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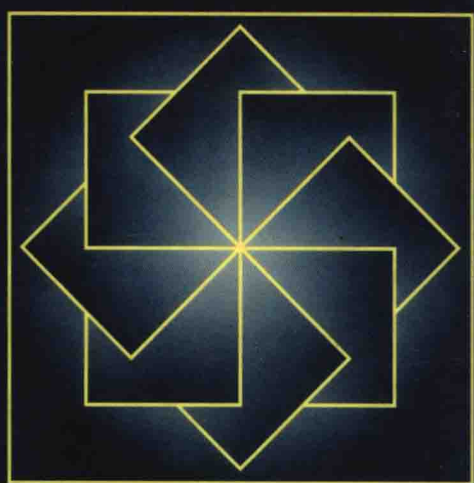
# 拓扑学

(英文版 · 第2版)

# TOPOLOGY

SECOND EDITION



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Pearson Education

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# Topology

(Second Edition)

本书作者在拓扑学领域享有盛誉。

本书分为两个独立的部分。第一部分普通拓扑学，讲述点集拓扑学的内容：前4章作为拓扑学的引论，介绍作为核心题材的集合论、拓扑空间、连通性、紧性以及可数性和分离性公理；后4章是补充题材。第二部分代数拓扑学，讲述与拓扑学核心题材相关的主题，其中包括基本群和覆盖空间及其应用。

本书最大的特点在于对理论的清晰阐述和严谨证明，力求让读者能够充分理解。对于疑难的推理证明，将其分解为简化的步骤，不给读者留下疑惑。此外，书中还提供了大量练习，可以巩固加深学习的效果。严格的论证、清晰的条理、丰富的实例，让深奥的拓扑学变得轻松易学。





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# Preface

This book is intended as a text for a one- or two-semester introduction to topology, at the senior or first-year graduate level.

The subject of topology is of interest in its own right, and it also serves to lay the foundations for future study in analysis, in geometry, and in algebraic topology. There is no universal agreement among mathematicians as to what a first course in topology should include; there are many topics that are appropriate to such a course, and not all are equally relevant to these differing purposes. In the choice of material to be treated, I have tried to strike a balance among the various points of view.

**Prerequisites**. There are no formal subject matter prerequisites for studying most of this book. I do not even assume the reader knows much set theory. Having said that, I must hasten to add that unless the reader has studied a bit of analysis or "rigorous calculus," much of the motivation for the concepts introduced in the first part of the book will be missing. Things will go more smoothly if he or she already has had some experience with continuous functions, open and closed sets, metric spaces, and the like, although none of these is actually assumed. In Part II, we do assume familiarity with the elements of group theory.

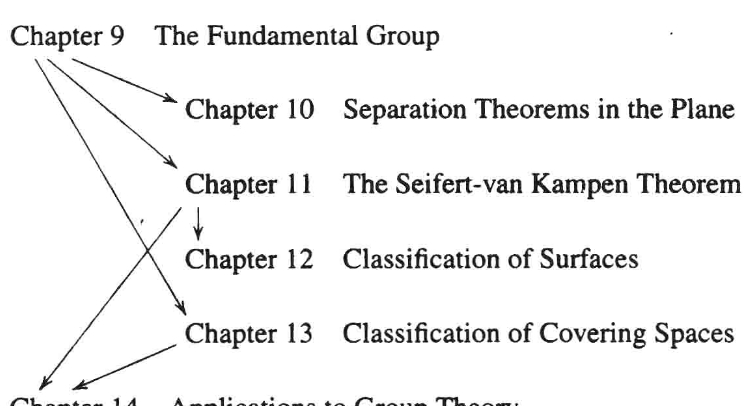
Most students in a topology course have, in my experience, some knowledge of the foundations of mathematics. But the amount varies a great deal from one student to another. Therefore, I begin with a fairly thorough chapter on set theory and logic. It starts at an elementary level and works up to a level that might be described as "semi-sophisticated." It treats those topics (and only those) that will be needed later in the book. Most students will already be familiar with the material of the first few sections, but many of them will find their *expertise* disappearing somewhere about the middle of the chapter. How much time and effort the instructor will need to spend on this chapter will thus depend largely on the mathematical sophistication and experience of the students. Ability to do the exercises fairly readily (and correctly!) should serve as a reasonable criterion for determining whether the student's mastery of set theory is sufficient for the student to begin the study of topology.

Many students (and instructors!) would prefer to skip the foundational material of Chapter 1 and jump right in to the study of topology. One ignores the foundations, however, only at the risk of later confusion and error. What one *can* do is to treat initially only those sections that are needed at once, postponing the remainder until they are needed. The first seven sections (through countability) are needed throughout the book; I usually assign some of them as reading and lecture on the rest. Sections 9 and 10, on the axiom of choice and well-ordering, are not needed until the discussion of compactness in Chapter 3. Section 11, on the maximum principle, can be postponed even longer; it is needed only for the Tychonoff theorem (Chapter 5) and the theorem on the fundamental group of a linear graph (Chapter 14).

**How the book is organized.** This book can be used for a number of different courses. I have attempted to organize it as flexibly as possible, so as to enable the instructor to follow his or her own preferences in the matter.

Part I, consisting of the first eight chapters, is devoted to the subject commonly called general topology. The first four chapters deal with the body of material that, in my opinion, should be included in any introductory topology course worthy of the name. This may be considered the "irreducible core" of the subject, treating as it does set theory, topological spaces, connectedness, compactness (through compactness of finite products), and the countability and separation axioms (through the Urysohn metrization theorem). The remaining four chapters of Part I explore additional topics; they are essentially independent of one another, depending on only the core material of Chapters 1–4. The instructor may take them up in any order he or she chooses.

Part II constitutes an introduction to the subject of Algebraic Topology. It depends on only the core material of Chapters 1–4. This part of the book treats with some thoroughness the notions of fundamental group and covering space, along with their many and varied applications. Some of the chapters of Part II are independent of one another; the dependence among them is expressed in the following diagram:



Certain sections of the book are marked with an asterisk; these sections may be omitted or postponed with no loss of continuity. Certain theorems are marked similarly. Any dependence of later material on these asterisked sections or theorems is indicated at the time, and again when the results are needed. Some of the exercises also depend on earlier asterisked material, but in such cases the dependence is obvious.

Sets of supplementary exercises appear at the ends of several of the chapters. They provide an opportunity for exploration of topics that diverge somewhat from the main thrust of the book; an ambitious student might use one as a basis for an independent paper or research project. Most are fairly self-contained, but the one on topological groups has as a sequel a number of additional exercises on the topic that appear in later sections of the book.

**Possible course outlines**. Most instructors who use this text for a course in general topology will wish to cover Chapters 1–4, along with the Tychonoff theorem in Chapter 5. Many will cover additional topics as well. Possibilities include the following: the Stone-Čech compactification (§38), metrization theorems (Chapter 6), the Peano curve (§44), Ascoli's theorem (§45 and/or §47), and dimension theory (§50). I have, in different semesters, followed each of these options.

For a one-semester course in algebraic topology, one can expect to cover most of Part II.

It is also possible to treat both aspects of topology in a single semester, although with some corresponding loss of depth. One feasible outline for such a course would consist of Chapters 1–3, followed by Chapter 9; the latter does not depend on the material of Chapter 4. (The non-asterisked sections of Chapters 10 and 13 also are independent of Chapter 4.)

**Comments on this edition**. The reader who is familiar with the first edition of this book will find no substantial changes in the part of the book dealing with general topology. I have confined myself largely to "fine-tuning" the text material and the exercises. However, the final chapter of the first edition, which dealt with algebraic topology, has been substantially expanded and rewritten. It has become Part II of this book. In the years since the first edition appeared, it has become increasingly common to offer topology as a two-term course, the first devoted to general topology and the second to algebraic topology. By expanding the treatment of the latter subject, I have intended to make this revision serve the needs of such a course.

**Acknowledgments**. Most of the topologists with whom I have studied, or whose books I have read, have contributed in one way or another to this book; I mention only Edwin Moise, Raymond Wilder, Gail Young, and Raoul Bott, but there are many others. For their helpful comments concerning this book, my thanks to Ken Brown, Russ McMillan, Robert Mosher, and John Hemperly, and to my colleagues George Whitehead and Kenneth Hoffman.

The treatment of algebraic topology has been substantially influenced by the excellent book by William Massey [M], to whom I express appreciation. Finally, thanks are due Adam Lewenberg of MacroTeX for his extraordinary skill and patience in setting text and juggling figures.

But most of all, to my students go my most heartfelt thanks. From them I learned at least as much as they did from me; without them this book would be very different.

J.R.M.

# A Note to the Reader

Two matters require comment—the exercises and the examples.

Working problems is a crucial part of learning mathematics. No one can learn topology merely by poring over the definitions, theorems, and examples that are worked out in the text. One must work part of it out for oneself. To provide that opportunity is the purpose of the exercises.

They vary in difficulty, with the easier ones usually given first. Some are routine verifications designed to test whether you have understood the definitions or examples of the preceding section. Others are less routine. You may, for instance, be asked to generalize a theorem of the text. Although the result obtained may be interesting in its own right, the main purpose of such an exercise is to encourage you to work carefully through the proof in question, mastering its ideas thoroughly—more thoroughly (I hope!) than mere memorization would demand.

Some exercises are phrased in an "open-ended" fashion. Students often find this practice frustrating. When faced with an exercise that asks, "Is every regular Lindelöf space normal?" they respond in exasperation, "I don't know what I'm supposed to do! Am I suppose to prove it or find a counterexample or what?" But mathematics (outside textbooks) is usually like this. More often than not, all a mathematician has to work with is a conjecture or question, and he or she doesn't know what the correct answer is. You should have some experience with this situation.

A few exercises that are more difficult than the rest are marked with asterisks. But none are so difficult but that the best student in my class can usually solve them.

Another important part of mastering any mathematical subject is acquiring a repertoire of useful examples. One should, of course, come to know those major examples from whose study the theory itself derives, and to which the important applications are made. But one should also have a few counterexamples at hand with which to test plausible conjectures.

Now it is all too easy in studying topology to spend too much time dealing with "weird counterexamples." Constructing them requires ingenuity and is often great fun. But they are not really what topology is about. Fortunately, one does not need too many such counterexamples for a first course; there is a fairly short list that will suffice for most purposes. Let me give it here:

$\mathbb{R}^J$ the product of the real line with itself, in the product, uniform, and box topologies.

$\mathbb{R}_\ell$ the real line in the topology having the intervals $[a, b)$ as a basis.

$S_\Omega$ the minimal uncountable well-ordered set.

$I_o^2$ the closed unit square in the dictionary order topology.

These are the examples you should master and remember; they will be exploited again and again.

# Contents

## Part II ALGEBRAIC TOPOLOGY

# Part I
# GENERAL TOPOLOGY