Computational Methods in Potential Aerodynamics

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To my mother, Maria A. Corbo.

PREFACE

This volume is based on edited Proceedings of a recent ICTS course, held in Amalfi, Italy, which dealt with Computational Methods in Potential Aerodynamics. The field of Computational Aerodynamics covers many areas in which the methodologies are markedly different. For instance, the integral-equation methods (also known as panel methods or boundary element methods) are widely accepted for linear (subsonic and supersonic) flows whereas the nonlinear (transonic) flows are typically approached by the finite-difference method. Also, different methodologies are used to solve steady-state flows, oscillatory (complex exponential) flows and transient flows. Other differences occur in the formulations for airplanes vs. helicopters (fixed wing vs. rotary wing aerodynamics) and in analysis vs. design. The objective of the course was to bring together widely recognized specialists from all these areas in order to compare and contrast the different approaches to the problems (linear and nonlinear aerodynamics, steady and unsteady flows, fixed wings and rotary wings, analysis and design). The speakers included representatives from industry, government and universities in order to achieve a proper balance between theory and applications, current methodology and future trends.

This volume is divided into seven parts.

I Introduction
II Steady Subsonic and Supersonic Flows
III Unsteady Subsonic and Supersonic Flows
IV Steady Transonic Flows
V Unsteady Transonic Flows
VI Wake Analysis
VII Recent Developments

This division reflects the actual organization of the course. Some of the papers span more than one category. Therefore, it is appropriate to elaborate further on the organization of the material. Part I of the volume is an introduction of the formulation of potential flows. In most of the following papers it is assumed that the reader is familiar with this classical material, which is included here for the sake of completeness.

Part II covers steady subsonic and supersonic flows. The methodology for the computation of these flows is in a mature stage of development. Panel methods are now classical and widely accepted in the aerospace community. Two papers by J L Hess and by E N Tinoco and P E Rubbert cover a historical review as well as recent developments.

Part III, unsteady subsonic and supersonic flows, includes a historical review by E C Yates, Jr, and papers on the three main methodologies in the field, which are all of the integral-equation type. The classical approach, the lifting surface method is covered by W S Rowe. The doublet-lattice method, widely used in industry, is covered by J P Giesing. The Green's function method is covered by M I Freedman; applications are found in the paper by K Tseng in Part V. In the last paper in this group, I tried to show how the above methods are all closely related to the concepts of Green's function and Green's theorem.

The material covered in Part IV, steady transonic flows, has been the subject of major research activity in the past few years and is still under development. The finite-difference method, the standard technique in this area, is reviewed in the paper by D Nixon. Application to analysis and design are presented in the papers by C Boppe, by J W Slooff and by E N Tinoco and P E Rubbert.

Unsteady transonic flows are covered in Part V, which includes an overall review by E C Yates Jr, a paper on finite difference by F X Caradonna (with emphasis on helicopter aerodynamics) and one on integral equations by K Tseng.

Formulations for steady and unsteady, incompressible and compressible, wake analysis are covered in Part VI. The vortex-layer approach is reviewed in the paper by O A Kandil whereas the doublet-layer approach is reviewed in the paper by S R Sipcic and myself; the link between these two formulations is discussed in my paper in Part III.

Finally recent developments are covered in Part VII. A paper by J W Slooff covers integral equation methods for subsonic and transonic flows. A second paper by myself, M I Freeman, D J Deutsch and S R Sipcic presents a new integral-equation formulation for a frame of reference moving in arbitrary motion (with emphasis on helicopter rotors).

In addition, original contributions (presented at the course in the session on Recent and Current Development) are found in the paper by J P Giesing in Part III (section on unsteady transonic flows), in my paper in Part III (appendix on two dimensional supersonic flows) and in the paper by Sipcic and myself in Part VI (appendix on wake generation and trailing edge condition).

It is apparent from the above remarks that this volume is unusual in that equal emphasis is given to steady as well as unsteady aerodynamics. This clearly reflects my own preference, since most of my work in the past twenty years has been in the field of unsteady aerodynamics. However, I believe there are more important reasons to justify this choice. The first and foremost being that introductory books on recent developments in the field of unsteady aerodynamics are not available. These developments may be of interest even to steady-state aerodynamicists: in my experience, the effort required to understand unsteady aerodynamics yields a deeper insight into the steady-state problem as well; furthermore, methodology originally developed for unsteady aerodynamics has found its way in steady-state computer codes, such as PANAIR. Moreover, unsteady aerodynamics is not only a fascinating field of research (eg new development on the interplay between aerodynamics and acoustics of helicopter rotors), but is also an essential tool in aircraft design, aeroelasticity and helicopters in forward flights are only two examples of practical applications. I hope that this volume will help steady-state aerodynamicists appreciate the complexities of unsteady aerodynamics and stimulate renewed interest in this field.

I wish to express my appreciation to Professor Giovanni Lanzara, Director of ICTS, for providing me with the unique opportunity to coordinate the course (in the unforgettable setting of Amalfi) and to prepare this volume, and to Dr Paolo Fadda for the superb organization of the course and the continuous support that I received in completing this volume. I also want to thank the speakers for their enthusiastic response, their outstanding presentations, and their equally outstanding manuscripts. My thanks also go to all people who helped me in preparing this volume, in particular Laura Stockar for her extreme patience in transforming into legible material the manuscripts that I wrote, and to Dr. Carlos A. Brebbia and his associates at Computational Mechanics International for their superb work in producing this volume.

Finally I want to thank my daughters Federica and Francesca for their patience during the preparation of the manuscripts and, above all, my wife Nancy for her understanding and support which were essential for the completion of this volume.

Luigi Morino Boston, 1985

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FOUNDATION OF POTENTIAL FLOWS

by

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ABSTRACT

The fundamental equations governing the motion of perfect fluid are presented in this section. Specifically, continuity equation, Euler equations, and entropy equation are obtained starting from the fundamental principles of conservation of mass, momentum and energy, along with Gibbs thermodynamics with the restriction that fluid is perfect (that is, inviscid and adiabatic). These equations are used to prove that, if a flow field is initially isentropic and irrotational and no shocks arise, then the field remains irrotational, except for the points emanating from the trailing edge (wake). The equation for the velocity potentional is then obtained.

FOUNDATIONS OF INVISCID FLOWS

THE FUNDAMENTAL EQUATIONS governing the motion of perfect fluid are presented in this section. Specifically, continuity equation, Euler equations, and entropy equation are obtained starting from the fundamental principles of conservation of mass, momentum and energy, along with Gibbs thermodynamics with the restriction that the fluid is perfect (that is, inviscid and adiabatic).

MATHEMATICAL PRELIMINARIES-The ∇ (del) symbol

$$\nabla = \frac{\partial}{\partial x_1}\bar{i}_1 + \frac{\partial}{\partial x_2}\bar{i}_2 + \frac{\partial}{\partial x_3}\bar{i}_3 \tag{1.1}$$

will be used extensively in this paper. Note that

$$\text{grad } F = \nabla F \tag{1.2}$$

$$\text{div } \bar{w} = \nabla\cdot\bar{w} \tag{1.3}$$

$$\text{curl } \bar{w} = \nabla\times\bar{w} \tag{1.4}$$

Also, four important integral theorems will be used. Stokes theorem states that if σ is an arbitrary surface with contour C (covered in counter-clockwise fashion with respect to the normal $\bar{n}$), then for any differentiable vector field $\bar{w}$

$$\iint_\sigma \text{curl } \bar{w}\cdot\bar{n}\, d\sigma = \oint_C \bar{w}\cdot d\bar{x} \tag{1.5}$$

Gauss or divergence theorem states that if σ is the boundary of V and $\bar{n}$ is the outer normal of σ, then for any differentiable vector field $\bar{w}$

$$\iiint_V \text{div } \bar{w}\, dV = \oiint_\sigma \bar{w}\cdot\bar{n}\, d\sigma \tag{1.6}$$

In particular, if $\bar{w} = F\bar{i}_k$

$$\iiint_V \frac{\partial F}{\partial x_k} dV = \oiint_\sigma F n_k\, d\sigma \tag{1.7}$$

This yields the gradient theorem

$$\iiint_V \nabla F\, dV = \oiint_\sigma F\bar{n}\, d\sigma \tag{1.8}$$

BASIC ASSUMPTIONS-Let V_M be a material volume, that is, a volume composed of the same material particles at all times. The conservation principles for mass, momentum and energy are

Mass

$$\frac{d}{dt}\iiint_{V_M} \rho\, dV = 0 \tag{1.9}$$

Momentum

$$\frac{d}{dt}\iiint_{V_M} \rho\bar{v}\, dV = \iiint_{V_M} \rho\bar{f}\, dV + \oiint_\sigma \bar{t}\, d\sigma \tag{1.10}$$

Energy

$$\frac{d}{dt}\iiint_{V_M} \rho(\frac{v^2}{2} + e)dV = \iiint_{V_M} \rho\bar{f}\cdot\bar{v}\ dV + \oint\!\!\oint_{\sigma} \bar{t}\cdot\bar{v}\ d\sigma - \oint\!\!\oint_{\sigma} q\ d\sigma \qquad (1.11)$$

In the above equations, σ is the boundary of V_M, ρ is the density, $\bar{v}$ is the velocity, $\bar{f}$ is the force per unit mass, $\bar{t}$ is the force per unit surface, e is the internal energy, and q is the heat flux per unit surface.

The assumption of perfect (that is, inviscid and adiabatic) fluid implies

$$\bar{t} = -p\bar{n} \qquad \text{(inviscid)} \qquad (1.12)$$

$$q = 0 \qquad \text{(adiabatic)} \qquad (1.13)$$

TIME DERIVATIVE OF VOLUME INTEGRAL-In order to evaluate the time derivative of a volume integral, it is convenient to introduce material (convected) coordinates, ξ^α. These coordinates move with a material point (i.e., ξ^α = const corresponds to the same material point at all times). The motion of a particle identified by the coordinates ξ^α is given by

$$\bar{x} = \bar{x}(\xi^\alpha, t) \qquad (1.14)$$

Therefore, if V_0 is the (time independent) volume of the space ξ^α corresponding to the volume V_M,

$$\begin{aligned} \frac{d}{dt}\iiint_{V_M} F\ dV &= \frac{d}{dt}\iiint_{V_M} FJ\ d\xi^1 d\xi^2 d\xi^3 \\ &= \iiint_{V_M} \frac{D}{Dt}(FJ)\ d\xi^1 d\xi^2 d\xi^3 \qquad (1.15) \\ &= \iiint_{V_M} (\frac{DF}{Dt} J + F\frac{DJ}{Dt})\ d\xi^1 d\xi^2 d\xi^3 \end{aligned}$$

where

$$\begin{aligned} \frac{DF}{Dt} &= \frac{d}{dt} F(\bar{x}(\xi^\alpha, t), t) = \\ &= \frac{\partial F}{\partial t} + \frac{\partial F}{\partial x_1} v_1 + \frac{\partial F}{\partial x_2} v_2 + \frac{\partial F}{\partial x_3} v_3 \qquad (1.16) \\ &= (\frac{\partial}{\partial t} + \bar{v}\cdot\nabla)F \end{aligned}$$

is the material (substantial) derivative and

$$J = \frac{\partial(x_1, x_2, x_3)}{\partial(\xi^1, \xi^2, \xi^3)} \qquad (1.17)$$

is the Jacobian of the transformation $x(\xi^\alpha)$. It may be shown (see Serrin[1], p. 131) that

$$\frac{DJ}{Dt} = J\ \nabla\cdot\bar{v} \tag{1.18}$$

Hence

$$\begin{aligned}\frac{d}{dt}\iiint_{V_M} F dV &= \iiint_{V_0} \left(\frac{\partial F}{\partial t} + \bar{v}\cdot\nabla F + F\ \nabla\cdot\bar{v}\right) J\ d\xi^1 d\xi^2 d\xi^3 \\ &= \iiint_{V_M} \left(\frac{\partial F}{\partial t} + \nabla\cdot(F\bar{v})\right)\ dV \\ &= \iiint_{V_M} \frac{\partial F}{\partial t}\ dV + \oiint_{\sigma} F\ \bar{v}\cdot\bar{n}\ d\sigma\end{aligned} \tag{1.19}$$

which gives the familiar expression of the derivative of a volume integral in terms of the derivative of the integrand and the flux through the boundary.

THE CONTINUITY EQUATION-The continuity equation is obtained from the principle of conservation of mass, Eq. 1.9:

$$\begin{aligned}\frac{d}{dt}\iiint_{V_M} \rho\ dV &= \frac{d}{dt}\iiint_{V_0} \rho J\ d\xi^1 d\xi^2 d\xi^3 \\ &= \iiint_{V_0} \frac{D}{Dt}\ (\rho J)\ d\xi^1 d\xi^2 d\xi^3 = 0\end{aligned} \tag{1.20}$$

This implies (because of the arbitrariness of the volume V_M)

$$\frac{D}{Dt}(\rho J) = 0 \tag{1.21}$$

or, using Eq. 1.18,

$$\frac{D\rho}{Dt} + \rho\ \nabla\cdot\bar{v} = 0 \tag{1.22}$$

i.e.,

$$\frac{\partial\rho}{\partial t} + \bar{v}\cdot\nabla\rho + \rho\nabla\cdot\bar{v} = 0 \tag{1.23}$$

or

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\bar{v}) = 0 \tag{1.24}$$

REYNOLDS TRANSPORT THEOREM-Equation 1.11 may be used to obtain the important Reynolds transport theorem

$$\frac{d}{dt}\iiint_{V_M} \rho F dV = \iiint_{V_M} \rho\ \frac{DF}{Dt} dV \tag{1.25}$$

The proof is immediate if one notes that (using Eq. 1.21) the above equation may be written as

$$\frac{d}{dt}\iiint_{V_0} \rho F J \, d\xi^1 d\xi^2 d\xi^3 = \iiint_{V_0} \rho \frac{DF}{Dt} J \, d\xi^1 d\xi^2 d\xi^3 \qquad (1.26)$$

EULER EQUATIONS–Starting from the principle of conservation of momentum, Eq. 1.10, and using the assumption of inviscid flows, Eq. 1.12, Reynolds transport theorem, Eq. 1.25 and the gradient theorem, Eq.1.6, one obtains

$$\iiint_{V_M} \rho \frac{D\overline{v}}{Dt} dV = \iiint_{V_M} \rho \overline{f} dV - \iiint_{V_M} \nabla p \, dV \qquad (1.27)$$

This yields (because V_M is arbitrary) Euler equations

$$\frac{D\overline{v}}{Dt} = f - \frac{1}{\rho} \nabla p \qquad \text{(Euler)} \qquad (1.28)$$

CONSERVATION OF MECHANICAL ENERGY–Euler equations may be used to obtain the theorem of conservation of mechanical energy. Multiplying the terms of Eq. 1.28 scalarly by $\overline{v}$ and integrating over the volume V_M yields

$$\iiint_{V_M} \rho \frac{D\overline{v}}{Dt} \cdot \overline{v} \, dV = \iiint_{V_M} \rho \overline{f} \cdot \overline{v} \, dV - \iiint_{V_M} \nabla p \cdot \overline{v} \, dV \qquad (1.29)$$

Note that, using Gauss theorem, Eq. 1.6,

$$\begin{aligned} - \iiint_{V_M} \nabla p \cdot \overline{v} \, dV &= - \iiint_{V_M} [\nabla \cdot (p\overline{v}) - p \nabla \cdot \overline{v}] \, dV = \\ &= - \oiint_{\sigma} p \overline{n} \cdot \overline{v} \, d\sigma + \iiint_{V_M} p \nabla \cdot \overline{v} \, dV \end{aligned} \qquad (1.30)$$

Hence, using Reynolds transport theorem, Eq. 1.25, and Eq. 1.12, one obtains the theorem of conservation of mechanical energy:

$$\frac{d}{dt}\iiint_{V_M} \frac{1}{2}\rho v^2 dV = \iiint_{V_M} \rho \overline{f} \cdot \overline{v} \, dV + \oiint_{\sigma} \overline{t} \cdot \overline{v} \, d\sigma + \iiint_{V_M} p \, \nabla \cdot \overline{v} \, dV \quad (1.31)$$

CONSERVATION OF THERMODYNAMIC ENERGY–Comparing the principle of conservation of energy, Eq. 1.11, with the theorem of conservation of mechanical energy, Eq. 1.31, and using Reynolds transport theorem, Eq. 1.25, one obtains the theorem of conservation of thermodynamic energy:

$$\iiint_{V_M} \rho \frac{De}{Dt} dV = -\iiint_{V_M} p \, \nabla \cdot \overline{v} \, dV + \oiint_{\sigma} q d\sigma \qquad (1.32)$$

Using the assumption of adiabatic flows, Eq. 1.13, continuity equation, Eq. 1.22, and the arbitrariness of the volume V_M, one obtains

$$\frac{De}{Dt} = - \frac{1}{\rho} p \, \nabla \cdot \overline{v} = \frac{1}{\rho^2} p \frac{D\rho}{Dt} = - p \frac{D}{Dt}\left(\frac{1}{\rho}\right) \qquad (1.33)$$