

LARGE STRAIN



finite element method

A
PRACTICAL
COURSE

Antonio Munjiza, Esteban Rougier and Earl E. Knight



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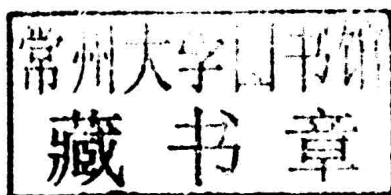
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LARGE STRAIN FINITE ELEMENT METHOD

Antonio Munjiza would like to dedicate this book to Jasna and Boney.

*Esteban Rougier would like to dedicate this book to his wife Sole
and to his sons Ignacio and Matias.*

*Earl E. Knight would like to dedicate this book to the love of his life,
his best friend and confidante, Cheryl Marie.*

Preface

The conventional finite element method is based on the assumption that structural system displacements under load are small and that the structural material does not stretch much under that load. Arguably, the small strain, small displacements-based finite element method is not of much use in modern scientific, engineering and technological applications. Even in classic structural engineering applications, the conventional finite element method is hardly applicable. This shift has occurred because design codes and standards have changed in recent years to include the ultimate limit state, i.e., considerations of structural collapse. As a consequence, one now has to consider both large strains (plastic strains) and large displacements. In other state of the art applications of the finite element method, finite element simulations are increasingly becoming an integral part of the so-called virtual experimentation, examples of these are biological, medical science, material science, process engineering, military and many other applications of the finite element method. In these applications the finite element simulation has to reproduce reality (as opposed to approximating reality), together with possible emergent properties such as flow, damage, failure, collapse, yield, etc.

In this context, not even the higher order theories and their finite element realizations are suitable representations of the physical realities involved. The answer is an exact formulation that encompasses an exact representation of large displacements, large strains, and material properties including anisotropy. Such a theory, when implemented in a finite element software package, must cover 2D solids, 3D solids, and 2.5D shell and membrane static and dynamic simulations.

Theoretical aspects of these formulations were resolved in the 1960s and 1970s. The finite element adaptation of these theoretical formulations has mostly taken place during the 1990s and early years of the 21st century. This work has resulted in a large body of scientific papers that have described it as the next generation of finite element packages. Nevertheless, the subject has remained a mystery for undergraduate students, postgraduate students, practicing engineers and scientist and even for users and developers of finite element software.

This book is written with the key objective of “demystifying” the subject, making it easy for students, engineers and software developers to master the minute details of the finite element method that incorporates large strains, large displacements, and material nonlinearity.

The book is written in such a way that it provides a pathway to master all the method's related subjects starting with matrices, systems of equations, scalar and vectors and progressing onto tensors of the first order, and tensors of the second order. With this knowledge base in hand, the book provides an engineering-based approach to deformation kinematics that avoids the often confusing mathematical jargon yet concentrates on the physics and uses mathematics only when necessary. At this stage, the reader is made familiar with a generalized framework for developing large strains based nonlinear material laws. This is done without any reference to the finite element method, having in mind, for example, a material modeler whose job is to solely develop material laws.

Finally, the book presents the large strain large displacement based finite element method including 2D solid, 3D solid, 2.5D membrane, plate and shell problems. These are explained in such detail that they contain all the necessary mathematical equations, algorithms and formulae that can be readily implemented into the finite element method. As such, they should be of great value for developers of finite element packages. They will also provide users of finite element packages with an enhanced understanding of the algorithmic, theoretical and formulation aspects of the finite element software they are using.

The authors hope that the book will ultimately benefit practicing engineers, scientists, undergraduate students, master students and PhD students in diverse fields of related applied subjects.

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Contents

Preface	xiii
Acknowledgements	xv
 PART ONE FUNDAMENTALS	 1
1 Introduction	3
1.1 Assumption of Small Displacements	3
1.2 Assumption of Small Strains	6
1.3 Geometric Nonlinearity	6
1.4 Stretches	8
1.5 Some Examples of Large Displacement Large Strain Finite Element Formulation	8
1.6 The Scope and Layout of the Book	13
1.7 Summary	13
 2 Matrices	 15
2.1 Matrices in General	15
2.2 Matrix Algebra	16
2.3 Special Types of Matrices	21
2.4 Determinant of a Square Matrix	22
2.5 Quadratic Form	24
2.6 Eigenvalues and Eigenvectors	24
2.7 Positive Definite Matrix	26
2.8 Gaussian Elimination	26
2.9 Inverse of a Square Matrix	28
2.10 Column Matrices	30
2.11 Summary	32

3	Some Explicit and Iterative Solvers	35
3.1	The Central Difference Solver	35
3.2	Generalized Direction Methods	43
3.3	The Method of Conjugate Directions	50
3.4	Summary	63
4	Numerical Integration	65
4.1	Newton-Cotes Numerical Integration	65
4.2	Gaussian Numerical Integration	67
4.3	Gaussian Integration in 2D	70
4.4	Gaussian Integration in 3D	71
4.5	Summary	72
5	Work of Internal Forces on Virtual Displacements	75
5.1	The Principle of Virtual Work	75
5.2	Summary	78
	PART TWO PHYSICAL QUANTITIES	79
6	Scalars	81
6.1	Scalars in General	81
6.2	Scalar Functions	81
6.3	Scalar Graphs	82
6.4	Empirical Formulas	82
6.5	Fonts	83
6.6	Units	83
6.7	Base and Derived Scalar Variables	85
6.8	Summary	85
7	Vectors in 2D	87
7.1	Vectors in General	87
7.2	Vector Notation	91
7.3	Matrix Representation of Vectors	91
7.4	Scalar Product	92
7.5	General Vector Base in 2D	93
7.6	Dual Base	94
7.7	Changing Vector Base	95
7.8	Self-duality of the Orthonormal Base	97
7.9	Combining Bases	98
7.10	Examples	104
7.11	Summary	108
8	Vectors in 3D	109
8.1	Vectors in 3D	109
8.2	Vector Bases	111
8.3	Summary	114
9	Vectors in n-Dimensional Space	117
9.1	Extension from 3D to 4-Dimensional Space	117
9.2	The Dual Base in 4D	118

9.3	Changing the Base in 4D	120
9.4	Generalization to n -Dimensional Space	121
9.5	Changing the Base in n -Dimensional Space	124
9.6	Summary	127
10	First Order Tensors	129
10.1	The Slope Tensor	129
10.2	First Order Tensors in 2D	131
10.3	Using First Order Tensors	132
10.4	Using Different Vector Bases in 2D	134
10.5	Differential of a 2D Scalar Field as the First Order Tensor	137
10.6	First Order Tensors in 3D	141
10.7	Changing the Vector Base in 3D	142
10.8	First Order Tensor in 4D	143
10.9	First Order Tensor in n -Dimensions	147
10.10	Differential of a 3D Scalar Field as the First Order Tensor	149
10.11	Scalar Field in n -Dimensional Space	152
10.12	Summary	153
11	Second Order Tensors in 2D	155
11.1	Stress Tensor in 2D	155
11.2	Second Order Tensor in 2D	158
11.3	Physical Meaning of Tensor Matrix in 2D	159
11.4	Changing the Base	161
11.5	Using Two Different Bases in 2D	163
11.6	Some Special Cases of Stress Tensor Matrices in 2D	167
11.7	The First Piola-Kirchhoff Stress Tensor Matrix	168
11.8	The Second Piola-Kirchhoff Stress Tensor Matrix	169
11.9	Summary	174
12	Second Order Tensors in 3D	175
12.1	Stress Tensor in 3D	175
12.2	General Base for Surfaces	179
12.3	General Base for Forces	182
12.4	General Base for Forces and Surfaces	184
12.5	The Cauchy Stress Tensor Matrix in 3D	186
12.6	The First Piola-Kirchhoff Stress Tensor Matrix in 3D	186
12.7	The Second Piola-Kirchhoff Stress Tensor Matrix in 3D	188
12.8	Summary	189
13	Second Order Tensors in nD	191
13.1	Second Order Tensor in n -Dimensions	191
13.2	Summary	200
PART THREE	DEFORMABILITY AND MATERIAL MODELING	201
14	Kinematics of Deformation in 1D	203
14.1	Geometric Nonlinearity in General	203
14.2	Stretch	205
14.3	Material Element and Continuum Assumption	208

14.4	Strain	209
14.5	Stress	213
14.6	Summary	214
15	Kinematics of Deformation in 2D	217
15.1	Isotropic Solids	217
15.2	Homogeneous Solids	217
15.3	Homogeneous and Isotropic Solids	217
15.4	Nonhomogeneous and Anisotropic Solids	218
15.5	Material Element Deformation	221
15.6	Cauchy Stress Matrix for the Solid Element	225
15.7	Coordinate Systems in 2D	227
15.8	The Solid- and the Material-Embedded Vector Bases	228
15.9	Kinematics of 2D Deformation	229
15.10	2D Equilibrium Using the Virtual Work of Internal Forces	231
15.11	Examples	235
15.12	Summary	238
16	Kinematics of Deformation in 3D	241
16.1	The Cartesian Coordinate System in 3D	241
16.2	The Solid-Embedded Coordinate System	241
16.3	The Global and the Solid-Embedded Vector Bases	243
16.4	Deformation of the Solid	244
16.5	Generalized Material Element	246
16.6	Kinematic of Deformation in 3D	247
16.7	The Virtual Work of Internal Forces	249
16.8	Summary	255
17	The Unified Constitutive Approach in 2D	257
17.1	Introduction	257
17.2	Material Axes	259
17.3	Micromechanical Aspects and Homogenization	260
17.4	Generalized Homogenization	263
17.5	The Material Package	264
17.6	Hyper-Elastic Constitutive Law	265
17.7	Hypo-Elastic Constitutive Law	266
17.8	A Unified Framework for Developing Anisotropic Material Models in 2D	267
17.9	Generalized Hyper-Elastic Material	267
17.10	Converting the Munjiza Stress Matrix to the Cauchy Stress Matrix	274
17.11	Developing Constitutive Laws	279
17.12	Generalized Hypo-Elastic Material	288
17.13	Unified Constitutive Approach for Strain Rate and Viscosity	292
17.14	Summary	293
18	The Unified Constitutive Approach in 3D	295
18.1	Material Package Framework	295
18.2	Generalized Hyper-Elastic Material	295
18.3	Generalized Hypo-Elastic Material	299

18.4 Developing Material Models	302
18.5 Calculation of the Cauchy Stress Tensor Matrix	302
18.6 Summary	312
PART FOUR THE FINITE ELEMENT METHOD IN 2D	315
19 2D Finite Element: Deformation Kinematics Using the Homogeneous Deformation Triangle	317
19.1 The Finite Element Mesh	317
19.2 The Homogeneous Deformation Finite Element	317
19.3 Summary	326
20 2D Finite Element: Deformation Kinematics Using Iso-Parametric Finite Elements	327
20.1 The Finite Element Library	327
20.2 The Shape Functions	327
20.3 Nodal Positions	330
20.4 Positions of Material Points inside a Single Finite Element	331
20.5 The Solid-Embedded Vector Base	332
20.6 The Material-Embedded Vector Base	334
20.7 Some Examples of 2D Finite Elements	337
20.8 Summary	340
21 Integration of Nodal Forces over Volume of 2D Finite Elements	343
21.1 The Principle of Virtual Work in the 2D Finite Element Method	343
21.2 Nodal Forces for the Homogeneous Deformation Triangle	348
21.3 Nodal Forces for the Six-Noded Triangle	352
21.4 Nodal Forces for the Four-Noded Quadrilateral	353
21.5 Summary	355
22 Reduced and Selective Integration of Nodal Forces over Volume of 2D Finite Elements	357
22.1 Volumetric Locking	357
22.2 Reduced Integration	358
22.3 Selective Integration	359
22.4 Shear Locking	362
22.5 Summary	364
PART FIVE THE FINITE ELEMENT METHOD IN 3D	365
23 3D Deformation Kinematics Using the Homogeneous Deformation Tetrahedron Finite Element	367
23.1 Introduction	367
23.2 The Homogeneous Deformation Four-Noded Tetrahedron Finite Element	368
23.3 Summary	377
24 3D Deformation Kinematics Using Iso-Parametric Finite Elements	379
24.1 The Finite Element Library	379
24.2 The Shape Functions	379

24.3	Nodal Positions	381
24.4	Positions of Material Points inside a Single Finite Element	382
24.5	The Solid-Embedded Infinitesimal Vector Base	383
24.6	The Material-Embedded Infinitesimal Vector Base	386
24.7	Examples of Deformation Kinematics	387
24.8	Summary	392
25	Integration of Nodal Forces over Volume of 3D Finite Elements	393
25.1	Nodal Forces Using Virtual Work	393
25.2	Four-Noded Tetrahedron Finite Element	396
25.3	Reduce Integration for Eight-Noded 3D Solid	399
25.4	Selective Stretch Sampling-Based Integration for the Eight-Noded Solid Finite Element	400
25.5	Summary	401
26	Integration of Nodal Forces over Boundaries of Finite Elements	403
26.1	Stress at Element Boundaries	403
26.2	Integration of the Equivalent Nodal Forces over the Triangle Finite Element	404
26.3	Integration over the Boundary of the Composite Triangle	407
26.4	Integration over the Boundary of the Six-Noded Triangle	408
26.5	Integration of the Equivalent Internal Nodal Forces over the Tetrahedron Boundaries	409
26.6	Summary	412
PART SIX	THE FINITE ELEMENT METHOD IN 2.5D	415
27	Deformation in 2.5D Using Membrane Finite Elements	417
27.1	Solids in 2.5D	417
27.2	The Homogeneous Deformation Three-Noded Triangular Membrane Finite Element	419
27.3	Summary	438
28	Deformation in 2.5D Using Shell Finite Elements	439
28.1	Introduction	439
28.2	The Six-Noded Triangular Shell Finite Element	440
28.3	The Solid-Embedded Coordinate System	441
28.4	Nodal Coordinates	442
28.5	The Coordinates of the Finite Element's Material Points	443
28.6	The Solid-Embedded Infinitesimal Vector Base	444
28.7	The Solid-Embedded Vector Base versus the Material-Embedded Vector Base	447
28.8	The Constitutive Law	449
28.9	Selective Stretch Sampling Based Integration of the Equivalent Nodal Forces	449
28.10	Multi-Layered Shell as an Assembly of Single Layer Shells	455
28.11	Improving the CPU Performance of the Shell Element	456
28.12	Summary	462
Index		463