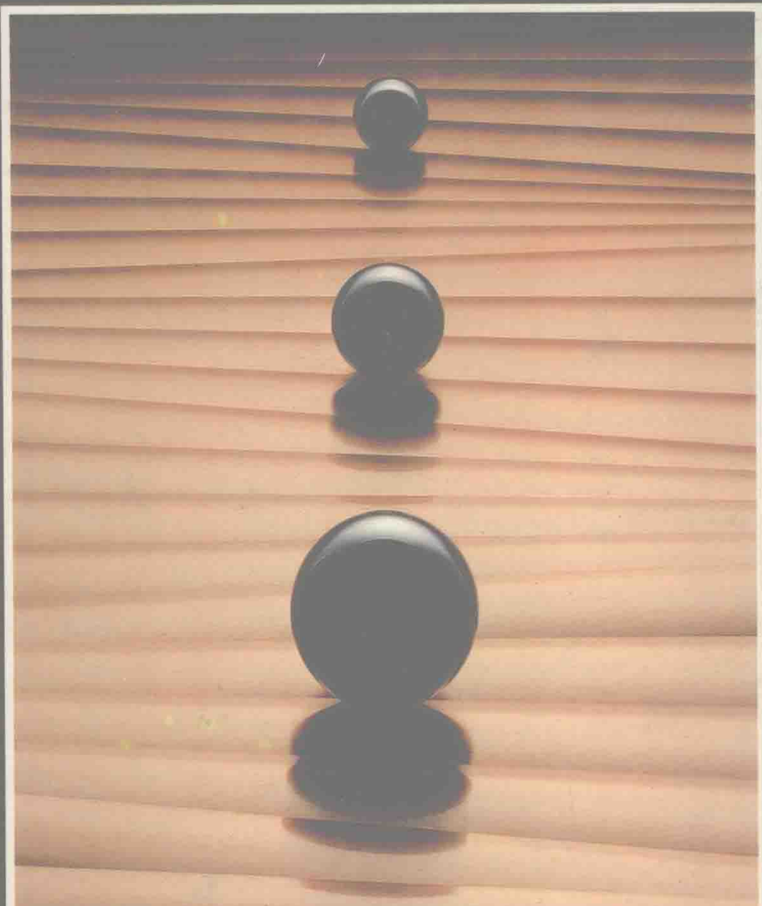
Fundamentals of Algebra and Trigonometry



Dennis T. Christy

Fundamentals of Algebra and Trigonometry

Dennis T. Christy
Nassau Community College

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Preface

Audience

This book is intended for students who need a concrete approach to mathematics. The presentation assumes that the student has completed two years of high school algebra or a college course in intermediate algebra. However, a detailed review of all necessary ideas is given early in the text so that students whose basic skills need improvement have a wealth of helpful material.

Approach

Problem-Solving Approach

My experience is that students who take college algebra and trigonometry learn best by "doing." Examples and exercises are crucial since it is usually in these areas that the students' main interactions with the material take place. The problem-solving approach contains brief, precisely formulated paragraphs, followed by many detailed examples. In most cases the discussion proceeds from the specific to the general, and each section has many exercises for both in-class practice and homework. The problem sets are carefully graded and contain an unusual number of routine manipulative problems. (There is nothing more frustrating than being stuck on the beginning exercises!) So that each student may be challenged fairly, difficult problems have been included. There is also a wide variety of questions and discussions that show the student the usefulness of mathematics. Many reviewers felt that the readability of the text and the quality and abundance of the exercises and examples were outstanding features.

Functions Theme

The organization takes a functional approach to algebra and trigonometry in that after developing some basic algebra, the function concept plays the unifying role in the study of polynomial, rational, exponential, logarithmic, and trigonometric functions.

Algebra Review

The first two chapters, which review basic algebra, are particularly detailed to help ensure proficiency in basic skills.

Trigonometry Development

The trigonometric functions are presented in order of abstraction and sophistication, and the text proceeds from the right triangle definitions to the general angle definitions and then to the unit circle definitions.

Calculus Preparation

To prepare students for calculus, there is an emphasis on explaining how to deal with some of the expressions encountered in a calculus course. For instance, Chapter 1 shows how to simplify algebraic expressions that will be found in calculus problems. Chapter 3 expands the discussion of slope to include the difference quotient, Chapter 6 shows problems dealing with inverse trigonometric functions that are important for integration by trigonometric substitution, and Chapter 10 considers the sense in which certain infinite geometric series converge to a sum.

Calculator Use

The text encourages the use of calculators and discusses how they can be used effectively. It is assumed in the discussion that students have scientific calculators that use the algebraic operating system (AOS). Calculator illustrations show primarily the keystrokes required on a Texas Instruments TI-30 SLR+. Tables are included and discussed in the text in case students choose not to use a calculator.

Features

- Problem-solving approach with intuitive concept developments
- Over 4,300 exercises and 500 examples
- Problem sets of graduated difficulty with enough routine problems to give students experience, confidence, and skill with basic procedures
- Extensive and varied applications problems
- Student-oriented and mathematically sound explanations
- Boxes with margin labels for important definitions and rules
- Second color to highlight important ideas
- Avoidance of awkward page and line breaks
- Chapter introductions that include an interest-getting problem that is solved as an example in the text
- Unique chapter overviews that highlight key concepts to review at the end of each chapter
- Abundant chapter review exercises
- Anticipates the needs of students who continue to calculus and other higher level math courses
- Emphasis on functions and graphs
- Right triangle introduction to trigonometry
- Instructions on calculator use
- Important formulas and tables on endsheets
- Complete instructional package

Pedagogy

Chapter Introductions

In the spirit of problem solving, each chapter opens with a problem that should quickly involve students and teachers in a discussion of an important chapter concept. Some of the problems are applications, some are puzzles, and one is a proof. None of the problems requires a lot of sophisticated mathematics and it is hoped that students will try to solve the problem either initially or after covering the relevant section in the text. These problems are later worked out as examples in each chapter.

Chapter Overviews and Review Exercises

At the end of each chapter there is a detailed list of key concepts to review that is organized in a section-by-section format so the material does not seem overwhelming. There is also a collection of review exercises that not only review the basic ideas but also expose students to slightly different question wording and formats, including multiple choice questions. From these exercises it should be easy for instructors to choose questions for review assignments or for sample tests at an appropriate level of difficulty.

Instructional Package

- *Student Study Guide and Solutions Manual*
- *Instructor's Manual* (includes transparency masters)
- *Instructor's Solutions Manual*
- wcb Math TestPak
- *Test Item File*

Student Study Guide and Solutions Manual

Students for whom the textbook is not enough need more than just a lot of solved problems in a solutions manual. Primarily they need help focusing on the key objectives and concepts in the course. To provide some of this help, an accompanying study guide and solutions manual is available that covers the following key aspects for each section in the text.

1. Specific objectives for the section.
2. A list of important terms.
3. A summary of the key rules and formulas.
4. Detailed solutions to selected even-numbered exercises with at least one example from each exercise group. Exercise numbers for these problems are printed in color in the text for easy identification.
5. Margin exercises matched to the solved problems so students can check their progress.

The *Student Study Guide and Solutions Manual* also contains sample test questions (with answers) for each chapter in the text.

Instructor's Manual

The *Instructor's Manual* contains three tests for each chapter, three final exams, and the answers to the even-numbered problems in the section exercise sets. Transparency masters of the chapter introductory problems and important theorems and definitions are also included in this manual. The problems in the three tests for each chapter and in the three final exams also appear in wcb Math TestPak.

Instructor's Solutions Maual

This manual contains solutions to every problem in the text. These solutions are intended for the use of the instructor only and are basic outlines of possible problem solutions.

wcb Math TestPak

wcb Math TestPak was developed expressly for wcb math texts. It is a free computerized testing service with two convenient options. First, you may use your own Apple IIe, IIc, Macintosh, or IBM PC (both 5¼″ and 3½″ disks) to produce your tests by using items available in the test bank or by editing these items, deleting them, or adding your own. Questions may be chosen by number or at random. Secondly, you may use the call-in service offered by the publisher. Contact your local wcb sales representative for details.

Test Item File

The printed *Test Item File* in an 8½″ × 11″ format contains all of the questions on the wcb Math TestPak. It will serve as a ready-reference if you use your own computer to generate tests. The *Test Item File* contains the problems in the *Instructor's Manual* along with additional items that have been incorporated into the Math TestPak bank.

Acknowledgments

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Dennis T. Christy

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Calculator Use

To the Student

A scientific hand-held calculator is now standard equipment for precalculus mathematics and beyond. These ten dollar wonders provide you with the benefits of electronic computation that is fast, accurate, and easy to learn. Most important, efficient calculator use helps you focus on important mathematical ideas. To understand and apply mathematical concepts is our fundamental aim and calculators are marvelous aids in attaining this goal. Tables are included at the back of the book in case you choose not to use a calculator. But, since calculators are inexpensive, easy to use, and a significant learning aid, we recommend you obtain one.

A scientific calculator (the type you need) contains at least the following special features: algebraic keys x^2, $\sqrt{x}$, $1/x$, y^x or x^y, $\sqrt[x]{y}$; trigonometric keys sin, cos, tan, $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, degree and radian angular modes; logarithmic and exponential keys log, ln, 10^x, e^x; parentheses keys (,); a scientific notation key EE or EXP; and one memory that can store and recall.

In this book we also assume a scientific calculator using the algebraic operating system (AOS). Texas Instruments, Sharp, and Casio produce scientific calculators using this system. With AOS you can key in the problem exactly as it appears, and the calculator is programmed to use the order of operations discussed in Section 1.2. For example, since multiplication is done before addition 2 [+] 3 [×] 4 [=] 14. If your calculator displays 20 when you key in this sequence, it is operating on left to right logic. You must then be careful to key in the problem so the correct order of operations is followed. Calculator illustrations in this text show primarily the key strokes required on a Texas Instruments TI-30-SLR+. In any case, you should read the owner's manual that comes with your calculator to familiarize yourself with its specific keys and limitations.

One other introductory note—a calculator COMPUTES, that's all. You do the important part—you THINK. You analyze the problem, decide on the significant relationships, and determine if the solution makes sense in the "real world." It's nice not to get bogged down in certain calculations and tables, but that never was the major problem.

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Fundamentals of Algebra and Trigonometry

1 Fundamentals of Algebra

A race car driver must average at least 150 mi/hour for two laps around a track to qualify for the finals. The driver averages 180 mi/hour on the first lap, but mechanical trouble reduces the average speed on the second lap to only 120 mi/hour. Does the driver qualify for the finals? (See Example 10 of Section 1.4. *Hint:* average speed = total distance/total time.)

A common student lament goes something like, "I understand the new concepts, but the algebra is killing me!" In this chapter we hope to remedy this problem by reviewing *in detail* some basic rules in algebra about real numbers, exponents, factoring, fractions, and radicals. Success here will go a long way toward success in this course and in higher mathematics.

In this text we take a problem-solving approach which emphasizes that one learns mathematics by *doing* mathematics, while *thinking* mathematically. That is, you need to actively work through the problems (with pencil and paper), while *focusing on the definitions, relationships, and procedures* that link together all steps in the solution. In this spirit of problem solving we open each chapter with a problem. Some are applications, some are puzzles, and one is a proof. Taken together, they illustrate the varied nature of problem solving. Since none of them require a lot of sophisticated mathematics, we hope you will take a stab at an answer either initially or after covering the relevant section in the text.

1.1 REAL NUMBERS

Mathematics is a basic tool in analyzing concepts in every field of human endeavor. In fact, the primary reason you have studied this subject for at least a decade is that mathematics is the most powerful instrument available in the search to understand the world and to control it. Mathematics is essential for full comprehension of technological and scientific advances, economic policies and business decisions, and the complexities of social and psychological issues. At the heart of this mathematics is algebra. Calculus, statistics, and computer science are but a few of the areas in which a knowledge of algebraic concepts and manipulations is necessary.

Algebra is a generalization of arithmetic. In arithmetic we work with specific numbers, such as 5. In algebra we study numerical relations in a more general way by using symbols, such as x, that may be replaced by a number from some collection of numbers. Since the symbols represent numbers, they behave according to the same rules that numbers must follow. Consequently, instead of studying specific numbers, we study symbolic representations of numbers and try to define the laws that govern them.

We begin our study of algebra by giving specific names to various sets* of numbers. The collection of the counting numbers, zero, and the negatives of the counting numbers is called the **integers.** Thus, the set of integers may be written as

$$\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}.$$

The set of fractions with an integer in the top of the fraction (numerator) and a nonzero integer in the bottom of the fraction (denominator) is called the **rational numbers.** Symbolically, a rational number is a number that may be written in the form a/b, where a and b are integers, with b not equal to ($\neq$) zero. The numbers $\sqrt{2}/3$ and $2/\pi$ are fractions but they are not rational numbers because they cannot be written as the quotient of two integers. All integers are rational numbers because we can think of each integer as having a 1 in its denominator. (Example: $4 = 4/1$.)

Our definition for rational numbers specified that the denominator cannot be zero. To see why, you need to know that

$$\frac{8}{2} = 4 \text{ is equivalent to saying that } 8 = 4 \cdot 2 \text{ and}$$

$$\frac{55}{11} = 5 \text{ is equivalent to saying that } 55 = 5 \cdot 11.$$

If $8/0 = a$, where a is some rational number, this would mean that $8 = a \cdot 0$. But $a \cdot 0 = 0$ for any rational number. There is no rational number a such that $a \cdot 0 = 8$. Thus, we say that $8/0$ is *undefined.*

Now consider $0/0 = a$. This is equivalent to $0 = a \cdot 0$. But $a \cdot 0 = 0$ for *any* rational number. Thus, not just one number a will solve the equation—any a will. Since $0/0$ does not name a particular number, it is also undefined. Consequently, division by zero is undefined in every case, so the denominator in a rational number cannot be zero.

To define our next set of numbers, we now consider the decimal representation of numbers. We may convert rational numbers to decimals by long division. Consider the following examples of repeating decimals. A bar is placed above the portion of the decimal that repeats.

$$\frac{3}{4} = \begin{matrix} 0.7500\ldots \\ \text{or} \\ 0.75\overline{0} \end{matrix} \qquad \frac{2}{3} = \begin{matrix} 0.6666\ldots \\ \text{or} \\ 0.\overline{6} \end{matrix} \qquad \frac{8}{7} = 1.\overline{142857}$$

*A **set** is simply a collection of objects, and we may describe a set by listing the objects or members of the collection within braces.

The decimals repeat because at some point we must perform the same division and start a cycle. For example, when converting $\frac{8}{7}$, the only possible remainders are 0, 1, 2, 3, 4, 5, and 6. In performing the division, as shown in Figure 1.1, we had remainders of 1, 3, 2, 6, 4, and 5. In the next step we must obtain one of these remainders a second time and start a cycle, or obtain 0 as the remainder, which results in repeating zeros. Thus, if a/b is a rational number, it can be written as a repeating decimal.

$$
\begin{array}{r}
1.\overline{142857} \\
7\overline{)8.000000} \\
\underline{7} \\
1\ 0 \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
1
\end{array}
$$

Figure 1.1

It is also true that any repeating decimal may be converted to a ratio between two integers, as shown in Example 1.

EXAMPLE 1 Express the repeating decimal $0.\overline{17}$ as the ratio of two integers.

Solution First, let $x = 0.1717...$ Multiplying both sides of this equation by 100 moves the decimal two places to the right, so we obtain

$$
\begin{array}{rl}
100x &= 17.1717\ldots \\
x &= 0.1717\ldots \\ \hline
\end{array}
$$

now subtracting yields $\quad 99x = 17 \quad$ or $x = \dfrac{17}{99}$.

Thus, the repeating decimal $0.\overline{17}$ is equivalent to the fraction $\frac{17}{99}$. □