets, as basic mathematical structures, are explicitly or implicitly used for building virtually all other mathematical constructions. One type of such constructions is presented by sequences, arrays and chains of named sets, which have many useful applications in mathematics and beyond. Mathematicians utilize mostly set-theoretical named sets in the form of relations, graphs, functions and functionals, as well as named set sequences in the form of exact sequences, chain complexes, cochain complexes and logical inference. People often meet named sets and their chains in everyday life.

There are different types and kinds of structure integration. The main concern of Chapter 4 is invertible integration of set-theoretical named sets and their sequences, arrays and chains, i.e., such integration the result of which allows one to reconstruct the initial structures. Two types of invertible structure integration are explored: relation invertible structure integration and complete invertible structure integration. The first type of the studied integration allows reconstruction of all ties in the initial structure, e.g., in a named set sequence or in a named set array. The second type of the studied integration allows reconstruction of the whole initial structure (named set sequence or named set array). It is demonstrated that many properties, such as normality, functionality or disconnectedness, of named sets, their chains and arrays are preserved by invertible structure integration. Other properties of structure integrals of named set sequences, arrays and chains are also obtained.

The next chapter further develops integration of extrafunctions. In comparison with conventional functions, extrafunctions provide more powerful means for solving differential equations, exploration of random processes and building models of physical phenomena (Burgin, 2012). All applications need operations with extrafunctions. In many cases, it is possible to take an operation with ordinary functions and to perform similar operations with extrafunctions applying these functional operations separately to each coordinate. Operations performed in this manner are called *regular*. It is proved that it is possible to extend several operations with functions to regular operations with extrafunctions. Examples of such operations are addition of real functions and multiplication of real functions by numbers. However, there are operations with functions the extension of which by coordinates does not work because their application is not invariant with respect to representations of extrafunctions. One of such operations is integration, which is important for calculus, differential equations and many applications. The goal of Chapter 5, by Mark Burgin, is to improve this situation by developing a method of regularization of irregular operations. Then this method is applied to integration. The main constructions are put together in the context of fiber bundles over hyperspaces of integral vector spaces and integral algebras.

In Chapter 6, Maslina Darus and Rabha Ibrahim define and study new families of differential and integral operators, as well as classes of functions defined by these operators. The authors find various properties of these classes of functions and families of operators.

In Chapter 7, Issidore Fleischer gives a very compressed description of an abstract theory of integrals developed by him without explicit assumptions about the integration domain. His schema of integration encompasses several known approaches to integration.

Different authors have studied satisfiability of the conditions from the Saks-Henstock lemma. This lemma, actually, a theorem, tells us when it is possible to build Henstock integral or MacShane integral using primitive functions. In Chapter 8, Toshiharu Kawasaki demonstrates that every Henstock-Kurzweil integrable mapping that takes values either in a complete topological Hausdorff vector space X or in a complete locally pseudoconvex space X satisfies the conditions from the Saks-Henstock lemma if and only if every unconditionally convergent series in X is absolutely convergent.

In the second part of the book, applications of integration in mathematics are studied.

In Chapter 9, Mark Burgin studies integration of nonlinear partial differential equations (PDE). It is known that it is impossible to find solutions in the set of differentiable functions for many important differential equations. In the development of the theory of differential equations, this impossibility, at first, led to the introduction of weak solutions of differential equations, and then to the creation of a more general theory – the theory of generalized functions or distributions. The development of distribution theory immensely expanded the scope of solvable partial differential equations. Nevertheless, this did not completely solve the problem, since many partial differential equations (some of them are linear and very simple) cannot have solutions in the set of distributions.

At the same time, as it is demonstrated in (Burgin and Ralston, 2004) and in Chapter 9, the theory of extrafunctions allows one to find generalized solutions to a much larger set of nonlinear PDE in comparison with distributions. In a sense, the approach developed here follows the traditional method of solving PDE by series of functions originated with Newton and Leibniz. This succeeds because in spaces of extrafunctions, all series of ordinary real or complex functions are convergent. Thus, finding a solution in the form of a formal series allows one to have the solution in the form of extrafunctions for the same equation or system of equations. Extrafunction solutions have many advantages because properties of extrafunction spaces are better than properties of spaces of formal series, e.g., extrafunction spaces have better topology.

Chapter 9 consists of two parts. In the first part, multiplication for hypernumbers and extrafunctions is defined by extending the system of hypernumbers to the system of **E**-hypernumbers. We need to do this because there is no regular ways to define multiplication for hypernumbers and extrafunctions. At the same time, there is another way to build multiplication for hypernumbers and extrafunctions based on the regularization technique developed in (Burgin, 2010) for integration and differentiation of extrafunctions.

Introduction of multiplication provides a possibility to study nonlinear transformations and nonlinear dynamical systems for hypernumbers proving extended forms of an abstract nonlinear Cauchy-Kowalewski theorem in the second part of Chapter 9, establishing existence of solutions for very general classes of nonlinear PDEs. There are several reasons why we need general theorems of the Cauchy-Kowalewski type about existence and/or uniqueness of solutions for differential equations.

First, general theorems of existence have always been of great interest in mathematics as they provided unifying properties of mathematical objects. One of the first theorems of this kind is the theorem about existence of infinitely many prime numbers. Very often mathematicians achieved existence of solutions for some classes of problems by extending the class in which such solutions are looked for. After obtaining a general existence theorem for an extended class of objects, it is possible to investigate when such solutions belong to a definite more restricted class with better properties.

Second, now computers are used for a search of solutions of many differential equations. If we know that a solution exists, then the computational search is grounded. Without this knowledge, such search looks, for example, like the search for solution in radicals for algebraic equations of the fifth degree, while we know such solutions do not exist as Abel and Galois proved. In addition, proofs of general theorems of existence and/or uniqueness of solutions are often constructive, giving methods of approximate solution for these differential equations. Thus, general theorems of existence and/or uniqueness of solutions form foundations for numerical solutions of differential equations. Utilization of existence/uniqueness theorems helps to eliminate misconceptions and wrong conclusions.

Third, now the qualitative theory of differential equations is an important mathematical field with many applications in physics. As we know, only few differential equations have explicit solutions. However, this is not an essential problem for applications. As wrote Poincare (1908), an engineer does not really need an explicit solution of a differential equation but its essential properties. However, it is possible to study properties of solutions only if these solutions exist. Consequently, general theorems of existence and/or uniqueness of solutions form foundations for the qualitative theory of differential equations. Only existence of solutions can ground correct application of qualitative methods. History knows many situations when scientists discussed non-existing physical entities (for example, non-existing elementary particles or flogiston).

The class of fractional operator equations and inclusions of various types plays a significant role not only in mathematics but also in physics, control systems, dynamical systems and engineering. Thus, the problem of solving such equations and inclusions is important for all these areas. In Chapter 10, Rabha Ibrahim establishes existence of solutions for integral inclusion of fractional order for set-valued (multi-valued) function with convex and non-convex values. Proofs are based on fixed points theorems. The developed approach and obtained results are illustrated by examples.

In Chapter 11, Karunakaran and Bhuvaneswari study integral characteristics of analytic functions in the open disc U of the complex plane.

The classical theory of integral transforms on function spaces is a well established area in mathematics. Recently, this theory has been extended to generalized function spaces, such as Boehmian spaces and generalized quotient spaces. In Chapter 12, Karunakaran and Chella Rajathi introduce and study a new Boehmian space is in the context of the Laplace transform in this space. Besides extending the classical results, this theory unifies the various theories of Laplace transforms developed for classical functions, generalized functions and generalized quotient spaces.

James (1964) introduced nonsquare constants characterizing normed linear spaces. Different authors have studied these characteristic for different spaces. In Chapter 13, Z.D. Ren studies normed linear spaces and their important special case - the Orlicz sequence spaces equipped with the Orlicz norm - and obtains (Theorem 1) more exact than previously known boundaries for the nonsquare constants of Orlicz sequence spaces.

Mikusiński and Mikusiński (1981) introduced Boehmians as a generalization of distributions by regular operators. Later an abstract construction of a Boehmian space was given in (Mikusiński, 1983) with two notions of convergence. Thereafter various Boehmian spaces have been defined and various integral transforms have been constructed in these spaces. In Chapter 14, R. Roopkumar constructs two suitable Boehmian spaces and defines the Stieltjes transform of a Boehmian as another Boehmian. He proves that the extended Stieltjes transform is consistent with the distributional Stieltjes transform and satisfies many conventional properties, such as linearity, bijectivity and continuity.

In the third part of this book, integration of random process characteristics is studied.

Random phenomena have interested people from the earliest times. Their fundamental nature has been a point of attention for philosophy and the methodology of science, while their intrinsic regularities have been brought to light by mathematics, statistics and empirical sciences. Consequently, regardless of the difficulty in rigorously pinpointing the causes and nature of chance/randomness, there has long been a strong endeavor to find the regularities inherent in them, to describe them in mathematical terms and to analyze them quantitatively. This is what is done by probability theory and mathematical statistics, which are vast and still expanding fields that are mathematically oriented and closely linked to a diverse number of different applications.

Probability theory constructs and analyzes general models of random phenomena, whereby probability is an integral characterization of a process giving a quantitative measure for the potential of specific random events occurring in this process. Taking probability as the basic integral characteristic, the further development involved integrals, which are used to obtain mathematical descriptions of random phenomena, such as: expectation, variance and standard deviation of random variables.

Building his system of axioms for probabilities, Renyi (1955) did this for conditional probability, taking it as the basic concept. In a similar way, some authors took conditional probability to be the primitive notion, and axiomatized it directly (cf., for example, (Popper, 1959; van Fraassen, 1976; Spohn, 1986; Roeper and Leblanc 1999)). In addition, Mušicki-Kovačević (1986) studied nonstandard conditional probabilities. In Chapter 15, Mark Burgin and Alan Krinik introduce and study conditional hyperprobabilities based on the theory of hypernumbers and extrafunctions (Burgin, 2012). After explaining some constructions from the theory of hypernumbers and extrafunctions, the concept of hyperprobability is introduced in a theoretical axiomatic framework, as well as in a practical frequency interpretation setting. While probabilities take values as real numbers from the interval [0, 1], hyperprobabilities take values as real hypernumbers from the interval [0, 1] and it was demonstrated that conventional probabilities determined by Kolmogorov axioms form a special class of hyperprobabilities (Burgin and Krinik, 2009). To further develop hyperprobability theory, conditional hyperprobabilities are defined, their properties are examined and examples are presented. In Chapter 15, an extended Bayes Theorem for conditional hyperprobabilities is proved and various properties of conditional hyperprobabilities are examined. At the end, additional examples of random events and random variables having conditional hyperprobabilities are considered.

The next chapter also extends the concept of probability but in a different direction. By its classical definition, probability is a positive function. In spite of this, many prominent physicists, such as Paul Dirac, Werner von Heisenberg, Eugene Wigner, and Richard Feyman, as well as some economists have successfully used negative probabilities in their research. At the same time, history of science shows that physicists and economists often apply concepts that are practically validated but mathematically ungrounded. The same happened to negative probability. That is why in Chapter 16, Mark Burgin and Gunter Meissner develop mathematical foundations for negative probabilities in the form of an axiomatic system. The second goal of this chapter is extending applications of negative probabilities to problems in finance. According to this orientation, at first, an axiomatic system for extended probabilities, which include both classical probabilities and negative probabilities, is constructed and various general properties of negative probabilities and extended probabilities are derived. In particular, it is proved that when the extended probability takes only non-negative values, it coincides with the classical probability. Then it is demonstrated how negative probabilities can be utilized for modeling financial options such as swapoptions. In trading practice, these options are typically valued in a Black-Scholes-Merton framework assuming a log-normal distribution for the underlying interest rate. However, in some cases, such as the 2007/2008 financial crisis, interest rates can get negative. Then the log-normal distribution becomes inapplicable. To remedy this situation, negative probabilities are applied to evaluation of interest rate options in a log-normal framework, providing an adequate model for negative interest rates.

In Chapter 17, J. Gani and Randall Swift consider natural cell mutation or assisted mutations that are caused by external factors such as chemicals or plasmids. They begin by reviewing deterministic models of each type of mutation as originally put forward by Luria and Delbrück's (1943). These models are next considered in a stochastic setting by assuming the previous deterministic variables are now continuous-time Markov chains. Gani and Swift solve Komogorov's forward equation for the probability generating function of these processes by solving the associated partial differential equation. The authors obtain an explicit formula for this probability generating function which has the interesting form of a product of probability generating functions of Poisson and negatively binomial distributions. Using their general expression for this probability generating function, the authors deduce important information about $X(t)$ and $Y(t)$. In the last section of their chapter, Gani and Swift turn their attention to the stochastic assisted cell mutation model.

Chapter 18 is written by Lilinoe Harbottle, Blake Hunter and Alan Krinik with both mathematical and pedagogical objectives in mind. The authors present a systematic approach to determine the transient probability functions of the general, three or four state irreducible

Markov process. The form of the transient probability functions depends upon the discriminate of the system's characteristic equation which, in turn, depend upon the infinitesimal state transition rates. Expressions for the steady state distribution are also derived and appear as part of the general transient probability formulas.

It is well known that sequential hypothesis test procedures can have appreciable cost savings compared to fixed sample size test plans. The first sequential hypothesis procedure was developed by Wald for one-parameter families of distributions and later extended by Bartlett to handle the case of nuisance parameters. However, Bartlett's procedure requires independent and identically distributed observations. In contrast to this, it is common for data in ecological applications to exhibit spatial correlations. The goal of Judy Li, Daniel Jeske, Jesús Lara and Mark Hoddle in Chapter 19 is to show how to incorporate the existence of spatial correlation into the sequential hypothesis testing framework so that applications, such as pest management, can improve the accuracy of their treat or no-treat decisions. The existence of spatial correlations in pest count data is illustrated by analyzing the spatial structure in a data set of mite counts.

In the fourth part of the book, general applications of integration are studied.

In Chapter 20, Yuichiro Kakihara, studies various topologies on the set *CC* of information channels and on the set *CO* of channel operators. Here an information channel is a function from the direct product of two Baire σ–algebras into the interval [0, 1]. Information operators are specific positive bounded linear operators between linear spaces of Baire measures. The space *CC* of all information channels is regarded as a subspace of the space *CO*. Kakihara explores basic topological properties, such as continuity of channels and operators, of these two spaces *CC* and *CO*, as well as of their subsets. In particular, approximations of operators from *CO* by sequences of channels from *CC* with respect to the pointwise weak* topology is considered.

In Chapter 21, Vladimir Lerner analyzes recent results for the entropy functional on trajectories of a controlled diffusion process and presents a base of the information path functional (IPF) approach. In this approach, the information path functional of an observed controllable diffusion process is analyzed, considering the IPF variation principle, the extremal equations, the invariant relations, and the identification of the process' information macrodynamic model. The author formulates a variation problem allowing a *dynamic* approximation of the entropy functional by the corresponding information path functional. An illustration of the IPF variation problem solution is given using both the Kolmogorov equation for the functional of a Markov process and the Jacobi-Hamilton equation for the IPF dynamics. The author studies the IPF extremal equations and the invariant relations oriented at solving important mathematical and applied problems in information macrodynamics. The information path functional, which is similar to the Feynman path integral, allows evaluating the information content of a random process, building its dynamic macromodel, the model cooperating information network, and revealing the process's information code. Studying the IPF connection to irreversible thermodynamics and the observed physical parameters allows employing the IPF approach to a variety of complex objects with superimposing processes. The information path functional is also applied to establishing an information analogy between the Schrödinger equation and equations of informational macrodynamics, which define an information wave function. This function is applied to problems of the financial markets in (Choustova, 2009; Haven, 2009).

Structural plate systems stiffened by beams are widely used in buildings, bridges, ships, aircrafts and machines due to the economic and structural advantages of such systems. Stiffening of the plate is used to increase its load carrying capacity and to prevent buckling especially in case of in-plane loading. Stiffened reinforced concrete plate structures are efficient, economical and functional, while construction using precast beams is the quickest, most familiar and economical method for long bridges or for long span slabs. The extensive use of plate structures demands their rigorous analysis. Consequently, many researchers studied these problems.

In Chapter 22, by Sapountzakis, an improved model for the static analysis of reinforced concrete plates stiffened by arbitrarily placed parallel reinforced concrete or steel beams with deformable connection taking into account the influence of creep and shrinkage effects relative with the time of the casting and the time of the loading of the plate and the beams is presented. According to the proposed model, the stiffening beams are isolated from the plate by sections in the lower outer surface of the plate, making the hypothesis that the plate and the beams can slip in all directions of the connection without separation (i.e. uplift neglected) and taking into account the arising tractions in all directions at the fictitious interfaces. These tractions are integrated with respect to each half of the interface width resulting two interface lines, along which the loading of the beams as well as the additional loading of the plate is defined. Their unknown distribution is established by applying continuity conditions in all directions at the interfaces taking into account their relation with the interface slip through the shear connector stiffness. The utilization of two interface lines for each beam enables the nonuniform distribution of the interface transverse shear forces and the nonuniform torsional response of the beams to be taken into account describing better in this way the actual response of the plate - beams system. The analysis of both the plate and the beams is accomplished on their deformed shape taking into account second-order effects. Six boundary value problems are formulated and solved using the Analog Equation Method (AEM), a BEM based method. The solution of the aforementioned plate and beam problems, which are nonlinearly coupled, is achieved using iterative numerical methods. The adopted model describes better the actual response of the plate - beams system and permits the evaluation of the shear forces at the interfaces in both directions, the knowledge of which is very important in the design of prefabricated ribbed plates. Numerical examples of practical situations are presented.

To conclude, it is necessary to remark that this volume is based on papers published in the second volume of the international journal *INTEGRATION*: *Mathematical Theory and Applications*. Some of these papers were updated by the authors. Besides, a part of this book is based upon research presented at the organized by Mark Burgin and Alan Krinik Special Session on *Mathematical Models of Random Phenomena* at the Sectional Meeting of the American Mathematical Society (AMS) in Los Angeles, October 9-10, 2010. Finally, it is necessary to thank all the authors for contributing and all reviewers, who worked hard to ensure the high quality of this book.

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